1996
VCE
MATHEMATICAL METHODS
CAT 2

DETAILED SUGGESTED SOLUTIONS

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Victorian Certificate of Education
1996

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination (Facts, skills and applications task)

Friday 8 November 1996: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOKLET

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this booklet.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer booklet (Part II) and hand them in.
You may retain this question booklet.

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
33	33	33

Directions to students

Materials
Question booklet of 15 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the booklet.
An approved calculator may be used.
You should have at least one pencil and an eraser.

The task
Detach the formula sheet from the centre of this booklet during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for Part I.
All questions should be answered on the answer sheet provided for multiple-choice questions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer booklet (Part II) and hand them in.
You may retain this question booklet.

Specific instructions to students

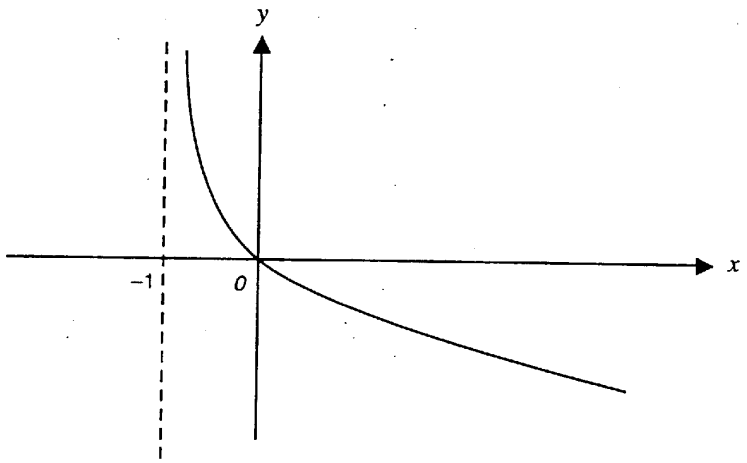
This part consists of 33 questions.
Answer **all** questions in this part on the answer sheet provided for multiple-choice questions.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers. You should attempt every question.
No credit will be given for a question if two or more letters are marked for that question.

Question 1

The equations of the vertical and horizontal asymptotes of the graph whose equation is $y = \frac{1}{x+1} - 4$ are respectively

A. $x = 1, y = -4$
B. $x = -1, y = -4$
C. $x = -4, y = -1$
D. $x = -4, y = 1$
E. $x = -1, y = 4$

Question 2

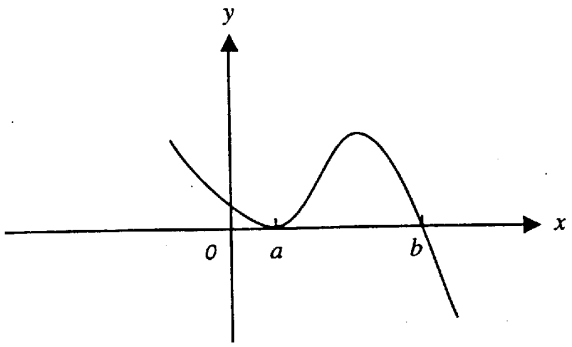


The graph shown could be that of the relation

A. $y = e^{-x} - 1$
B. $y = -e^{x} + 1$
C. $y = -\log_e(x) + 1$
D. $y = -\log_e(x + 1)$
E. $y = -\log_e(x) - 1$

Question 3

The graph shown could be that of the relation



A. $y = -(x-a)^2(x-b)$

B. $y = (x-a)(x-b)^2$

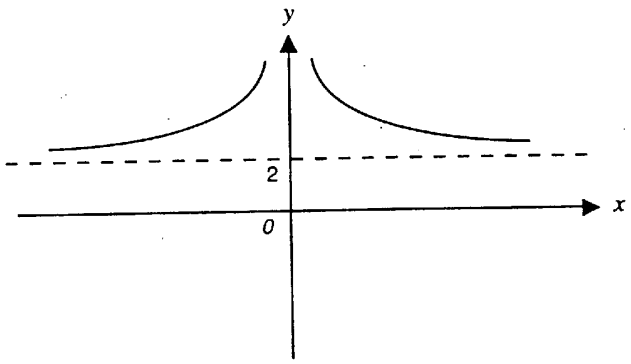
C. $y = (x-a)^2(x-b)$

D. $y = -(x+a)^2(x+b)$

E. $y = -x(x-a)(x-b)$

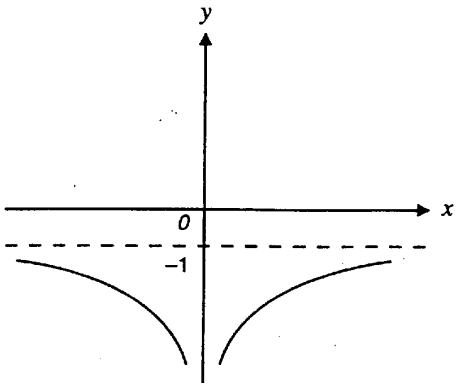
Question 4

The graph whose equation is $y = f(x)$ is shown below.

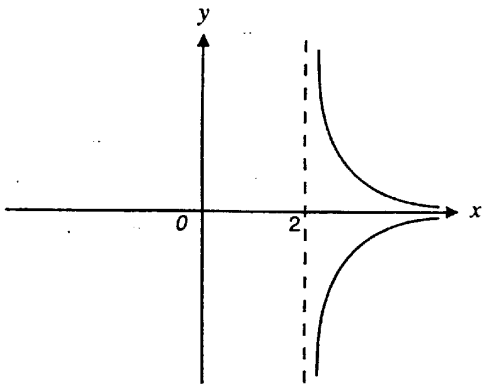


Which one of the following represents the graph whose equation is $y = 1 - f(x)$?

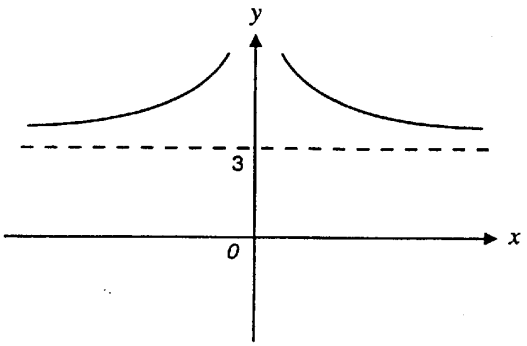
A.



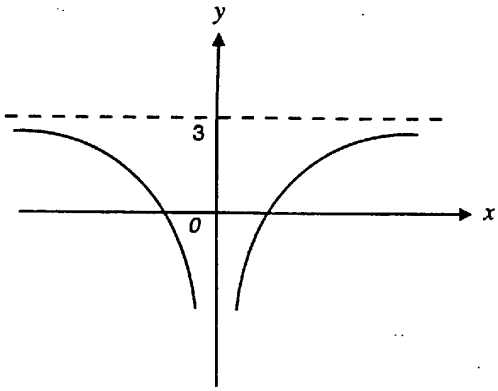
B.



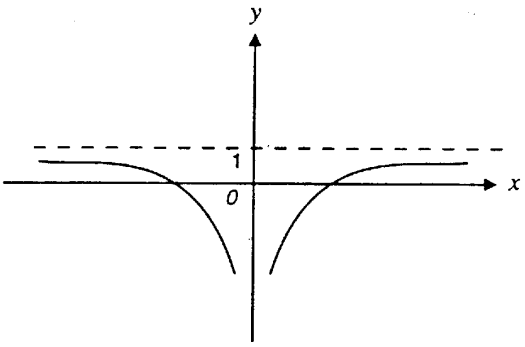
C.



D.

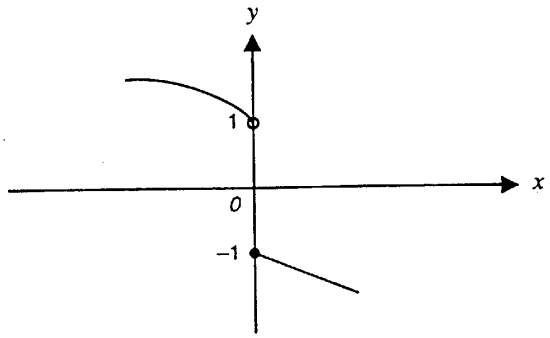


E.



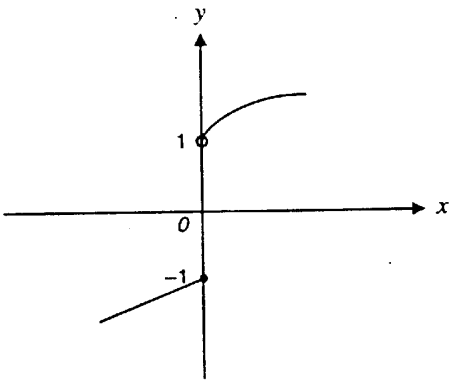
Question 5

The graph of the function f is shown below.

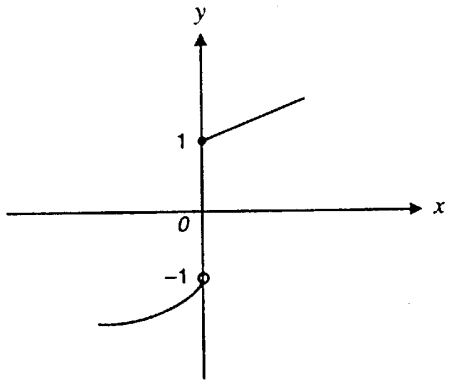


Which one of the following could be the graph of the inverse function f^{-1}?

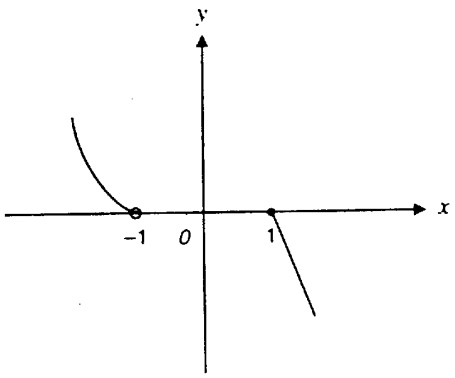
A.



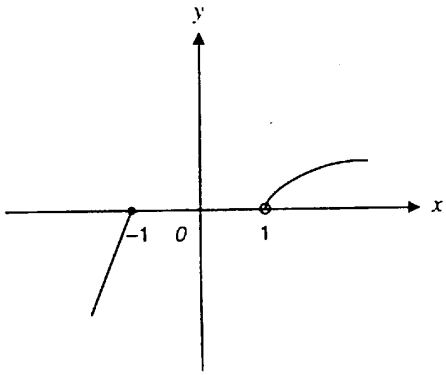
B.



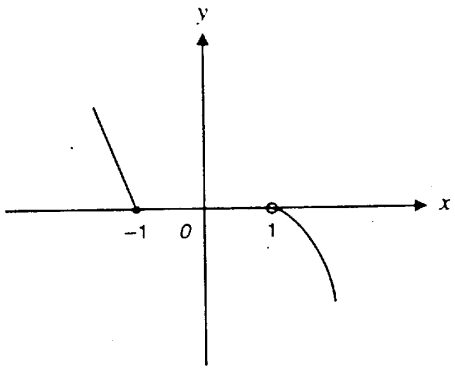
C.



D.



E.



Question 6

The amount of substance, m grams, present in a tank after t hours was recorded as in the following table.

t	1	2	3	4
m	0.68	1.85	5.02	13.65

There was some of the same substance present in the tank initially (that is, at time $t = 0$) but the exact initial amount was not recorded. The data may best be modelled by an equation of the form (where a and b are positive constants)

A. $m = at^2$

B. $m = a\log_e(bt)$

C. $m = ae^{bt}$

D. $m = at + b$

E. $m = at^{\frac{1}{2}}$

Question 7

The function $f : [0, \pi] \to R, f(x) = 3\cos\left(x - \frac{\pi}{2}\right)$ has range

A. R

B. [0,6]

C. [0,3]

D. [–3,0]

E. [–3,3]

Question 8

A trigonometric function is given by

$$f : R \to R, f(x) = 2\sin(3x + \pi) + 2$$

The amplitude, period and range of f are respectively

A. $2, \pi, [0, 4]$

B. $3, \frac{2\pi}{3}, R$

C. $2, \frac{2\pi}{3}, [0, 4]$

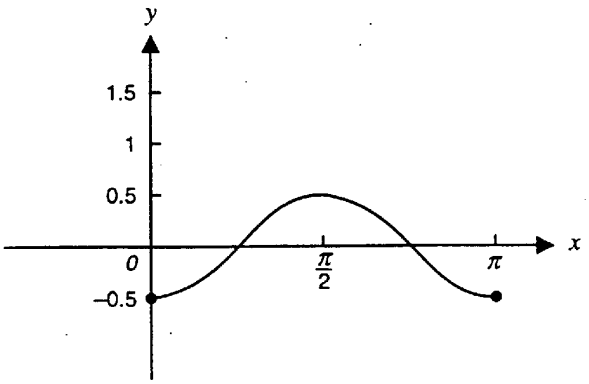
D. $3, \pi, [-2, 2]$

E. $2, \frac{2\pi}{3}, R$

Question 9

The graph shows one cycle of the curve whose equation is $y = a + b\cos(cx)$, where a, b and c are constants. The values of a, b and c respectively are

A. 0.5, 0.5, 2
B. −0.5, −0.5, 2
C. 0.5, 2, 0.5
D. 0, −0.5, 2
E. 0, 0.5, 0.5



Question 10

The **sum** of the solutions of the equation $\sin(4x) = 0.5,\ 0 \le x \le \frac{\pi}{2}$ is equal to

A. $\frac{\pi}{24}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. π

E. $\frac{3\pi}{2}$

Question 11

In the following equation a, b and c are **positive** constants. The equation $a\sin(x + b) = c$ is guaranteed to have at least one solution in the interval $0 \le x \le 2\pi$ provided only that

A. $c < a$

B. $c > a$

C. $b > \frac{\pi}{2}$

D. $b < \frac{\pi}{2}$

E. $c < 1$

Question 12

If $f(x) = \log_e(2x)$, then $f'(1)$ is equal to

A. 1

B. 0.5

C. $\log_e 2$

D. $2 \log_e 2$

E. 2

Question 13

If $y = \sin^2(4x)$, then $\frac{dy}{dx}$ is

A. $2 \sin(4x)$

B. $-2 \sin(4x) \cos(4x)$

C. $-8 \sin(4x) \cos(4x)$

D. $4 \cos^2(4x)$

E. $8 \sin(4x) \cos(4x)$

Question 14

Let $y = xe^x$, $x \in R$. The minimum value of y is

A. -1

B. $-\left(\frac{1}{e}\right)$

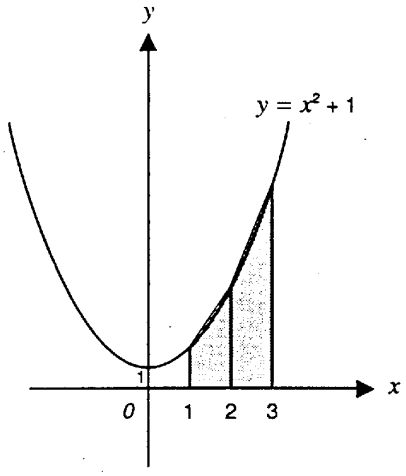
C. $\frac{1}{e}$

D. 1

E. e

Question 15

An approximate value of $\int_1^3 \left(x^2 + 1\right) dx$ is to be calculated using the area of two strips, each in the shape of a trapezium as shown in the diagram.



The value of this approximation is equal to

A. $10\frac{2}{3}$

B. 11

C. $11\frac{1}{2}$

D. 12

E. 15

Question 16

$\int_1^4 \left(2f(x) + 6\right) dx$ can be written as

A. $2\int_1^4 f(x)\,dx + 6$

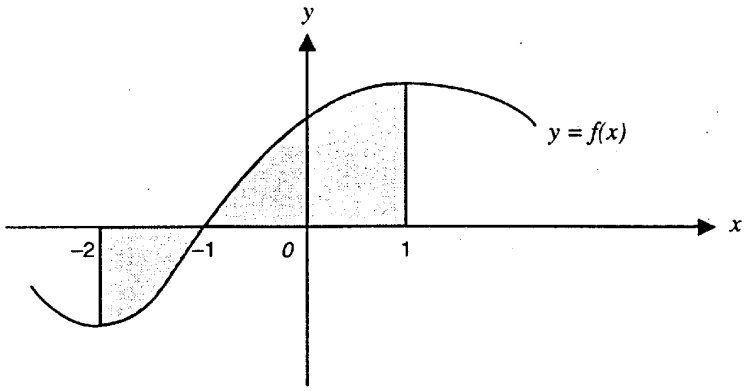
B. $2\int_1^4 \left(f(x) + 6\right) dx$

C. $2\int_1^4 f(x)\,dx + 18$

D. $2\int_1^4 f(x)\,dx + \int_1^4 3\,dx$

E. $2\int_1^4 f(x)\,dx + 6x$

Question 17



In the diagram shown, the total area of the two regions shaded is equal to

A. $\int_{1}^{-2} f(x)\,dx$

B. $\int_{-2}^{1} f(x)\,dx$

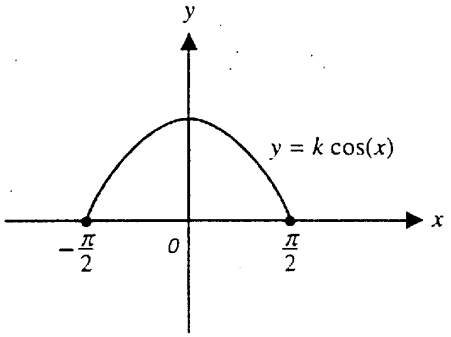
C. $\int_{1}^{-1} f(x)\,dx + \int_{-1}^{-2} f(x)\,dx$

D. $\int_{-1}^{1} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx$

E. $\int_{-1}^{1} f(x)\,dx + \int_{-2}^{-1} f(x)\,dx$

Question 18

The graph of $f : \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \to R, f(x) = k\cos(x)$ is shown. The area of the shaded region is equal to 1.



The value of k is

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. 1

E. 2

Question 19

$\int_0^1 \frac{1}{3x+1}\,dx = \log_e k$, where k is equal to

A. $1^{\frac{1}{3}}$

B. 2^2

C. $2^{\frac{2}{3}}$

D. 2^6

E. $3^{\frac{1}{3}}$

Question 20

An anti-derivative of $\frac{1}{(3x-4)^{\frac{5}{2}}}$ is equal to

A. $\frac{1}{(3x-4)^{\frac{3}{2}}}$

B. $\frac{-3}{2(3x-4)^{\frac{3}{2}}}$

C. $\frac{-9}{2(3x-4)^{\frac{3}{2}}}$

D. $\frac{3}{(3x-4)^{\frac{3}{2}}}$

E. $\frac{-2}{9(3x-4)^{\frac{3}{2}}}$

Question 21

Consider the function

$$f : [-3, \infty) \to R \text{ where } f(x) = \begin{cases} 2\left(1 - \frac{x^2}{9}\right) & \text{for } -3 \le x < 0 \\ 2\left(e^{-x} - 1\right) & \text{for } x \ge 0 \end{cases}$$

The range of f is

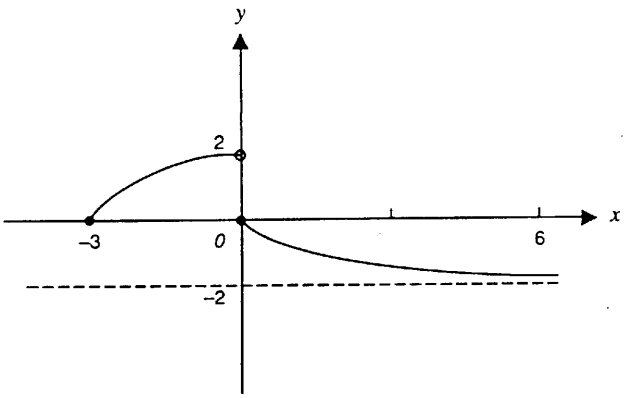
A. $(-2, 2]$

B. $(-2, 2)$

C. $(0, 2)$

D. $(-2, 0)$

E. $[-2, 2]$



Question 22

The linear factors of $x^4 - 9x^2$ are

A. $x, x, x^2 - 9$

B. $x, x, x - 9, x + 9$

C. $x, x - 3$

D. $x, x - 3, x - 3$

E. $x, x, x - 3, x + 3$

Question 23

The number of terms in the complete expansion of $(2x + 4y)^9$ is

A. 8

B. 9

C. 10

D. $2^9 + 4^9$

E. $2^9 \times 4^9$

Question 24

$\log_2 8 + 2 \log_2 16 - \log_2 4$ is equal to

A. 9

B. $2 \log_2 16 + \log_2 4$

C. $2 \log_2 20$

D. $2 \log_2 32$

E. $\log_2 36$

The following information refers to questions 25 and 26.

Over a thirty-day period a Year 12 student recorded the number of hours she spent studying each day to the nearest hour. The results are shown in the table below.

number of hours spent studying (x)	1	2	3	4	5	6	7
number of days when x hours were spent studying	1	5	2	5	8	4	5

Question 25

During this thirty-day period, the proportion of days on which more than four hours were spent studying is

A. $\frac{5}{30}$

B. $\frac{8}{30}$

C. $\frac{13}{30}$

D. $\frac{17}{30}$

E. $\frac{22}{30}$

Question 26

During this thirty-day period, the mean number of hours spent studying was

A. 0.93

B. 4

C. 4.53

D. 5

E. 136

Question 27

A manufacturing process produces silicon chips, 90 per cent of which are defective. Ten chips are selected at random from a large production run. The probability that more than eight of these chips are defective is

A. ${}^{10}C_9 (0.1)\,(0.9)^9$

B. $(0.9)^{10}$

C. ${}^{10}C_9 (0.1)\,(0.9)^9 + (0.9)^{10}$

D. $1 - (0.9)^{10}$

E. $1 - ((0.9)^{10} + {}^{10}C_9 (0.1)\,(0.9)^9)$

Question 28

For a particular binomial distribution with n independent trials, each with a probability of success p, the mean and variance are 5 and $3\frac{3}{4}$ respectively. Which one of the following gives the correct values for n and p?

A. $n = 20,\ p = \frac{1}{5}$

B. $n = 25,\ p = \frac{1}{5}$

C. $n = 25, p = \frac{4}{5}$

D. $n = 20, p = \frac{1}{4}$

E. $n = 20, p = \frac{3}{4}$

The following information refers to questions 29 and 30.

The mass of fruit jubes, in a packet labelled as containing 200 grams, has been found to be normally distributed with a mean of 205 grams and a standard deviation of 4 grams.

Question 29

The percentage of packets that contain less than 200 grams is, correct to one decimal place,

A. 2.0%

B. 10.6%

C. 21.2%

D. 78.8%

E. 89.4%

Question 30

The probability that the mass of a packet is between 200 grams and 207 grams is, correct to four decimal places,

A. 0.2029

B. 0.3968

C. 0.4141

D. 0.4599

E. 0.5859

Question 31

The random variable Z has a standard normal distribution with mean 0 and a standard deviation 1. If $\Pr(Z > a) = 0.1977$, then the value of a is, correct to two decimal places,

A. -0.85

B. -0.15

C. 0.15

D. 0.85

E. 1.70

Question 32

The best grapes for wine making are those whose mass is less than 3 grams. Farmer Grange randomly selected 200 grapes and found that 90 per cent had a mass less than 3 grams. The standard error of the sample proportion of grapes with a mass of less than 3 grams is closest to

A. 0.02

B. 0.04

C. 0.09

D. 0.1

E. 0.9

Question 33

A randomly selected group of 100 Victorian voters was surveyed about a proposal to increase the minimum driving age to 21. Seventy per cent of the sample agreed with the proposal. The approximate 95 per cent confidence interval for the proportion, p, of the population of all voters who agreed with the proposal is

A. $0.21 \leq p \leq 0.39$

B. $0.56 \leq p \leq 0.84$

C. $0.61 \leq p \leq 0.79$

D. $0.65 \leq p \leq 0.75$

E. $0.63 \leq p \leq 0.77$

Total 33 marks

END OF PART I MULTIPLE-CHOICE QUESTION BOOKLET

This table is provided for use with Part I Questions 29, 30 and 31 and Part II Question 5

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

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STUDENT NUMBER

Figures										Letter
Words										

Victorian Certificate of Education 1996

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination (Facts, skills and applications task)

Friday 8 November 1996: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART II

QUESTION AND ANSWER BOOKLET

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of a separate question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of the Part I question booklet.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer booklet (Part II) and hand them in.

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
6	6	17

Directions to students

Materials
Question and answer booklet of 10 pages, including one blank page for rough working.
You may use an approved calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the centre of the Part I booklet during reading time.
Ensure that you write your **student number** in the space provided on the cover of this booklet.
The marks allotted to each question are indicated at the end of the question.
There is a total of 17 marks available for Part II.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer booklet (Part II) and hand them in.

Specific instructions to students

Answer **all** questions in this part in the spaces provided.

Question 1

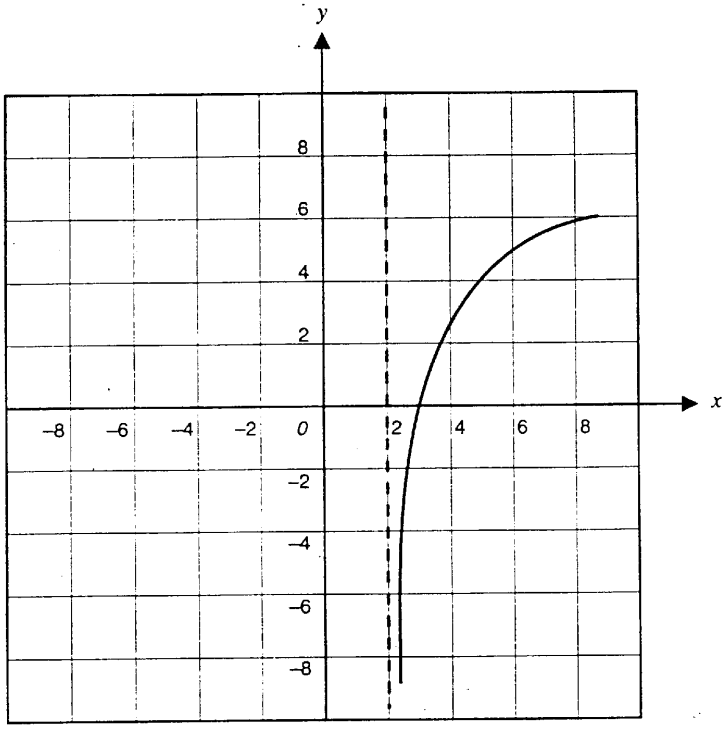
Evaluate the coefficient of x^2 in the expansion of $(2x-3)^5$.

1 mark

Question 2

The graph of the function

$f:(2,\infty)\rightarrow R, f(x)=4\log_e(x-2)$ is shown below.



Question 2 – continued

i. The graph passes through the point $(k, 2)$. Find k correct to three decimal places.

ii. On the same set of axes opposite, sketch the graph of the inverse of f.

2 + 1 = 3 marks

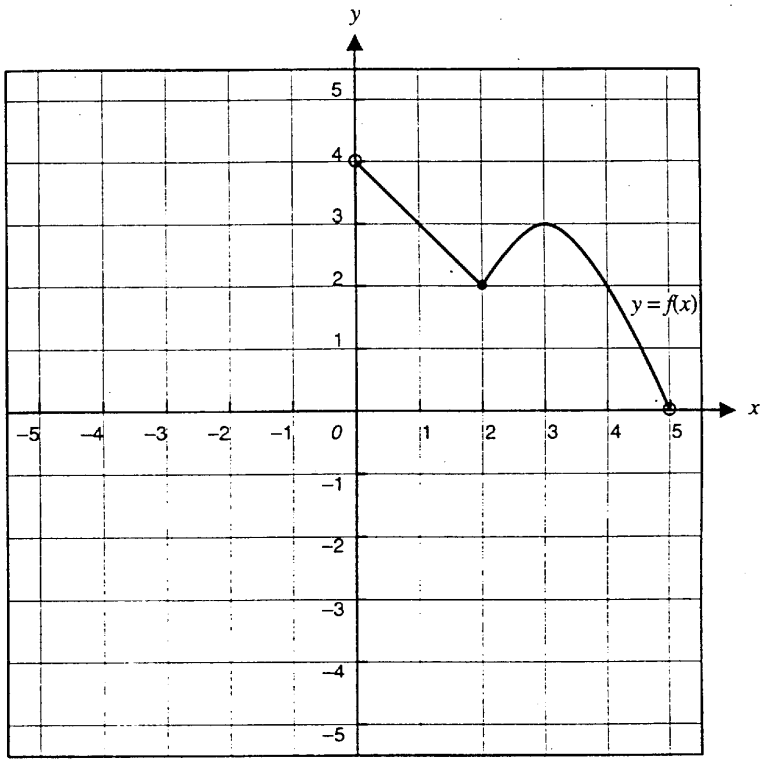
Question 3

For the graph whose equation is $y = -4\cos\left(\frac{\pi x}{8}\right) + 10$, find the equation of the tangent to the graph at the point where $x = 4$.

4 marks

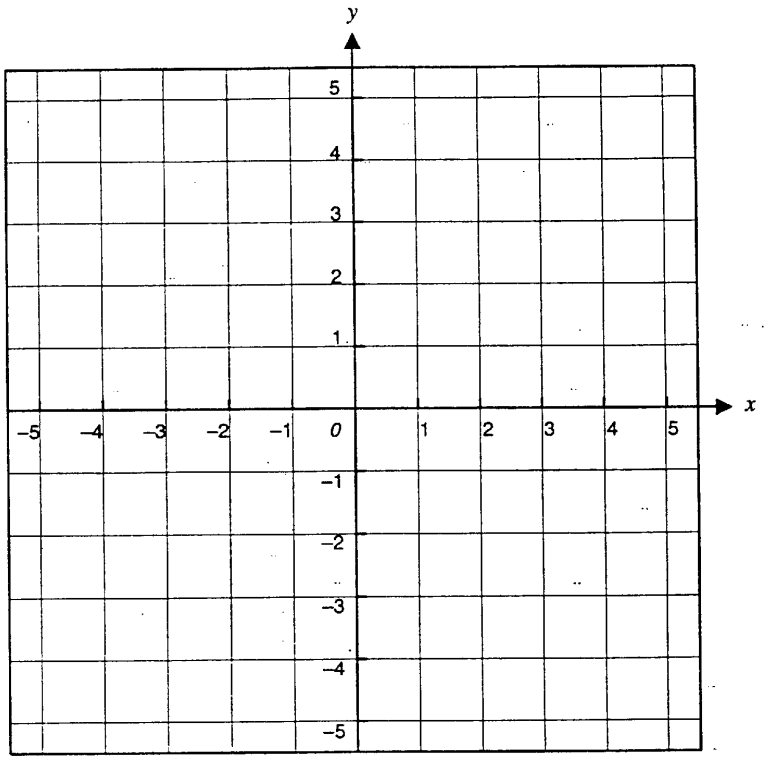
Question 4

The graph of the function $f : (0, 5) \rightarrow R$, is shown below.



i. On the set of axes below, sketch the graph of the derived function f'.

(Do **not** attempt to find the exact value of f' at $x = 2$ or $x = 5$.)



ii. State the domain of f'.

_____

2 + 1 = 3 marks

TURN OVER

Question 5

The time for Mollie to complete a bike race varies according to a normal distribution with a mean of 72 minutes and a standard deviation of 5 minutes.

i. Find the probability that Mollie completes a bike race in less than 75 minutes, correct to three decimal places.

ii. If Mollie completes a bike race in more than m minutes on 90 per cent of occasions, calculate the value of m, correct to one decimal place.

1 + 2 = 3 marks

Question 6

Find the derivative of $x \sin(x)$ and hence find an anti-derivative of $x \cos(x)$.

3 marks

Total 17 marks

This table is provided for use with Part I Questions 29, 30 and 31 and Part II Question 5

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

TURN OVER

Maths Methods CAT 2 1996 — SOLUTIONS

Q 1-5 Part 1

PART 1 : Multiple Choice

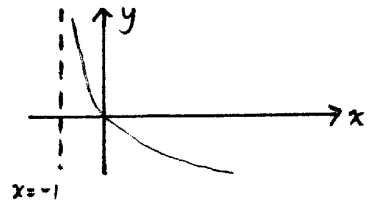
Q1

$y = \frac{1}{x+1} - 4$ ← horizontal asymptote : $y = -4$

└ vertical asymptote : $x = -1$

$\therefore$ **B**

Q2



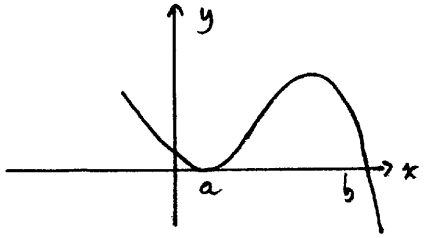
Vertical asymptote $\therefore$ log graph

$(0,0) \rightarrow x$ can $= 0$

$\therefore y = -\ln(x+1)$ $\therefore$ **D**

reflection of $y = \ln x$ in x-axis ✓

Q3



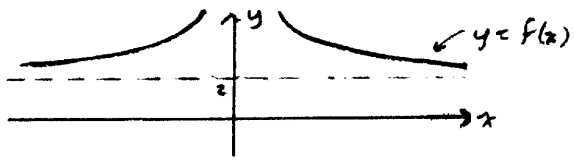
Graph is decreasing ↳ cubic → −

Repeated factor of $(x-a) \rightarrow (x-a)^2$

Single factor $(x-b) \rightarrow (x-b)$

$\therefore y = -(x-a)^2(x-b)$ $\therefore$ **A**

Q4

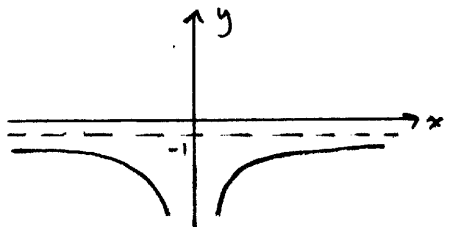


Graph of $y = 1 - f(x)$

$= -f(x) + 1$

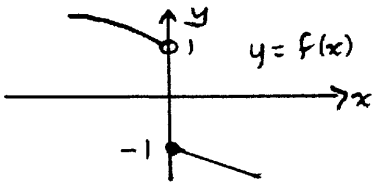
reflection in x-axis ↑ vertical translation ↑ 1

$\therefore$ graph of $y = 1 - f(x)$ is



$\therefore$ **A**

Q5



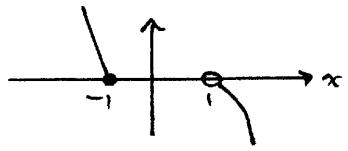
inverse has o at (1, 0)

• at (−1, 0)

Reflected to Quadrant 2

Reflected to Quadrant 4

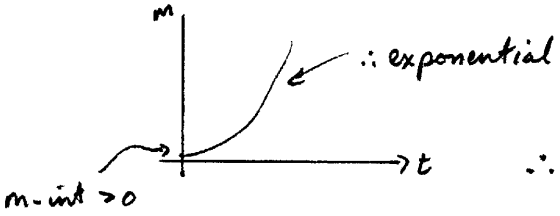
$\therefore$



$\therefore$ **E**

Q6//

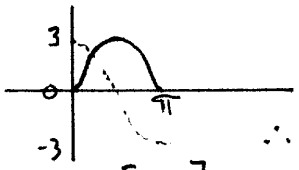
t	1	2	3	4
m	.68	1.85	5.02	13.65



$\therefore$ C

Q7// $f: [0, \pi] \to R$, $f(x) = 3\cos(x - \pi/2)$

($\frac{1}{2}$ cycle; amp = 3)



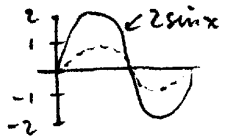
$\therefore$ C

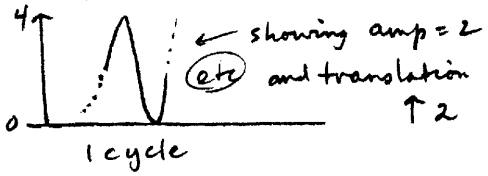
$\therefore$ Range $[0, 3]$

Q8// $f: R \to R$, $f(x) = 2\sin(3x + \pi) + 2$

$= 2\sin 3(x + \frac{\pi}{3}) + 2$

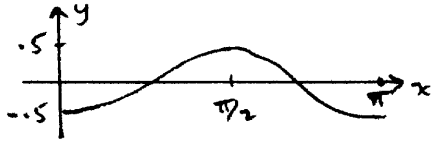
amp = 2; per = $\frac{2\pi}{3}$; Trans ↑2





$\therefore$ amp = 2, period = $\frac{2\pi}{3}$, range = $[0, 4]$ $\therefore$ C

Q9//



$y = a + b\cos(cx)$

$a = 0$ (no ↕ translation)

< 0 (reflection x-axis)

$b = -.5$ (amp)

period = $\pi = \frac{2\pi}{c}$ $\therefore$ $c = 2$ $\therefore$ D

Q10// $\sin(4x) = 0.5$, $0 \leq x \leq \pi/2$

$4x = \sin^{-1}(0.5)$ $\quad 0 \leq 4x \leq 2\pi$

$= \pi/6, 5\pi/6$

$x = \pi/24, 5\pi/24$

$\therefore$ sum = $\frac{\pi + 5\pi}{24} = \frac{6\pi}{24} = \frac{\pi}{4}$ $\therefore$ B

Q_{11} $a \sin(x+b) = c \qquad 0 \leq x \leq 2\pi \qquad a, b, c > 0$

$\sin(x+b) = \frac{c}{a}$

Now, since sine has a range of $[-1, 1]$ and means $\frac{c}{a} > 0$,

we know $0 \leq \sin(x+b) \leq 1$ $\therefore 0 < \frac{c}{a} \leq 1$

$\therefore 0 < c \leq a$

$\therefore c < a$ will be sufficient $\therefore$ A

Q_{12} $f(x) = \log_e(2x) \rightarrow f'(x) = 2 \times \frac{1}{2x} = \frac{1}{x}$

$\therefore f'(1) = \frac{1}{1}$

$= 1$ $\therefore$ A

Q_{13} $y = \sin^2(4x)$

$= (\sin(4x))^2$

Let $u = \sin 4x$

$y = u^2$

$dy/dx = dy/du \times du/dx$

$= 2u \times 4 \cos 4x$

$= 8 \sin(4x) \cos(4x)$ $\therefore$ E

Q_{14} $y = xe^x$

$dy/dx = xe^x + 1 \times e^x$ (Product Rule)

$= (x+1)e^x$

Min value found where $dy/dx = 0$

$\therefore (x+1)e^x = 0$

$x + 1 = 0$ (as $e^x \neq 0$)

$x = -1$

$\therefore x = -1$ produces a minimum value

$x < -1$, $dy/dx < 0$

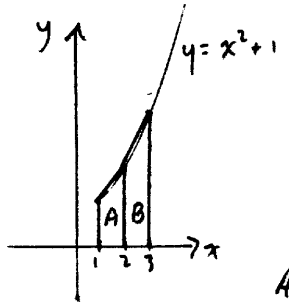
$x > 1$, $dy/dx > 0$

Min value $= -1 \times e^{-1}$

$= -e^{-1} = -\frac{1}{e}$ $\therefore$ B

15 – 18
Part 1

Q15



Trapezium	Left vertical side	Right vertical side
A	$1^2+1 = 2$	$2^2+1 = 5$
B	$2^2+1 = 5$	$3^2+1 = 10$

Area of trapezium A $= \frac{1}{2}(2+5)\times 1 = \frac{7}{2}$

Area of trapezium B $= \frac{1}{2}(5+10)\times 1 = \frac{15}{2}$

$\therefore$ Total area $= \frac{7+15}{2} = \frac{22}{2} = 11$ $\therefore$ **B**

Q16

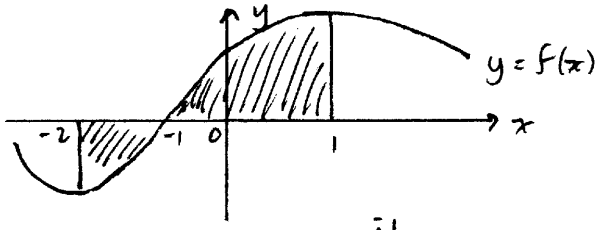
$$\int_1^4 (2f(x)+6)\,dx = \int_1^4 2f(x)\,dx + \int_1^4 6\,dx$$

$$= 2\int_1^4 f(x)\,dx + [6x]_1^4$$

$$= 2\int_1^4 f(x)\,dx + [24-6]$$

$$= 2\int_1^4 f(x)\,dx + 18 \quad \therefore \text{ C}$$

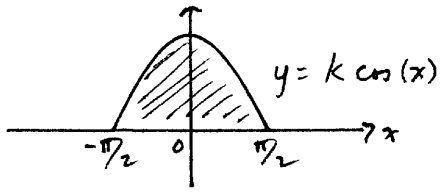
Q17



$$\text{Total area} = -\int_{-2}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx$$

$$= \int_{-1}^{1} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx \quad \therefore \text{ D}$$

Q18 $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R,\ f(x) = k\cos(x)$



$$\text{Area} = \int_{-\pi/2}^{\pi/2} k\cos x = 1$$

$$\therefore [k\sin x]_{-\pi/2}^{\pi/2} = 1$$

$$\therefore k\left[\sin\tfrac{\pi}{2} - \sin\left(-\tfrac{\pi}{2}\right)\right] = 1$$

$$k(1 - -1) = 1$$

$$2k = 1 \quad \therefore k = \tfrac{1}{2} \quad \therefore \text{ B}$$

Q19// $\int_0^1 \frac{1}{3x+1}\,dx = \log_e k$

$LHS = \frac{1}{3}\int_0^1 \frac{3}{3x+1}\,dx$

$= \frac{1}{3}\left[\log_e(3x+1)\right]_0^1$

$= \frac{1}{3}(\log_e 4 - \log_e 1)$

$= \frac{1}{3}\log_e 4$

$= \log_e 4^{1/3}$ $\quad \therefore k = 4^{1/3}$

$= (2^2)^{1/3}$

$= 2^{2/3}$ $\quad \therefore$ C

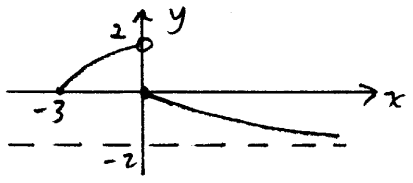
Q20// $\int \frac{1}{(3x-4)^{5/2}}\,dx = \int (3x-4)^{-5/2}\,dx$

$= \frac{(3x-4)^{-\frac{5}{2}+1}}{(-\frac{5}{2}+1)(3)}$ (assume $c=0$ here) "an" antiderivative

$= \frac{(3x-4)^{-\frac{3}{2}}}{-\frac{3}{2}\times 3}$

$= \frac{(3x-4)^{-3/2}}{-\frac{9}{2}} = \frac{-2}{9(3x-4)^{3/2}}$ $\quad \therefore$ E

Q21//



$-2 < y < 2$

$\therefore y \in (-2, 2)$

$\therefore$ B

Q22// $x^4 - 9x^2$

$= x^2(x^2-9)$

$= x^2(x-3)(x+3)$

$= x \times x \times (x-3) \times (x+3)$ $\quad \therefore$ E

Q23// Expansion of $(a+b)^n$ has $n+1$ terms

$\therefore$ Expansion of $(2x+4y)^9$ has $9+1$ or 10 terms $\therefore$ C

Q24// $\log_2 8 + 2\log_2 16 - \log_2 4$

$= 3 + 2 \times 4 - 2$

$= 3 + 8 - 2$

$= 9$ $\therefore$ A

Q25//

no. of hours (x)	1	2	3	4	5	6	7
no. of days	1	5	2	5	8	4	5

(5, 6, 7: more than 4 hours)

$\text{Proportion} = \frac{8+4+5}{30} = \frac{17}{30}$ $\therefore$ D

Q26// $\mu_X = E(X) = \{1 \times 1 + 2 \times 5 + 3 \times 2 + 4 \times 5 + 5 \times 8 + 6 \times 4 + 7 \times 5\}/30$

$= \{1 + 10 + 6 + 20 + 40 + 24 + 35\}/30$

$= 136/30$

$= 4.53\dot{}$ hours per day $\therefore$ C

Q27// Let X = no of defective chips in a sample of 10 chips

Binomial r.v., where $n = 10$, $p = .9$

$Pr(X > 8) = Pr(X = 9) + Pr(X = 10)$

$= \binom{10}{9}(.1)(.9)^9 + \binom{10}{10} \cdot 1^0 \cdot 9^{10}$

$= {}^{10}C_9 (.1)(.9)^9 + (.9)^{10}$ $\therefore$ C

Q28// Binomial r.v.

mean $= np = 5$

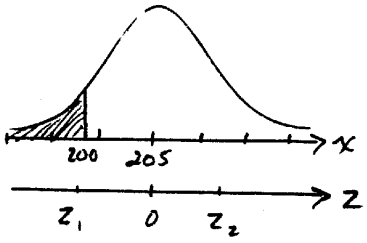
Variance $= np(1-p) = 3\frac{3}{4}$

$\therefore 5(1-p) = 3\frac{3}{4}$

$(1-p) = \frac{3}{4} \rightarrow p = \frac{1}{4}$ $n(\frac{1}{4}) = 5$ $\therefore$ D

$\therefore n = 20$

Q29//



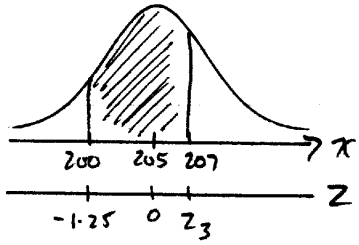
Let X = mass of fruit jubes

$\mu_X = 205, \quad \sigma_X = 4$

$$z_1 = \frac{200 - 205}{4}$$
$$= \frac{-5}{4}$$
$$= -1.25 \quad \therefore z_2 = 1.25$$

$Pr(X < 200)$
$= Pr(z_1 < -1.25)$
$= Pr(z_2 > 1.25)$
$= 1 - Pr(z_2 < 1.25)$
$= 1 - .8944$
$= .1056 \quad \therefore 10.56\% \sim 10.6\% \quad \therefore$ **B**

Q30//

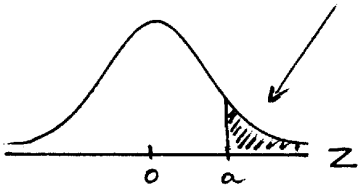


$$z_3 = \frac{207 - 205}{4}$$
$$= \frac{2}{4}$$
$$= 0.5$$

$Pr(200 < X < 207)$
$= Pr(-1.25 < Z < 0.5)$
$= Pr(Z < 0.5) - Pr(Z < -1.25)$
$= .6915 - .1056$ (from Q29)
$= 0.5859 \quad \therefore$ **E**

Q31//

$Pr(Z > a) = 0.1977 \quad \therefore Pr(Z < a) = 1 - 0.1977$
$= 0.8023$



Reading table inversely,
$a = 0.85 \quad \therefore$ **D**

Q_{32}, Sample size $= n = 200$
Sample proportion $= \hat{p} = .9$
$\therefore$ standard error $= \sqrt{\frac{.9 \times .1}{200}}$

$= 0.0212$

close to 0.02

$\therefore$ [A]

Q_{33}, Sample size $= n = 100$
Sample proportion $= \hat{p} = .7$

95% confidence interval for p is

$\hat{p} - 2\text{s.e.} < p < \hat{p} + 2\text{s.e.}$

Now, $\text{s.e.} = \sqrt{\frac{.7 \times .3}{100}}$

$= .0458$

$\therefore 2\text{s.e.} = .0916$

$\therefore$ 95% confidence interval is

$.7 - .0916 < p < .7 + .0916$

$.6083 < p < .7916$

$.61 < p < .79$ $\therefore$ [C]

MATHS METHODS CAT 2 1996

SOLUTIONS

PART II : SHORT ANSWER

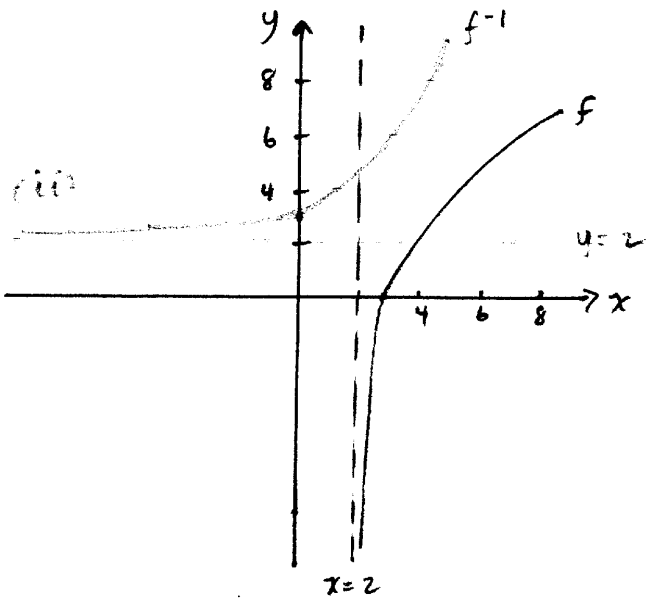
Q1,, $(2x-3)^5$ The x^2 term is $\binom{5}{3}(2x)^2(-3)^3$

$\therefore$ The coefficient is $\binom{5}{3}(2)^2(-3)^3$

$= 10 \times 4 \times -27$

$= \underline{-1080}$

Q2,, $f: (2,\infty) \to R,\ f(x) = 4\log_e(x-2)$



(i) $(k, 2)$

$2 = 4\log_e(k-2)$

$.5 = \log_e(k-2)$

$e^{.5} = k-2$

$k = e^{.5} + 2$

$\doteq \underline{3.649}$ (3 dp)

Part II

Q_{3ii} $y = -4\cos\left(\frac{\pi x}{8}\right) + 10$

Point: when $x = 4$, $y = -4\cos\left(\frac{\pi(4)}{8}\right) + 10$

$= -4\cos\left(\frac{\pi}{2}\right) + 10$

$= -4(0) + 10$

$= 10$ $\therefore$ $(4, 10)$ is point

Gradient: at any point,

gradient $= dy/dx$

$= -4x - \frac{\pi}{8}\sin\left(\frac{\pi x}{8}\right) + 0$

$= \frac{\pi}{2}\sin\left(\frac{\pi x}{8}\right)$

at $x = 4$, $dy/dx = \frac{\pi}{2}\sin\left(\frac{\pi(4)}{8}\right)$

$= \frac{\pi}{2}\sin\frac{\pi}{2}$

$= \frac{\pi}{2} \times 1$

$= \pi/2$ $\therefore$ $m = \pi/2$

Using point-gradient form of an equation of a line,

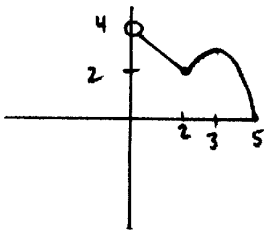
$y - y_1 = m(x - x_1)$

$y - 10 = \frac{\pi}{2}(x - 4)$

EQUATION OF TANGENT $\longrightarrow$ $y = \frac{\pi}{2}x - 2\pi + 10$

Part II

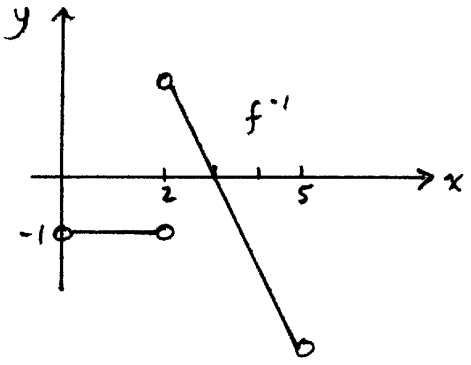
Q_4 $f: (0,5) \to R$



gradient is undefined for $(-\infty, 0]$
gradient $= -1$ for $(0, 2)$
gradient is undefined at $x = 2$
gradient is positive & decreasing for $(2, 3)$
gradient is 0 at $x = 3$
gradient is negative & decreasing for $(3, 5)$

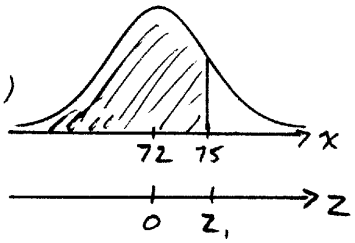
$\therefore$

(i)



(ii) Domain of f^{-1} is $(0,2) \cup (2,5)$

Q5 (i)



Let X = time for Mollie to complete a bike race (min)

$\mu = 72, \sigma = 5$

$z_1 = \frac{75-72}{5}$

$= \frac{3}{5}$

$= 0.6$

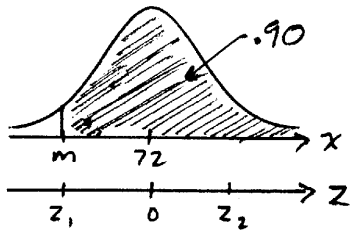
$Pr(X < 75)$

$= Pr(Z < 0.6)$

$= .7257$

$\approx \underline{0.726}$ (3 d.p.)

(ii)



Let $z_1 = -z_2$

$Pr(X > m) = .90$

$Pr(Z > z_1) = .90$

$Pr(Z < z_2) = .90$

From inverse table, $z_2 = 1.281$ or 1.282

$\therefore z_1 = -1.28$

Using $z = \frac{x-\mu}{\sigma}$

$-1.28 = \frac{m-72}{5}$

$-6.4 = m - 72$

$m = \underline{65.6}$

(She completes a bike race in more than 65.6 minutes, 90% of the time)

Q6ii $\quad d/dx\,(x\sin(x)) = x \times \cos(x) + 1 \times \sin x$ (Product Rule)

$= x\cos(x) + \sin x$

$\therefore \int (x\cos(x) + \sin x)\,dx = x\sin(x)$

$\int x\cos(x)\,dx + \int \sin(x)\,dx = x\sin(x)$

$\int x\cos(x)\,dx + (-\cos(x)) = x\sin(x)$

$\therefore \int x\cos(x)\,dx = \underline{x\sin(x) + \cos(x)}$

(assume $C = 0$: "an" antiderivative)

END OF SUGGESTED SOLUTIONS

1996 VCE MATHEMATICAL METHODS CAT 2

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