Victorian Certificate of Education
1997

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination
(Facts, skills and applications task)

Thursday 6 November 1997: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this book.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II) and hand them in.
You may retain this question book.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
33	33	33

Directions to students

Materials
Question book of 17 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the book.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
An approved scientific and/or graphics calculator may be used.
You should have at least one pencil and an eraser.

The task
Detach the formula sheet from the centre of this book during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for Part I.
All questions should be answered on the answer sheet provided for multiple-choice questions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II) and hand them in.
You may retain this question book.

Specific instructions to students

This part consists of 33 questions.

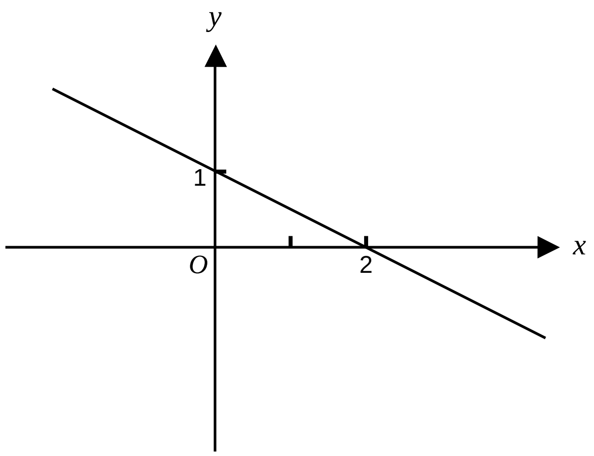
Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No credit will be given for a question if two or more letters are marked for that question.

Question 1

The gradient of a line which is perpendicular to the line shown is



A. –2

B. –1

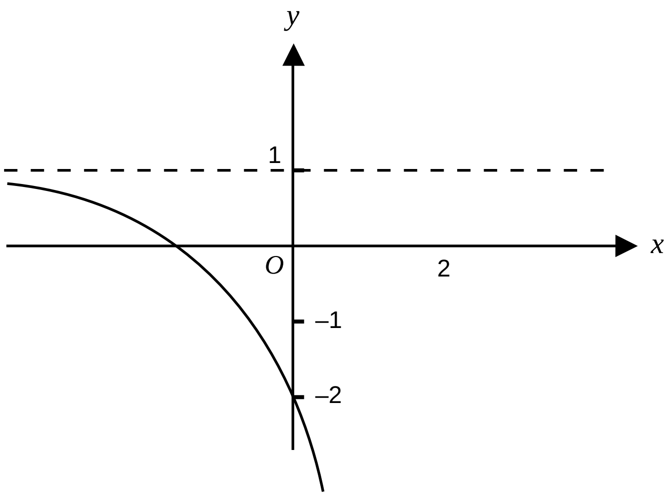
C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

Question 2

The graph whose equation is $y = A\,e^{x} + B$, where A and B are constants, is shown below.

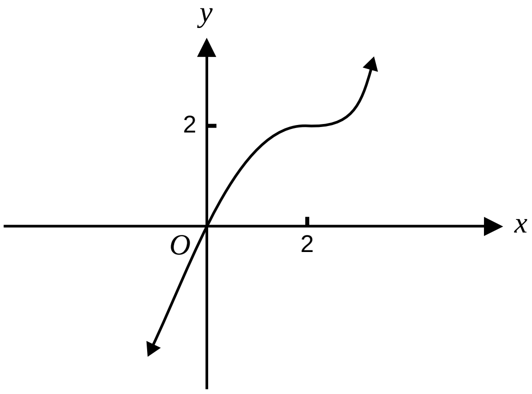


The values of A and B respectively are

A. $A = 1$ $B = -2$
B. $A = -2$ $B = 1$
C. $A = -1$ $B = -1$
D. $A = -3$ $B = 1$
E. $A = -1$ $B = -2$

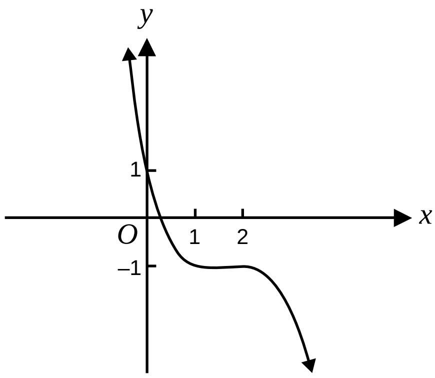
Question 3

The graph whose equation is $y = f(x)$ is shown below.

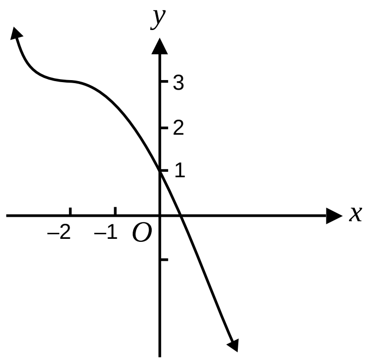


The graph whose equation is $y = 1 + f(-x)$ is

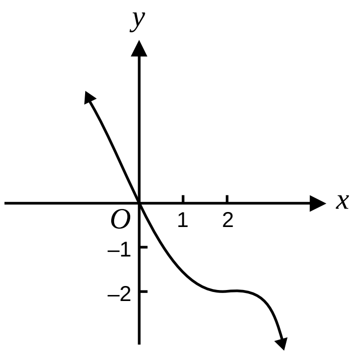
A.



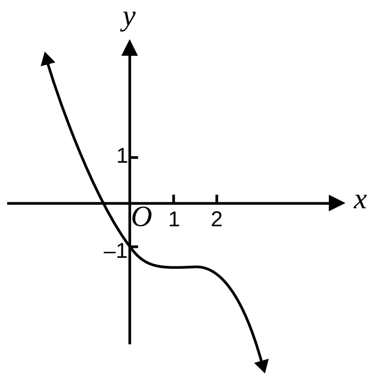
B.



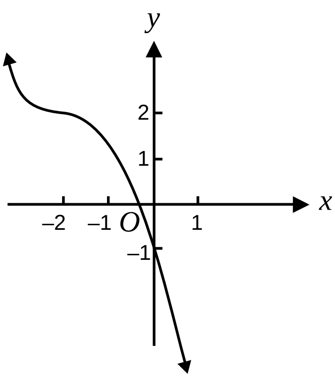
C.



D.



E.



Question 4

The parabola with equation $y = x^2$ is translated so that its image has its vertex at (–2, 5).

The equation of the image is

A. $y = -2x^2 + 5$

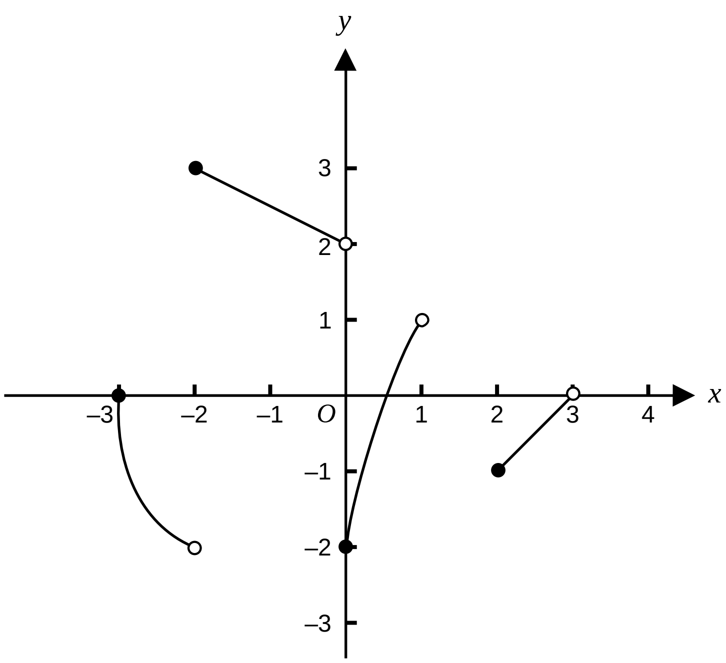
B. $y = (x - 2)^2 + 5$

C. $y = (x - 5)^2 + 2$

D. $y = (x + 2)^2 + 5$

E. $y = (x + 5)^2 - 2$

Question 5



The range of the function with graph as shown above is

A. $[-2, 3]$

B. $[-3, 3)$

C. $[-3, 1) \cup [2, 3)$

D. $[-2, 1) \cup (2, 3]$

E. $[-2, 0] \cup (2, 3]$

Question 6

The graph of a function f whose rule is $y = f(x)$ has exactly one asymptote whose equation is $y = 4$.

The graph of the inverse function f^{-1} will have

A. a horizontal asymptote with equation $y = 4$.

B. a horizontal asymptote with equation $y = \frac{1}{4}$.

C. a vertical asymptote with equation $x = 4$.

D. a vertical asymptote with equation $x = \frac{1}{4}$.

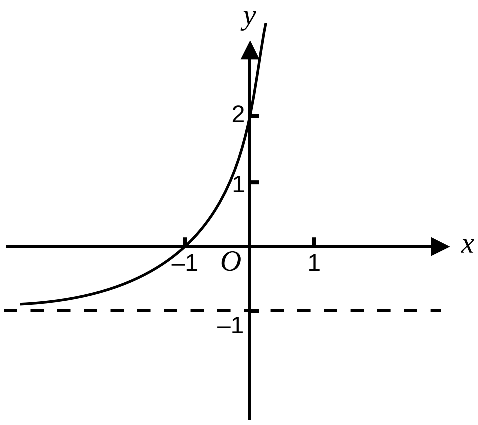
E. no asymptote.

Working space

TURN OVER

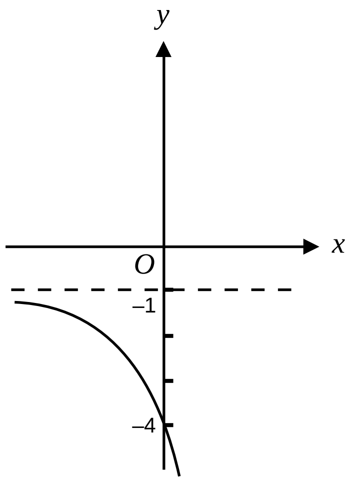
Question 7

A function f has an inverse function f^{-1}. The graph of f^{-1} is shown below.

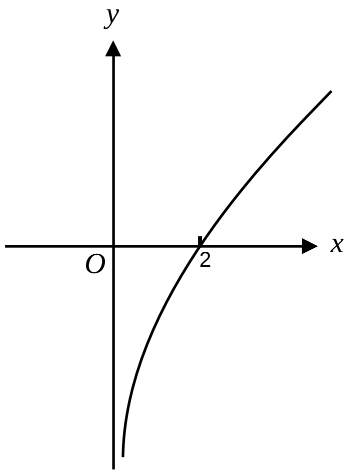


Which of the following is most likely to be the graph of f ?

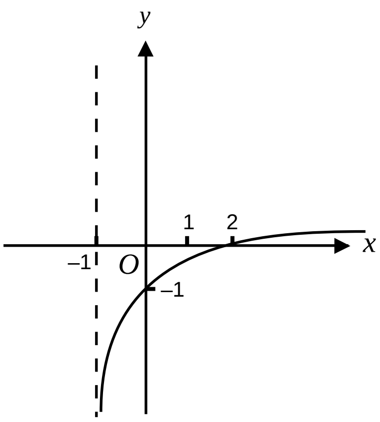
A.



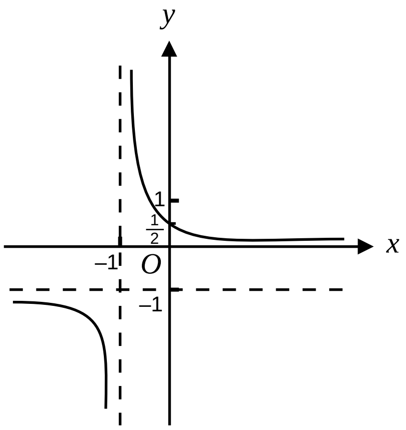
B.



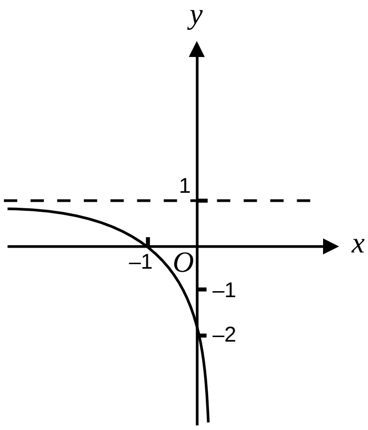
C.



D.



E.



Question 8

The function $f : R \rightarrow R,\ f(x) = 2\sin\left(\frac{x}{12}\right) + 1$ has amplitude and range respectively of

A. $\frac{1}{12}$, $[-2, 2]$

B. 2 , R

C. 4 , $[-1, 3]$

D. 2 , $[-1, 3]$

E. 4 , $[-2, 2]$

Question 9

The function $f : R \rightarrow R,\ f(x) = a\cos(bx) + c$, where a, b and c are positive constants, has period

A. a

B. b

C. $\frac{2\pi}{a}$

D. $\frac{2\pi}{b}$

E. $\frac{b}{2\pi}$

Question 10

A solution of the equation $\sin(3x) = a\cos(3x)$ is $\frac{\pi}{4}$. The value of a is

A. -3
B. -1
C. 0
D. 1
E. 3

Question 11

The graph of $y = \sin x$ is transformed into the graph $y = 3\sin(2x)$ by

A. a dilation in the y-direction by a scale factor of 3 and a translation in the x-direction of 2 units.

B. a dilation in the y-direction by a scale factor of 2 and a translation in the x-direction of $\frac{1}{3}$ units.

C. a dilation in the y-direction by a scale factor of 2 and a dilation in the x-direction by a scale factor of 3.

D. a dilation in the y-direction by a scale factor of 3 and a translation in the x-direction of π units.

E. a dilation in the y-direction by a scale factor of 3 and a dilation in the x-direction by a scale factor of $\frac{1}{2}$.

Question 12

The sum of the solutions of the equation $4\sin(2x) = 2$, in the interval $[0, 2\pi]$ is equal to

A. $\frac{\pi}{2}$

B. π

C. 2π

D. 3π

E. 4π

Question 13

If $f(x) = a\sin(2x)$, where a is a constant, and $f'(\pi) = 2$, then a is equal to

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

Question 14

If $y = x\log_e(2x)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{2}$

B. $\frac{1}{2x}$

C. $1 + \log_e(2x)$

D. $2 + \log_e(2x)$

E. $\frac{1}{2} + \log_e(2x)$

Question 15

If $y = \dfrac{x}{\sin(2x)}$, then $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{\sin(2x) - 2x\cos(2x)}{\sin^2(2x)}$

B. $\dfrac{1}{2\cos(2x)}$

C. $\dfrac{\sin(2x) + 2x\cos(2x)}{\sin^2(2x)}$

D. $\dfrac{2x\cos(2x) - \sin(2x)}{x^2}$

E. $\dfrac{2\sin(2x) + x\cos(2x)}{2\sin^2(2x)}$

Question 16

An anti-derivative of $\cos(3x) + 2e^{-2x}$ is

A. $\frac{1}{3}\sin(3x) - e^{-2x}$

B. $-3\sin(3x) - 4e^{-2x}$

C. $-\frac{1}{3}\sin(3x) - e^{-2x}$

D. $\frac{1}{3}\sin(3x) + e^{-2x}$

E. $\frac{1}{3}\sin(3x) - 4e^{-2x}$

Question 17

If $\dfrac{dy}{dx} = \dfrac{1}{(2x+3)^{\frac{3}{2}}}$ and c is a real constant, then y is equal to

A. $\dfrac{-2}{(2x+3)^{\frac{1}{2}}} + c$

B. $\dfrac{-1}{5(2x+3)^{\frac{5}{2}}} + c$

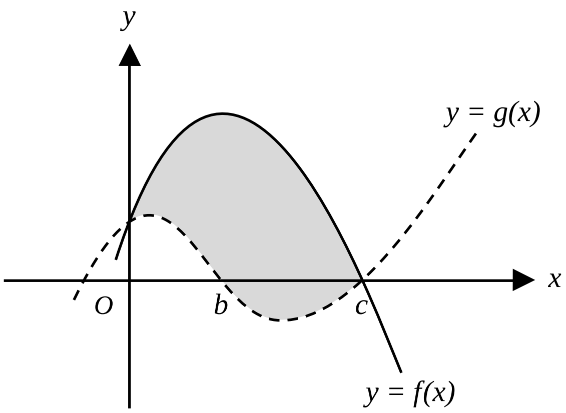
C. $\dfrac{-1}{(2x+3)^{\frac{1}{2}}} + c$

D. $\dfrac{2}{(2x+3)^{\frac{1}{2}}} + c$

E. $\dfrac{2}{5(2x+3)^{\frac{5}{2}}} + c$

Question 18

The area of the shaded region is given by



A. $\int_0^c \left(g(x) - f(x)\right) dx$

B. $\int_0^c \left(f(x) - g(x)\right) dx$

C. $\int_0^b \left(f(x) - g(x)\right) dx + \int_b^c \left(g(x) - f(x)\right) dx$

D. $\int_0^c f(x)\, dx - \int_0^b g(x)\, dx + \int_b^c f(x)\, dx$

E. $\int_0^c \left(f(x) + g(x)\right) dx$

Question 19

$\int_0^5 3\left(f(x)+2\right)\,dx$ can be written as

A. $\int_0^5 3f(x)\,dx + 2$

B. $3\int_0^5 \left(f(x)+6\right)\,dx$

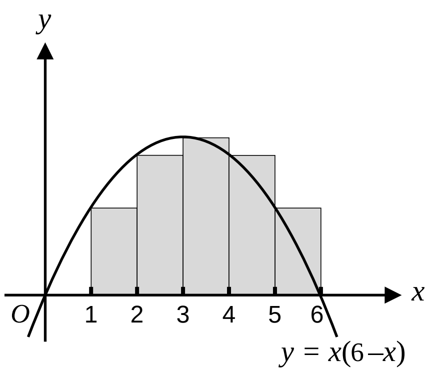
C. $3\int_0^5 f(x)\,dx + 30$

D. $3\int_0^5 f(x)\,dx + \int_0^5 2\,dx$

E. $3\int_0^5 f(x)\,dx + 6x$

Question 20

The area of the region bounded by the x-axis and by the curve whose equation is $y = x(6-x)$ can be approximated by the shaded area in the diagram below.



The exact area of the approximation is

A. 25

B. 30

C. 34

D. 35

E. 36

Question 21

The coefficient of x^2 in the expansion of $(3x - 2)^7$ is equal to

A. $288\,x^2$

B. -288

C. 6048

D. -6048

E. $-6048x^2$

Question 22

The linear factors of the polynomial $x^4 - 2x^3 - 5x^2 + 6x$ are

A.	$x - 1$,	$x + 1$,	$x - 2$,	$x + 3$
B.	x,	$x + 1$,	$x - 2$,	$x - 3$
C.	x,	$x + 1$,	$x - 2$,	$x + 3$
D.	x,	$x - 1$,	$x - 2$,	$x + 3$
E.	x,	$x - 1$,	$x + 2$,	$x - 3$

Question 23

Consider the polynomial $P(x) = (x - a)^2 (x + b) (x^2 + c)$ where $a > 0$, $b > 0$, and $c > 0$. The equation, $P(x) = 0$, has exactly

A. 1 distinct real solution.

B. 2 distinct real solutions.

C. 3 distinct real solutions.

D. 4 distinct real solutions.

E. 5 distinct real solutions.

Question 24

$3 \log_2 x + \log_2 (x^2) - \log_2 (x^5)$ is equal to

A. 0

B. $\log_2 (x^{10})$

C. $\log_2 x$

D. $\log_2 \left(\dfrac{x^2 + x^3}{x^5} \right)$

E. $\log_2 (x^2 + 3x - x^5)$

Question 25

The graph shown is that of the function $f: R \to R$, $f(x) = mx + 1$, where m is a constant.
The inverse, f^{-1}, is defined as $f^{-1}: R \to R$, $f^{-1}(x) = ax + b$, where a and b are constants.
Which one of the following statements is true?

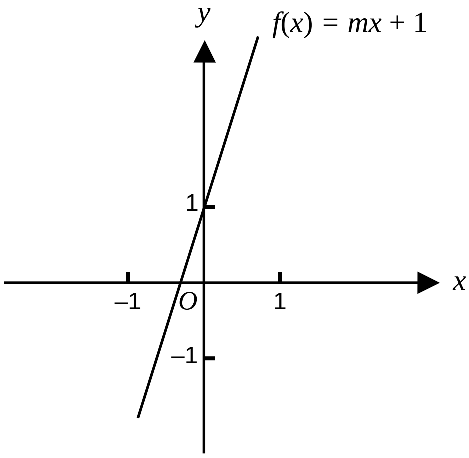
A. $a > 0, b < 0$

B. $a > 0, b > 0$

C. $a < 0, b > 0$

D. $a < 0, b < 0$

E. $a = \frac{1}{m}, b = -1$



Question 26

Jennifer constructs a spinner that will fall onto one of the numbers 1 to 5 with the following probabilities.

Number	1	2	3	4	5
Probability	0.3	0.2	0.1	0.1	0.3

If she spins the spinner once, the probability of obtaining an even number is

A. 0.02
B. 0.3
C. 0.4
D. 0.6
E. 0.7

Question 27

Ann has three chances to knock a coconut off a stand by throwing a ball. On each throw, the probability of success is $\frac{1}{5}$.

The probability that she will knock the coconut off the stand is

A. $\left(\frac{1}{5}\right)^3$

B. $1-\left(\frac{4}{5}\right)^3$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

E. $1-\left(\frac{1}{5}\right)^3$

Question 28

Which one of the following random variables is **discrete**?

A. The number of cans of cat food opened by a family during one week.

B. The area of a dairy farm in Victoria.

C. The weight of children in kindergarten in Victoria.

D. The volume of fuel used by Victorian motorists during one year.

E. The time that it takes a person to walk 2 km to the local railway station.

Question 29

Sometimes aeroplanes are fully booked but often do not carry a full passenger load due to last minute cancellations. For a 140 seat aircraft travelling between Melbourne and Canberra, the following proportions were established over a long period of time.

Number of passengers	136	137	138	139	140
Proportion of occasions	0.09	0.15	0.21	0.37	0.18

The mean number of passengers per trip is

A. 138.0

B. 138.1

C. 138.4

D. 139.0

E. 140.0

Question 30

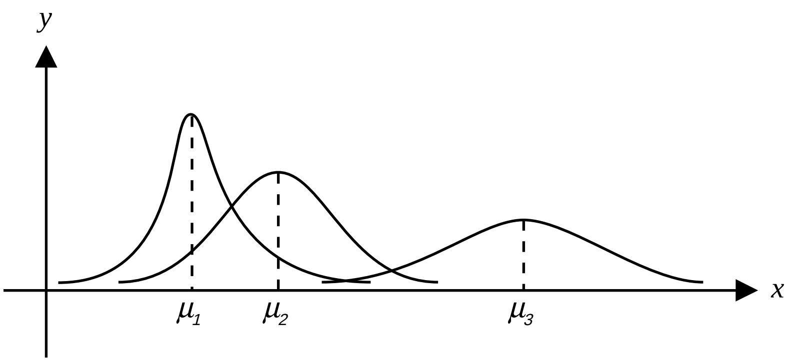
A die is loaded so that the probability of rolling a six is 0.2. The die is rolled twenty times. The mean and variance of the number of sixes is

	mean	variance
A.	3.3	2.78
B.	4	1.79
C.	4	3.2
D.	4	4
E.	16	3.2

Question 31

The diagram shows three normal distribution curves with means μ_1, μ_2, μ_3 and standard deviations σ_1, σ_2, σ_3, respectively.

Which one of the following responses contains **two correct** statements?



A. $\mu_2 = \frac{1}{2}(\mu_1 + \mu_3), \sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3)$

B. $\mu_3 > \mu_1$, $\sigma_1 = \sigma_3$

C. $\mu_3 > \mu_2$, $\sigma_2 = \sigma_3$

D. $\mu_2 > \mu_1$, $\sigma_2 < \sigma_1$

E. $\mu_2 > \mu_1$, $\sigma_2 > \sigma_1$

Question 32

The eggs laid by a particular breed of chicken have a mass which is normally distributed with a mean of 61g and a standard deviation of 2.5 g. The probability, correct to four decimal places, that a single egg has a mass between 60 g and 65 g is

A. 0.2000

B. 0.2898

C. 0.6006

D. 0.6826

E. 0.9452

Question 33

George is planning a study of the distribution of heights in centimetres among adults in his neighbourhood. He plans to measure the height of 100 people and calculate the mean m and the variance v of the heights in centimetres. He expects the heights to be normally distributed. He wants to make a statement of the form

‘90% of adults in this neighbourhood have a height h cm or greater’.

The formula that he will use to calculate h is

A. $h = m + 1.28v$

B. $h = m - 1.28v$

C. $h = m + 1.64\sqrt{v}$

D. $h = m - 1.64\sqrt{v}$

E. $h = m - 1.28\sqrt{v}$

END OF PART I MULTIPLE-CHOICE QUESTION BOOK