1997
VCE
MATHEMATICAL METHODS
CAT 2

DETAILED SUGGESTED SOLUTIONS

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Victorian Certificate of Education
1997

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination
(Facts, skills and applications task)

Thursday 6 November 1997: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this book.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II) and hand them in.
You may retain this question book.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
33	33	33

Directions to students

Materials
Question book of 17 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the book.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
An approved scientific and/or graphics calculator may be used.
You should have at least one pencil and an eraser.

The task
Detach the formula sheet from the centre of this book during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for Part I.
All questions should be answered on the answer sheet provided for multiple-choice questions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II) and hand them in.
You may retain this question book.

Specific instructions to students

This part consists of 33 questions.

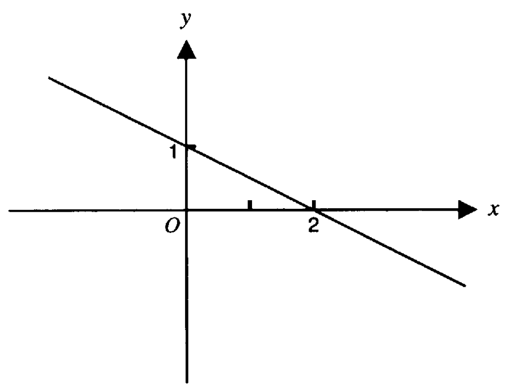
Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No credit will be given for a question if two or more letters are marked for that question.

Question 1

The gradient of a line which is perpendicular to the line shown is



A. -2

B. -1

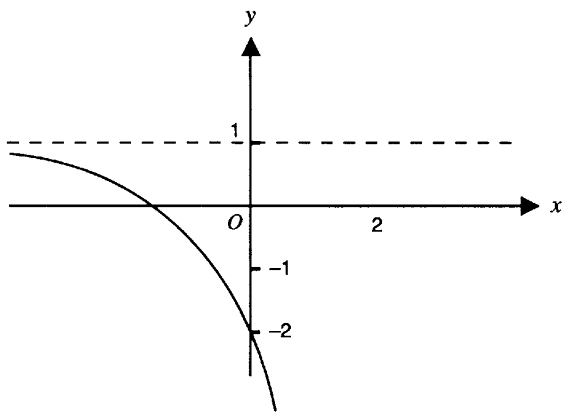
C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

Question 2

The graph whose equation is $y = A\,e^x + B$, where A and B are constants, is shown below.

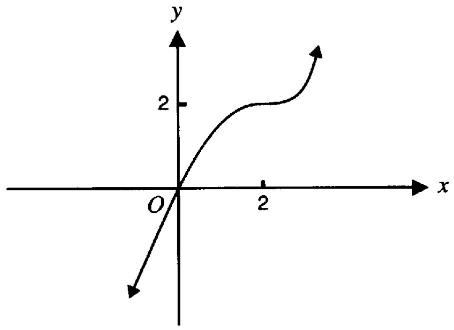


The values of A and B respectively are

A. $A = 1$ $B = -2$
B. $A = -2$ $B = 1$
C. $A = -1$ $B = -1$
D. $A = -3$ $B = 1$
E. $A = -1$ $B = -2$

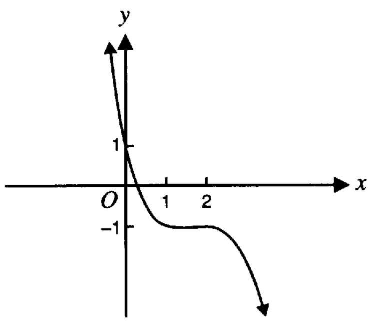
Question 3

The graph whose equation is $y = f(x)$ is shown below.

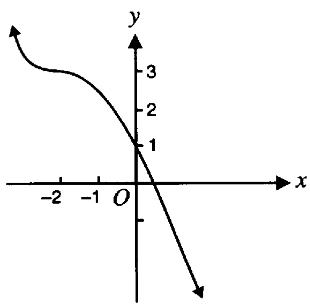


The graph whose equation is $y = 1 + f(-x)$ is

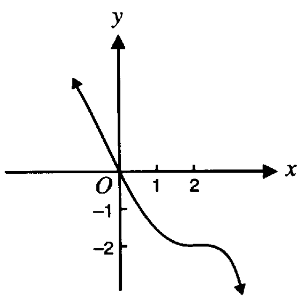
A.



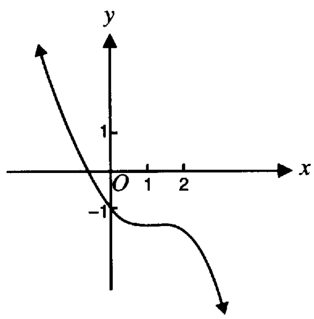
B.



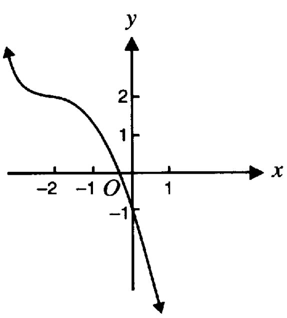
C.



D.



E.



Question 4

The parabola with equation $y = x^2$ is translated so that its image has its vertex at (–2, 5).
The equation of the image is

A. $y = -2x^2 + 5$

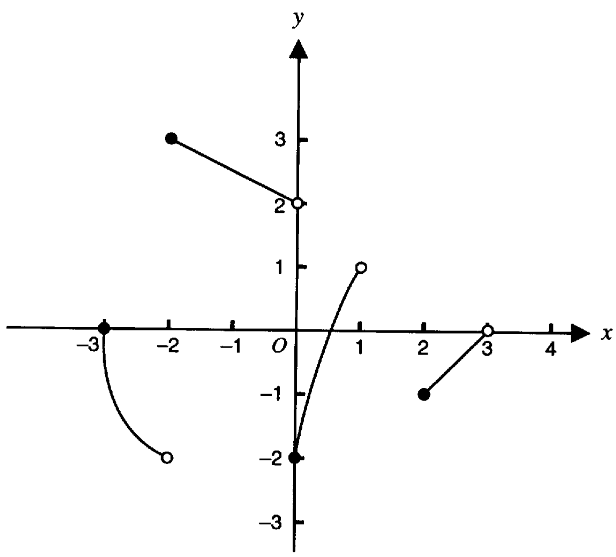
B. $y = (x-2)^2 + 5$

C. $y = (x-5)^2 + 2$

D. $y = (x+2)^2 + 5$

E. $y = (x+5)^2 - 2$

Question 5



The range of the function with graph as shown above is

A. [–2, 3]

B. [–3, 3)

C. $[-3, 1) \cup [2, 3)$

D. $[-2, 1) \cup (2, 3]$

E. $[-2, 0] \cup (2, 3]$

Question 6

The graph of a function f whose rule is $y = f(x)$ has exactly one asymptote whose equation is $y = 4$.
The graph of the inverse function f^{-1} will have

A. a horizontal asymptote with equation $y = 4$.

B. a horizontal asymptote with equation $y = \frac{1}{4}$.

C. a vertical asymptote with equation $x = 4$.

D. a vertical asymptote with equation $x = \frac{1}{4}$.

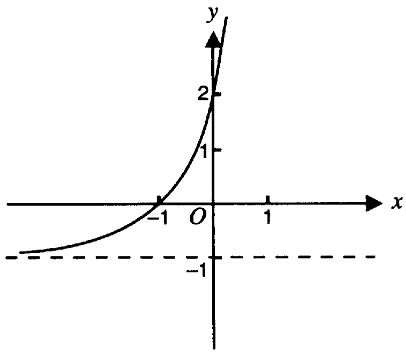
E. no asymptote.

Working space

TURN OVER

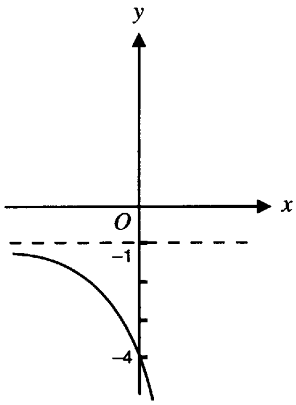
Question 7

A function f has an inverse function f^{-1}. The graph of f^{-1} is shown below.

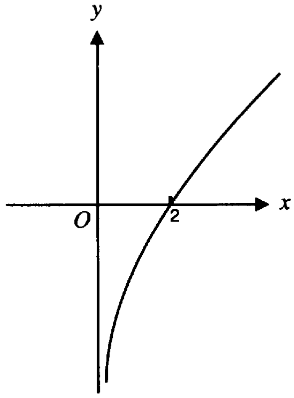


Which of the following is most likely to be the graph of f?

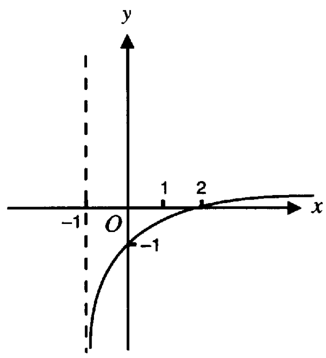
A.



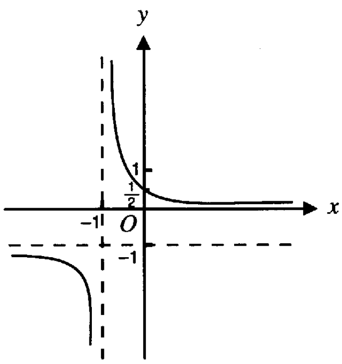
B.



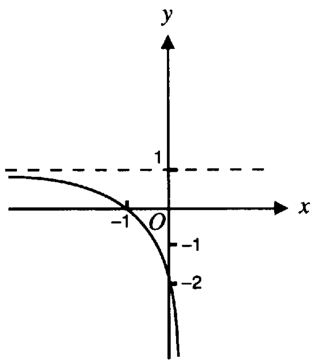
C.



D.



E.



Question 8

The function $f: R \rightarrow R, f(x) = 2 \sin\left(\frac{x}{12}\right) + 1$ has amplitude and range respectively of

A. $\frac{1}{12}$, $[-2, 2]$

B. 2 , R

C. 4 , $[-1, 3]$

D. 2 , $[-1, 3]$

E. 4 , $[-2, 2]$

Question 9

The function $f: R \rightarrow R, f(x) = a \cos(bx) + c$, where a, b and c are positive constants, has period

A. a

B. b

C. $\frac{2\pi}{a}$

D. $\frac{2\pi}{b}$

E. $\frac{b}{2\pi}$

Question 10

A solution of the equation $\sin(3x) = a \cos(3x)$ is $\frac{\pi}{4}$. The value of a is

A. -3
B. -1
C. 0
D. 1
E. 3

Question 11

The graph of $y = \sin x$ is transformed into the graph $y = 3 \sin (2x)$ by

A. a dilation in the y-direction by a scale factor of 3 and a translation in the x-direction of 2 units.

B. a dilation in the y-direction by a scale factor of 2 and a translation in the x-direction of $\frac{1}{3}$ units.

C. a dilation in the y-direction by a scale factor of 2 and a dilation in the x-direction by a scale factor of 3.

D. a dilation in the y-direction by a scale factor of 3 and a translation in the x-direction of π units.

E. a dilation in the y-direction by a scale factor of 3 and a dilation in the x-direction by a scale factor of $\frac{1}{2}$.

Question 12

The sum of the solutions of the equation $4 \sin (2x) = 2$, in the interval $[0, 2\pi]$ is equal to

A. $\frac{\pi}{2}$

B. π

C. 2π

D. 3π

E. 4π

Question 13

If $f(x) = a \sin (2x)$, where a is a constant, and $f'(\pi) = 2$, then a is equal to

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

Question 14

If $y = x \log_e(2x)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{2}$

B. $\frac{1}{2x}$

C. $1 + \log_e(2x)$

D. $2 + \log_e(2x)$

E. $\frac{1}{2} + \log_e(2x)$

MATHEMATICAL METHODS

Common Assessment Tasks 2 and 3

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)\,h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\frac{d}{dx}(\sin ax) = a\cos ax$

$\frac{d}{dx}(\cos ax) = -a\sin ax$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x}\,dx = \log_e x + c$, for $x > 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$

$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x(1-p)^{n-x}$	np	$np(1-p)$
Continuous distributions			
normal	If X is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $N(0, 1)$.		

sample mean: $\bar{x} = \frac{\Sigma x}{n}$

sample variance: $s^2 = \frac{1}{n-1}\Sigma(x-\bar{x})^2 = \frac{1}{n-1}(\Sigma x^2 - n\bar{x}^2)$

sample proportion	mean	variance	standard error
$\hat{p}$	$E(\hat{p}) = p$	$\operatorname{var}(\hat{p}) = \frac{p(1-p)}{n}$	$\operatorname{se}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

This table is provided for use with Part I Question 32

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Question 15

If $y = \dfrac{x}{\sin(2x)}$, then $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{\sin(2x) - 2x\cos(2x)}{\sin^2(2x)}$

B. $\dfrac{1}{2\cos(2x)}$

C. $\dfrac{\sin(2x) + 2x\cos(2x)}{\sin^2(2x)}$

D. $\dfrac{2x\cos(2x) - \sin(2x)}{x^2}$

E. $\dfrac{2\sin(2x) + x\cos(2x)}{2\sin^2(2x)}$

Question 16

An anti-derivative of $\cos(3x) + 2e^{-2x}$ is

A. $\frac{1}{3}\sin(3x) - e^{-2x}$

B. $-3\sin(3x) - 4e^{-2x}$

C. $-\frac{1}{3}\sin(3x) - e^{-2x}$

D. $\frac{1}{3}\sin(3x) + e^{-2x}$

E. $\frac{1}{3}\sin(3x) - 4e^{-2x}$

Question 17

If $\frac{dy}{dx} = \frac{1}{(2x+3)^{\frac{3}{2}}}$ and c is a real constant, then y is equal to

A. $\frac{-2}{(2x+3)^{\frac{1}{2}}} + c$

B. $\frac{-1}{5(2x+3)^{\frac{5}{2}}} + c$

C. $\frac{-1}{(2x+3)^{\frac{1}{2}}} + c$

D. $\frac{2}{(2x+3)^{\frac{1}{2}}} + c$

E. $\frac{2}{5(2x+3)^{\frac{5}{2}}} + c$

Question 18

The area of the shaded region is given by

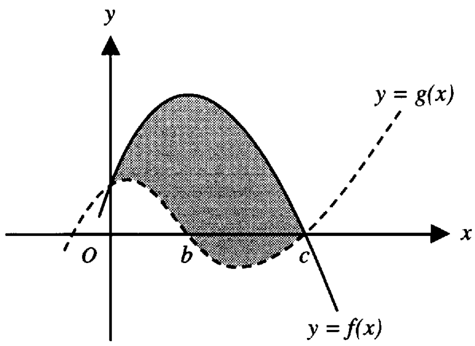
A. $\int_0^c (g(x) - f(x))\, dx$

B. $\int_0^c (f(x) - g(x))\, dx$

C. $\int_0^b (f(x) - g(x))\, dx + \int_b^c (g(x) - f(x))\, dx$

D. $\int_0^c f(x)\, dx - \int_0^b g(x)\, dx + \int_b^c f(x)\, dx$

E. $\int_0^c (f(x) + g(x))\, dx$



Question 19

$\int_0^5 3(f(x)+2)\,dx$ can be written as

A. $\int_0^5 3f(x)\,dx + 2$

B. $3\int_0^5 (f(x)+6)\,dx$

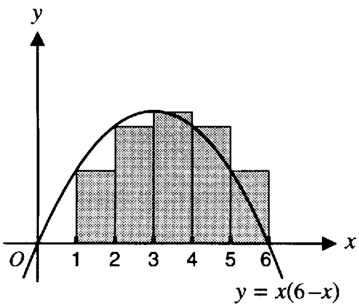
C. $3\int_0^5 f(x)\,dx + 30$

D. $3\int_0^5 f(x)\,dx + \int_0^5 2\,dx$

E. $3\int_0^5 f(x)\,dx + 6x$

Question 20

The area of the region bounded by the x-axis and by the curve whose equation is $y = x(6-x)$ can be approximated by the shaded area in the diagram below.



The exact area of the approximation is

A. 25
B. 30
C. 34
D. 35
E. 36

Question 21

The coefficient of x^2 in the expansion of $(3x-2)^7$ is equal to

A. $288x^2$

B. -288

C. 6048

D. -6048

E. $-6048x^2$

Question 22

The linear factors of the polynomial $x^4 - 2x^3 - 5x^2 + 6x$ are

A. $x-1$, $x+1$, $x-2$, $x+3$

B. x, $x+1$, $x-2$, $x-3$

C. x, $x+1$, $x-2$, $x+3$

D. x, $x-1$, $x-2$, $x+3$

E. x, $x-1$, $x+2$, $x-3$

Question 23

Consider the polynomial $P(x) = (x-a)^2(x+b)(x^2+c)$ where $a > 0$, $b > 0$, and $c > 0$. The equation, $P(x) = 0$, has exactly

A. 1 distinct real solution.

B. 2 distinct real solutions.

C. 3 distinct real solutions.

D. 4 distinct real solutions.

E. 5 distinct real solutions.

Question 24

$3\log_2 x + \log_2(x^2) - \log_2(x^5)$ is equal to

A. 0

B. $\log_2(x^{10})$

C. $\log_2 x$

D. $\log_2\left(\dfrac{x^2+x^3}{x^5}\right)$

E. $\log_2(x^2+3x-x^5)$

Question 25

The graph shown is that of the function $f: R \to R, f(x) = mx + 1$, where m is a constant.
The inverse, f^{-1}, is defined as $f^{-1}: R \to R, f^{-1}(x) = ax + b$, where a and b are constants.
Which one of the following statements is true?

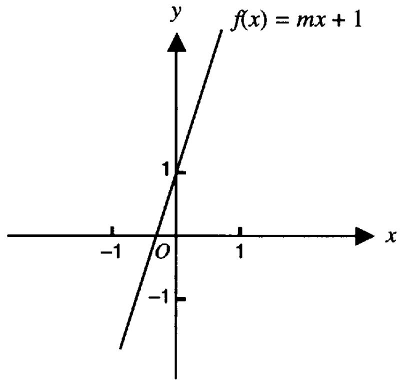
A. $a > 0, b < 0$

B. $a > 0, b > 0$

C. $a < 0, b > 0$

D. $a < 0, b < 0$

E. $a = \frac{1}{m}, b = -1$



Question 26

Jennifer constructs a spinner that will fall onto one of the numbers 1 to 5 with the following probabilities.

Number	1	2	3	4	5
Probability	0.3	0.2	0.1	0.1	0.3

If she spins the spinner once, the probability of obtaining an even number is

A. 0.02
B. 0.3
C. 0.4
D. 0.6
E. 0.7

Question 27

Ann has three chances to knock a coconut off a stand by throwing a ball. On each throw, the probability of success is $\frac{1}{5}$.

The probability that she will knock the coconut off the stand is

A. $\left(\frac{1}{5}\right)^3$

B. $1-\left(\frac{4}{5}\right)^3$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

E. $1-\left(\frac{1}{5}\right)^3$

Question 28

Which one of the following random variables is **discrete**?

A. The number of cans of cat food opened by a family during one week.

B. The area of a dairy farm in Victoria.

C. The weight of children in kindergarten in Victoria.

D. The volume of fuel used by Victorian motorists during one year.

E. The time that it takes a person to walk 2 km to the local railway station.

Question 29

Sometimes aeroplanes are fully booked but often do not carry a full passenger load due to last minute cancellations. For a 140 seat aircraft travelling between Melbourne and Canberra, the following proportions were established over a long period of time.

Number of passengers	136	137	138	139	140
Proportion of occasions	0.09	0.15	0.21	0.37	0.18

The mean number of passengers per trip is

A. 138.0

B. 138.1

C. 138.4

D. 139.0

E. 140.0

Question 30

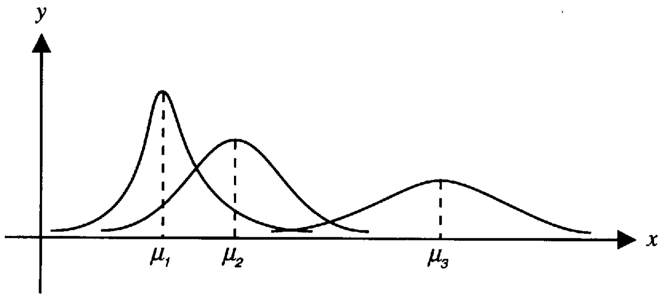
A die is loaded so that the probability of rolling a six is 0.2. The die is rolled twenty times. The mean and variance of the number of sixes is

	mean	variance
A.	3.3	2.78
B.	4	1.79
C.	4	3.2
D.	4	4
E.	16	3.2

Question 31

The diagram shows three normal distribution curves with means μ_1, μ_2, μ_3 and standard deviations σ_1, σ_2, σ_3, respectively.

Which one of the following responses contains **two correct** statements?



A. $\mu_2 = \frac{1}{2}(\mu_1 + \mu_3), \sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3)$

B. $\mu_3 > \mu_1, \sigma_1 = \sigma_3$

C. $\mu_3 > \mu_2, \sigma_2 = \sigma_3$

D. $\mu_2 > \mu_1, \sigma_2 < \sigma_1$

E. $\mu_2 > \mu_1, \sigma_2 > \sigma_1$

Question 32

The eggs laid by a particular breed of chicken have a mass which is normally distributed with a mean of 61g and a standard deviation of 2.5 g. The probability, correct to four decimal places, that a single egg has a mass between 60 g and 65 g is

A. 0.2000

B. 0.2898

C. 0.6006

D. 0.6826

E. 0.9452

Question 33

George is planning a study of the distribution of heights in centimetres among adults in his neighbourhood. He plans to measure the height of 100 people and calculate the mean m and the variance v of the heights in centimetres. He expects the heights to be normally distributed. He wants to make a statement of the form

'90% of adults in this neighbourhood have a height h cm or greater'.

The formula that he will use to calculate h is

A. $h = m + 1.28v$

B. $h = m - 1.28v$

C. $h = m + 1.64\sqrt{v}$

D. $h = m - 1.64\sqrt{v}$

E. $h = m - 1.28\sqrt{v}$

END OF PART I MULTIPLE-CHOICE QUESTION BOOK

ST NICHOLAS PLACE
15 PELHAM STREET
CARLTON VICTORIA 3053

ANSWER SHEET

USE PENCIL ONLY

- Write your name in the space provided above.
- Please enter your student number in the box provided and cross the squares as shown in the .
- Use a **PENCIL** for **ALL** entries. If you make a mistake, **ERASE** it – **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example: ✕

9 4 0 0 1 3 2 0 W

SUPERVISOR USE ONLY

USE PENCIL ONLY

Cross the " " box if the student was absent from the examination.

1	12	23
2	13	24
3	14	25
4	15	26
5	16	27
6	17	28
7	18	29
8	19	30
9	20	31
10	21	32
11	22	33

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

MATH MET CAT 2B

STUDENT NUMBER

Figures									
Words									

Letter

Victorian Certificate of Education 1997

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination (Facts, skills and applications task)

Thursday 6 November 1997: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART II

QUESTION AND ANSWER BOOK

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of the Part I question book.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II) and hand them in.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
6	6	17

Directions to students

Materials

Question and answer book of 9 pages, including one blank page for rough working.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the centre of the Part I book during reading time.
Ensure that you write your **student number** in the space provided on the cover of this book.
The marks allotted to each question are indicated at the end of the question.
There is a total of 17 marks available for Part II.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions. A decimal approximation will not be accepted if an exact answer is required to a question.
Calculus must be used to evaluate derivatives and definite integrals. A decimal value, no matter how accurate, will not be rewarded unless the appropriate working is shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
All written responses should be in English.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II) and hand them in.

Specific instructions to students

Answer **all** questions in this part in the spaces provided.

Question 1

Find the value of x for which $3e^{2x} = 1997$, giving your answer correct to two decimal places.

2 marks

Question 2

The temperature on a particular day can be modelled by the function

$$C = -4\cos\left(\frac{\pi t}{12}\right) + 16$$

where t is the time elasped, in hours, after 4:00 am and C is the temperature in degrees Celsius.

a. Calculate the temperature at 8:00 am.

1 mark

b. At what time is the temperature first 20°C?

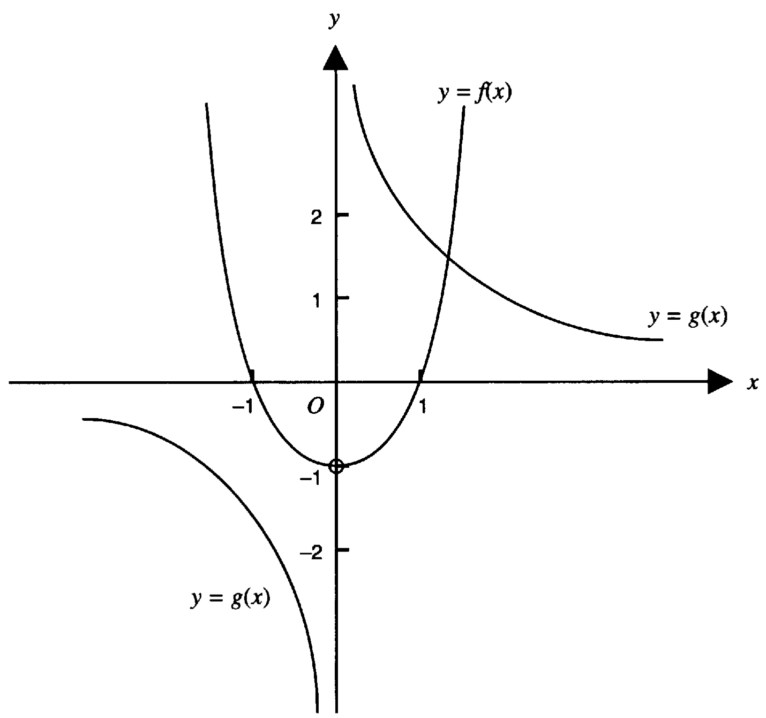
2 marks

Total 3 marks

TURN OVER

Question 3

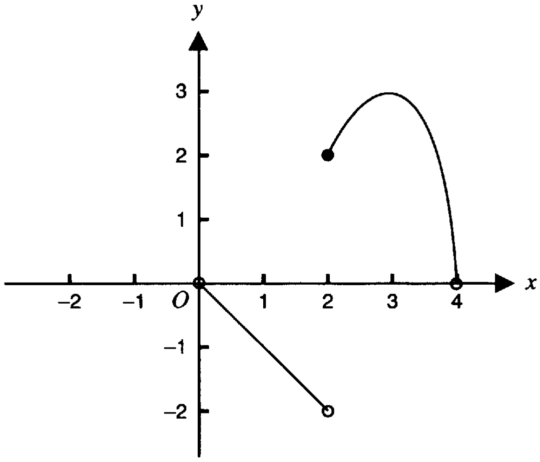
The graphs whose equations are $y = f(x)$ and $y = g(x)$ are shown in the diagram below. On the same set of axes, sketch the graph whose equation is $y = f(x) + g(x)$.



3 marks

Question 4

The graph of the function f is shown below.

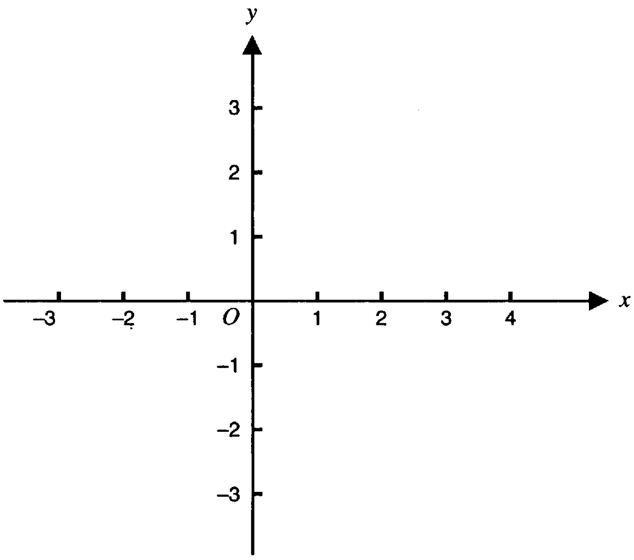


a. State the implied domain of f.

____

1 mark

b. Sketch the graph of the derived function f' on the set of axes below.



1 mark

c. State the domain of f'.

____

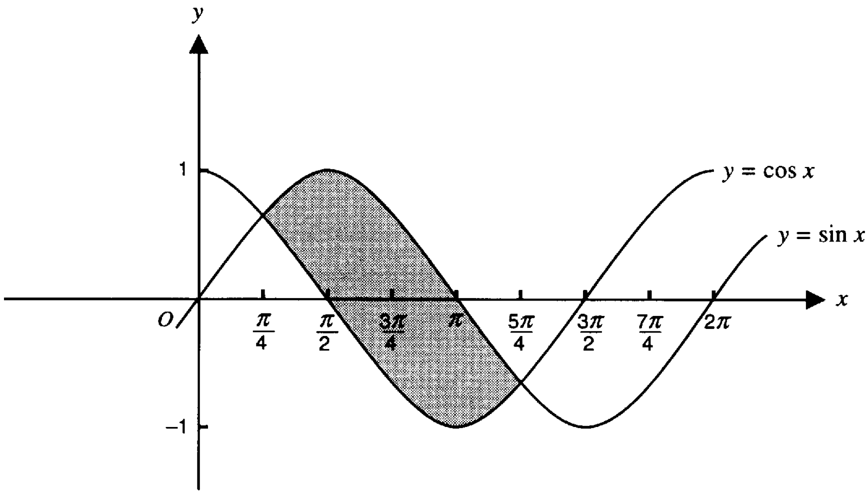
1 mark

Total 3 marks

TURN OVER

Question 5

Find the exact area of the shaded region in the diagram below.



3 marks

Question 6

Rodney rides a bicycle to work. Over a three-year period, he records the time it took him to ride to work on 1000 occasions. His results are given in the table below.

time (t minutes)	number of occasions
$t \le 20$	0
$20 < t \le 21$	3
$21 < t \le 22$	12
$22 < t \le 23$	122
$23 < t \le 24$	347
$24 < t \le 25$	355
$25 < t \le 26$	141
$26 < t \le 27$	18
$27 < t \le 28$	2
$t > 28$	0

If Rodney's trip takes longer than 25 minutes he is in danger of being late for work.

a. Calculate the proportion of occasions when he takes longer than 25 minutes.

1 mark

b. Calculate, correct to four decimal places, the standard error of this proportion.

1 mark

Question 6 – continued
TURN OVER

c. Find the approximate 95% confidence interval for the proportion, p, of occasions when Rodney takes longer than 25 minutes to ride to work. State your answer correct to four decimal places.

1 mark

Total 3 marks

Working space

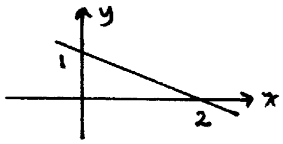
END OF PART II QUESTION AND ANSWER BOOK

Maths Methods CAT2 1997

SOLUTIONS

PART 1 : Multiple Choice

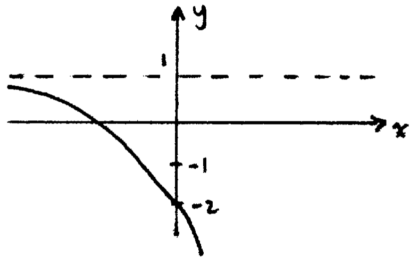
Q1.



gradient of given line $= -\frac{1}{2}$

$\therefore$ gradient of $\perp$ line $= 2$ $\therefore$ **E**

Q2.



$y = Ae^{x} + B$

use $(0,-2)$: $-2 = Ae^{0} + B$

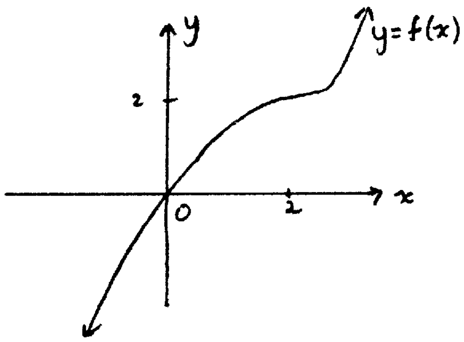
$\therefore A + B = -2$

as $x \to -\infty$, $e^{x} \to 0$, $Ae^{x} \to 0$

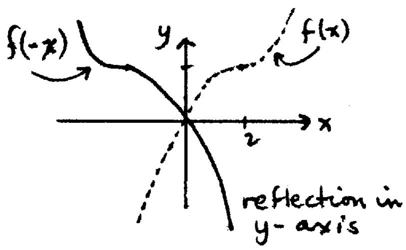
$\therefore$ as $x \to -\infty$, $y \to B$, $\therefore$ $\underline{B = 1}$

$A + B = -2$
$A + 1 = -2$
$\underline{A = -3}$ $\therefore$ **D**

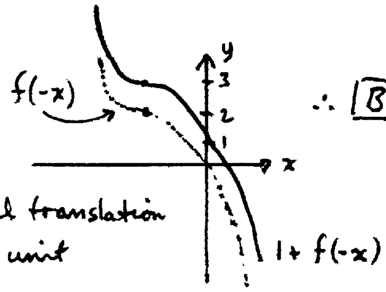
Q3.



①



②



Vertical translation ↑ 1 unit

$\therefore$ **B**

Q4. $y = x^{2}$ has vertex at $(0,0)$

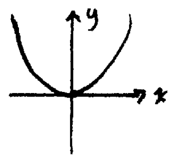


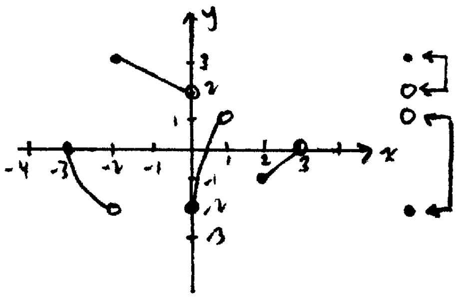
image has vertex at $(-2, 5)$

Using $y = (x-h)^{2} + k$

$h = -2$, $k = 5$

image is : $y = (x+2)^{2} + 5$ $\therefore$ **D**

Q5.



$2 < y \le 3$

$-2 \le y < 1$

$\therefore [-2, 1) \cup (2, 3]$

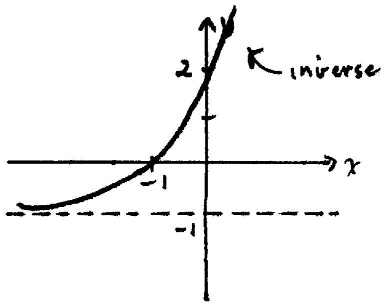
$\therefore$ **D**

Q6. Original $y = f(x)$ has asymptote $y = 4$ (horizontal)

As inverse requires x and y to "swap" roles,

Inverse $y = f^{-1}(x)$ has asymptote $x = 4$ (vertical) $\therefore$ **C**

Q7.



inverse	original
Asymptote $y = -1$	Asymptote $x = -1$
x-int $(-1, 0)$	y-int $(0, -1)$
y-int $(0, 2)$	x-int $(2, 0)$

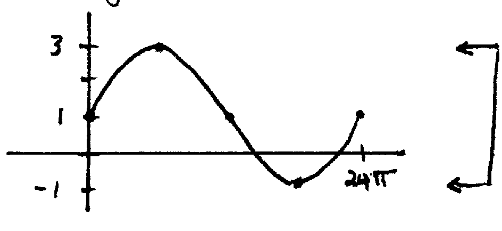
$\therefore$ **C**

Q8. $f: R \to R$, $f(x) = 2\sin\left(\frac{x}{12}\right) + 1$ ← Vertical translation ↑ 1

amp (2); $\frac{1}{12}x$

$$\text{Period} = \frac{2\pi}{\frac{1}{12}} = 24\pi$$

$\therefore$ One cycle



range is $[-1, 3]$

amp $= 2$

$\therefore$ **D**

Q9. $f: R \to R$, $f(x) = a\cos(bx) + c$ $a, b, c > 0$

$Per = \frac{2\pi}{b}$ ∴ D

Q10. $\sin(3x) = a\cos(3x)$ As $x = \frac{\pi}{4}$ is a solution,

$\sin\left(\frac{3\pi}{4}\right) = a\cos\left(\frac{3\pi}{4}\right)$

$\left(\div \text{ by } \cos\frac{3\pi}{4}\right)$ $\tan\frac{3\pi}{4} = a$

$\therefore a = -1$ ∴ B

Q11. $y = 3\sin 2x$ ← horizontal dilation $\times \frac{1}{2}$

↳ vertical dilation $\times 3$ ∴ E

Q12. $4\sin(2x) = 2$ $x \in [0, 2\pi]$

$\sin 2x = .5$ $2x \in [0, 4\pi]$

$2x = \sin^{-1}(.5)$

$= \pi/6,\ 5\pi/6,\ 13\pi/6,\ \frac{17\pi}{6}$ ($13\pi/6 = 2\pi + \frac{\pi}{6}$; $\frac{17\pi}{6} = 2\pi + \frac{5\pi}{6}$)

$x = \pi/12,\ 5\pi/12,\ 13\pi/12,\ 17\pi/12$

$\therefore$ Sum of solutions $= \frac{\pi + 5\pi + 13\pi + 17\pi}{12} = \frac{36\pi}{12} = 3\pi$ ∴ D

Q13. $f(x) = a\sin(2x)$

$f'(x) = 2a\cos(2x)$

$f'(\pi) = 2a\cos(2\pi)$

$= 2a(1)$

But since $f'(\pi) = 2$, $2 = 2a$ $\therefore a = 1$ ∴ E

Q14. $y = x\log_e(2x)$

Use product rule to differentiate:

$dy/dx = x \times 2 \times \frac{1}{2x} + 1 \times \log_e(2x)$

$= 1 + \log_e(2x)$ ∴ C

Q15. $y = \dfrac{x}{\sin(2x)}$ use quotient rule to differentiate:

$$\frac{dy}{dx} = \frac{\sin(2x) \times 1 - x \times 2\cos(2x)}{\sin^2(2x)}$$

$$= \frac{\sin(2x) - 2x\cos(2x)}{\sin^2(2x)} \qquad \therefore \boxed{A}$$

Q16. $\int [\cos(3x) + 2e^{-2x}]\, dx$

$$= \frac{1}{3}\sin(3x) + 2(-\tfrac{1}{2})e^{-2x} + c$$

$$= \frac{1}{3}\sin(3x) - e^{-2x} \quad (\text{Let } c = 0) \qquad \therefore \boxed{A}$$

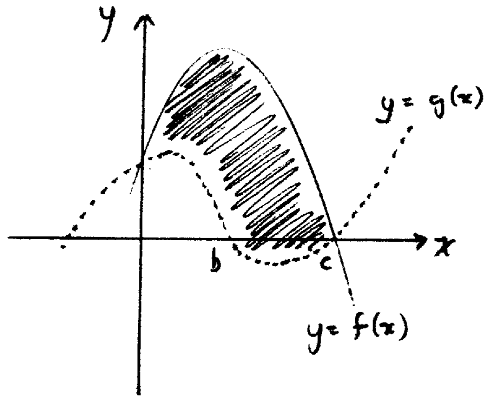
Q17. $\dfrac{dy}{dx} = \dfrac{1}{(2x+3)^{3/2}}$

$$y = \int \frac{1}{(2x+3)^{3/2}}\, dx$$

$$= \int (2x+3)^{-3/2}\, dx$$

$$= \frac{(2x+3)^{-\frac{3}{2}+1}}{(-\frac{3}{2}+1) \times 2} = \frac{(2x+3)^{-1/2}}{-\frac{1}{2} \times 2} = -\frac{1}{\sqrt{2x+3}} + c \qquad \therefore \boxed{C}$$

Q18.



Area = $\int (\text{upper} - \text{lower})$

$$= \int_0^c (f(x) - g(x))\, dx$$

$\therefore \boxed{B}$

Q19.

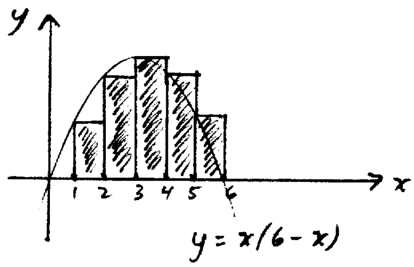
$$\int_0^5 3\,(f(x)+2)\,dx = 3\int_0^5 (f(x)+2)\,dx = 3\left[\int_0^5 f(x)\,dx + \int_0^5 2\,dx\right]$$

$$= 3\left[\int_0^5 f(x)\,dx + \left[2x\right]_0^5\right]$$

$$= 3\left[\int_0^5 f(x)\,dx + 10\right]$$

$$= 3\int_0^5 f(x)\,dx + 30 \quad \therefore \boxed{C}$$

Q20.



Left bound of rectangle	height
1	$1(6-1) = 5$
2	$2(6-2) = 8$
3	$3(6-3) = 9$
4	$4(6-4) = 8$
5	$5(6-5) = 5$

Width of each rectangle = 1

$\therefore$ Sum of areas $= 1\times5 + 1\times8 + 1\times9 + 1\times8 + 1\times5$

$= 5+8+9+8+5 = \underline{\underline{25}}$ $\quad \therefore \boxed{D}$

Q21. $(3x-2)^7$ $\quad x^2$ term : $\binom{7}{5}(3x)^2(-2)^5$

$\therefore$ coefficient $= \binom{7}{5}3^2(-2)^5$

$= 21 \times 9 \times -32$

$= \underline{\underline{-6048}}$ $\quad \therefore \boxed{D}$

Q22.

$x^4 - 2x^3 - 5x^2 + 6x$

$= \underline{\underline{x}}\,(x^3 - 2x^2 - 5x + 6)$ ← Let this $= P(x)$

$P(1) = 1 - 2 - 5 + 6 = 0$

$\therefore (x-1)$ is a factor

$P(x) = x^2(x-1) - x(x-1) - 6(x-1)$

$= (x-1)(x^2 - x - 6)$

$= \underline{\underline{(x-1)}}\,\underline{\underline{(x+2)}}\,\underline{\underline{(x-3)}}$

$\therefore$ factors are

x, $x-1$, $x+2$, and $x-3$

$\therefore \boxed{E}$

Also, G.C. shows x-intercepts at 0, 1, −2, and 3.

Q23. $P(x) = (x-a)^2(x+b)(x^2+c)$ $a,b,c > 0$

$P(x) = 0$ where $(x-a)^2 = 0$ $\therefore x = a$

where $(x+b) = 0$ $\therefore x = -b$

where $x^2 + c = 0$ $\therefore$ no solutions here as $c > 0$

$\therefore$ 2 solutions $\therefore$ **B**

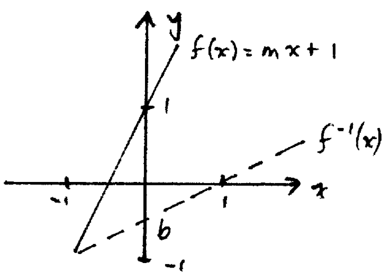
Q24. $3\log_2 x + \log_2(x^2) - \log_2(x^5)$

$= 3\log_2 x + 2\log_2 x - 5\log_2 x$

$= 0 \log_2 x$

$= 0$ $\therefore$ **A**

Q25.



$f^{-1}(x) = ax + b$

b: y-int of inverse, $\underline{b < 0}$ from graph

a: gradient of inverse, $\underline{a > 0}$ from graph $\therefore$ **A**

Q26.

No.	1	2	3	4	5
Prob.	·3	·2	·1	·1	·3

(Add the probabilities for 2 and 4)

$\Pr(\text{even}) = \cdot 2 + \cdot 1 = \underline{\cdot 3}$ $\therefore$ **B**

Q27. Let X = no of successful throws

Binomial Random variable $n = 3$, $p = \frac{1}{5}$

Pr(she knocks coconut off the stand)

= Pr(she has at least one successful throw)

$= \Pr(X > 0)$

$= 1 - \Pr(X = 0)$

$= 1 - \left(\frac{4}{5}\right)^3$ $\therefore$ **B**

Q28// Area, weight, volume, and time can theoretically be measured to unlimited accuracy and are ∴ continuous random variables, whereas the number of cans of cat food $\in \{0, 1, 2, 3, \ldots\}$

∴ **[A]**

Q29//

No. of passengers (X)	136	137	138	139	140
Prop. of occasions	0·09	0·15	0·21	0·37	0·18

$$\mu_X = E(X) = \cdot09(136) + \cdot15(137) + \cdot21(138) + \cdot37(139) + \cdot18(140)$$
$$= 12\cdot24 + 20\cdot55 + 28\cdot98 + 51\cdot43 + 25\cdot20$$
$$= 138\cdot4$$

∴ **[C]**

Q30// Let X = no. of sixes in 20 throws

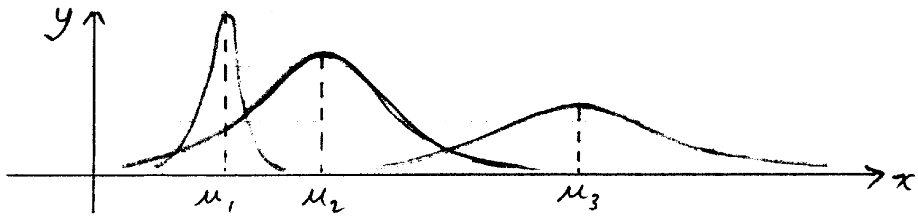
Binomial r.v. with $n = 20$, $p = \cdot2$

mean $= np = 20(\cdot2) = \underline{4}$

Variance $= np(1-p) = 20(\cdot2)(\cdot8) = \underline{\underline{3\cdot2}}$

∴ **[C]**

Q31//

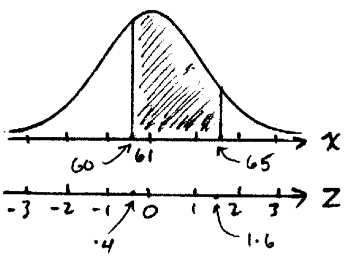


$\mu_1 < \mu_2 < \mu_3$

$\sigma_1 < \sigma_2 < \sigma_3$

∴ **[E]**

Q32//



Let X = mass of an egg (g) $\mu = 61$, $\sigma = 2\cdot5$

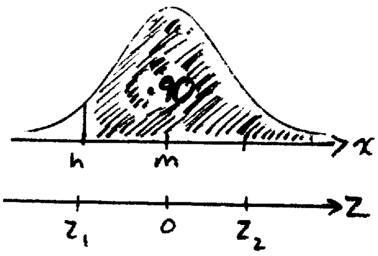
when $x = 60$, $Z = \frac{60-61}{2\cdot5} = -0\cdot4$

when $x = 65$, $Z = \frac{65-61}{2\cdot5} = 1\cdot6$

$$Pr(60 < x < 65) = Pr(-\cdot4 < Z < 1\cdot6)$$
$$= Pr(Z < 1\cdot6) - Pr(Z < -\cdot4)$$
$$= Pr(Z < 1\cdot6) - (1 - Pr(Z < 0\cdot4)) = \cdot9452 - (1 - \cdot6554)$$
$$= \underline{\underline{\cdot6006}}$$

∴ **[C]**

Q33.



Let X = height of a person

$Pr(X > h) = .9$

Reading table inversely,

$Pr(Z > z_1) = Pr(Z < z_2)$

means $z_2 = 1.2815$

$\therefore z_1 = -1.28$ (rounded)

Using $z = \frac{x - \mu}{\sigma}$

$$-1.28 = \frac{h - m}{\sqrt{v}}$$

$-1.28\sqrt{v} = h - m \quad \therefore h = m - 1.28\sqrt{v} \quad \therefore$ **E**

Part II : Short Answer

Q1.

$3e^{2x} = 1997$

$e^{2x} = 665.\dot{6}$

$2x = \ln(665.\dot{6})$

$= 6.500789$

$\therefore x = 3.25$ (2 d.p.)

Q2.

$C = -4\cos\left(\frac{\pi t}{12}\right) + 16 \qquad t$ = time (h) after 4am

(a) at 8am, $t = 4 \quad \therefore C = -4\cos\left(\frac{4\pi}{12}\right) + 16$

$= -4\cos\frac{\pi}{3} + 16$

$= -4\left(\frac{1}{2}\right) + 16$

$= -2 + 16$

$= 14 \qquad \therefore 14°C$

(b) When $C = 20$,

$20 = -4\cos\left(\frac{\pi t}{12}\right) + 16$

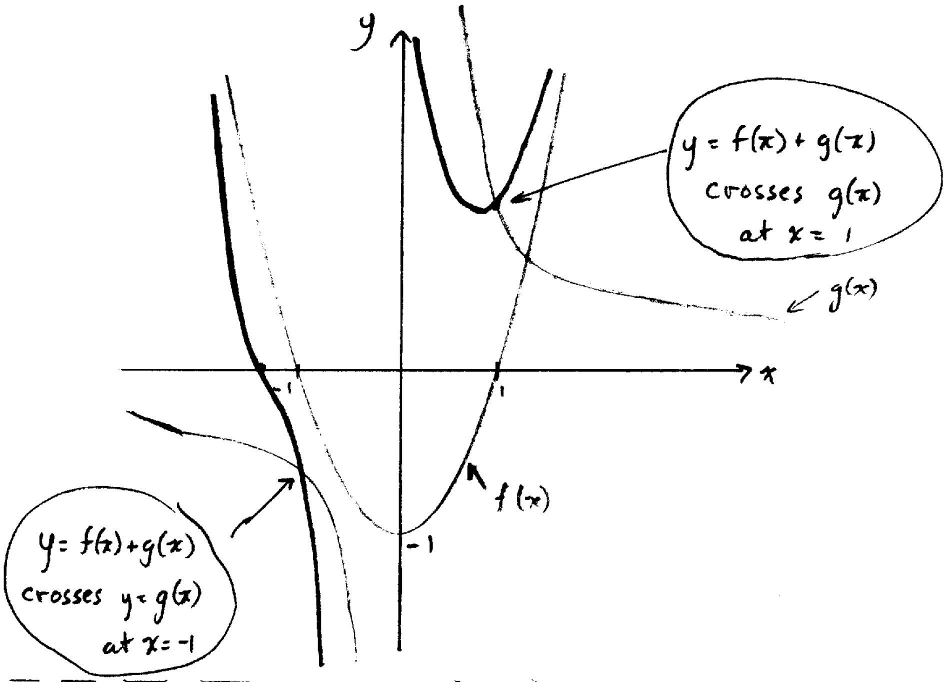
$4 = -4\cos\frac{\pi t}{12}$

$-1 = \cos\frac{\pi t}{12} \rightarrow \frac{\pi t}{12} = \pi, 3\pi, 5\pi, \ldots$

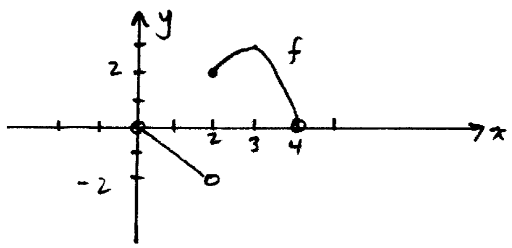
$t = 12, 36, 60, \ldots$

4pm (first time it's 20°C)

Q3.

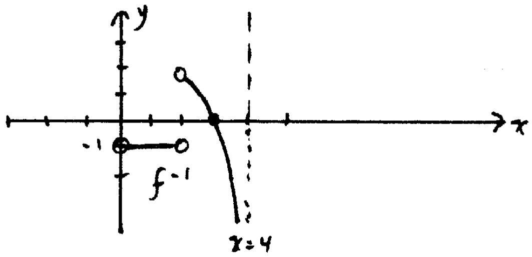


Q4.



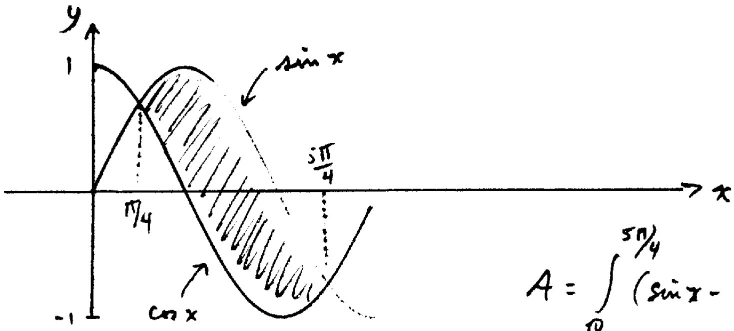
(a) Dom_f : $(0,4)$

b)



(c) $Dom_{f^{-1}}$: $(0,2) \cup (2,4)$

Q5//



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$$

$$= \left[-\cos x - \sin x \right]_{\pi/4}^{5\pi/4}$$

$$= -1 \left[\left(\cos 5\pi/4 + \sin 5\pi/4 \right) - \left(\cos \pi/4 + \sin \tfrac{\pi}{4} \right) \right]$$

$$= -1 \left[\left(-\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2} \right) - \left(\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2} \right) \right]$$

$$= -1 \left[-\sqrt{2} - \sqrt{2} \right]$$

$$= \underline{2\sqrt{2} \text{ units}^2}$$

Q6//

t	no. of occ
⋮	⋮
$24 < t \le 25$	355
$25 < t \le 26$	141
$26 < t \le 27$	18
$27 < t \le 28$	2
$t > 28$	0

(a) Proportion of occasions when $t > 25 = \frac{141 + 18 + 2}{1000}$

$$= \frac{161}{1000}$$

$$= \underline{\cdot 161} \text{ (or } \underline{16 \cdot 1\%}\text{)}$$

(b) $\hat{p} = \cdot 161$

$1 - \hat{p} = \hat{q} = \cdot 839$

$n = 1000$

$$\text{standard error} = \sqrt{\frac{\cdot 161 \times \cdot 839}{1000}}$$

$$= \underline{0 \cdot 0116} \ (4 \text{ d.p.})$$

(c) 95% confidence interval

$= \hat{p} \pm 2(\text{standard error})$

$= \cdot 161 \pm 2(0 \cdot 0116)$

$= \cdot 161 \pm \cdot 0232$

$\therefore \cdot 1378$ to $\cdot 1842$

or

$\underline{\cdot 1378 < p < \cdot 1842}$, with 95% confidence //

END OF SUGGESTED SOLUTIONS

1997 VCE MATHEMATICAL METHODS CAT 2

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