SUPERVISOR TO ATTACH PROCESSING LABEL HERE

**STUDENT NUMBER**

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|---|---|---|---|---|---|---|---|---|---|
| Figures | | | | | | | | | Letter: |
| Words | | | | | | | | | |

# Victorian Certificate of Education 1997

# MATHEMATICAL METHODS

## Common Assessment Task 3: Written examination (Analysis task)

**Monday 10 November 1997: 9.00 am to 10.45 am**
**Reading time: 9.00 am to 9.15 am**
**Writing time: 9.15 am to 10.45 am**
**Total writing time: 1 hour 30 minutes**

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Number of questions* | *Number of questions to be answered* |
|---|---|
| 4 | 4 |

**Directions to students**

**Materials**
Question and answer book of 14 pages.
There is a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the book.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set square and aids for curve sketching.

**The task**
Detach the formula sheet from the centre of this book during reading time.
Ensure that you write your **student number** in the space provided on the front of this book.
Answer **all** questions.
The marks allotted to each part of each question are indicated at the end of each part.
There is a total of 55 marks available for the task.
A decimal approximation will not be accepted if an exact answer is required to a question.
Calculus must be used to evaluate derivatives and definite integrals. A decimal value, no matter how accurate, will not be rewarded unless the appropriate working is shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
All written responses should be in English.

**At the end of the task**
Hand in this question and answer book.

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**Question 1**

Consider the function $f$: $\{t: t < a\} \rightarrow R$, $f(t) = -5 \log_e(8 - 0.1t)$, where $a$ has the largest value for which $f$ is defined.

**a.** What is the value of $a$?

1 mark

**b.** Find the exact values for the coordinates of the points where the graph of $f$ crosses each axis.

3 marks

**c.** Find the gradient of the tangent to the graph of $f$ at the point where $t = 70$.

2 marks

**Question 1** – continued

**d.** Find the rule of the inverse function $f^{-1}$.

3 marks

**e.** State the domain of the inverse function $f^{-1}$.

1 mark

**f.** Explain briefly what happens to the value of $f^{-1}(t)$ as $t \to \infty$.

1 mark

Total 11 marks

**Working space**

**Question 2**

Leigh works as a quality controller at a basketball factory. It is her job to decide whether the machines in use need to be reset. Machines are reset when an unacceptable number of defective basketballs are being packaged.

On a particular day, two machines are being used to produce basketballs and at the end of the day Leigh is given the following production data.

| | **Total number of basketballs produced** | **Number of defective basketballs produced** |
|---|---|---|
| machine A | 640 | 80 |
| machine B | 360 | 27 |

**a.** **i.** A basketball produced on this day by machine A is selected at random. What is the probability that it is defective?

1 mark

**ii.** A sample of 120 basketballs is selected at random from those produced on this day by machine A. What would be the expected number of defective basketballs in this sample?

1 mark

**iii.** A basketball is selected at random from all of the basketballs produced on this day by **both** machines and found to be **not** defective. What is the probability, correct to three decimal places, that this basketball was produced by machine A?

2 marks

**b.** Based on the figures in the above table, find the 95% confidence interval for the proportion of defective basketballs that machine B produces. Give your answers correct to three decimal places.

3 marks

**Question 2** – continued

As each basketball is produced by machine A or B, it is rolled onto a central conveyor belt. Basketballs from this belt are then packaged in boxes of 6.

**c.** **i.** On this day, Leigh selects a basketball at random from the conveyor belt. What is the probability that this basketball is defective? Give your answer correct to three decimal places.

1 mark

**ii.** What is the probability that a box of 6 basketballs produced on this day contains no defective basketballs? Give your answer correct to three decimal places.

2 marks

**iii.** What is the probability that a box of 6 basketballs produced on this day contains more than one defective basketball? Give your answer correct to three decimal places.

2 marks

**iv.** Leigh has decided that if 5% or more of the boxes contain more than 1 defective basketball, the machines need to be reset before the next day's production. Will the machines need to be reset? Justify your answer.

1 mark

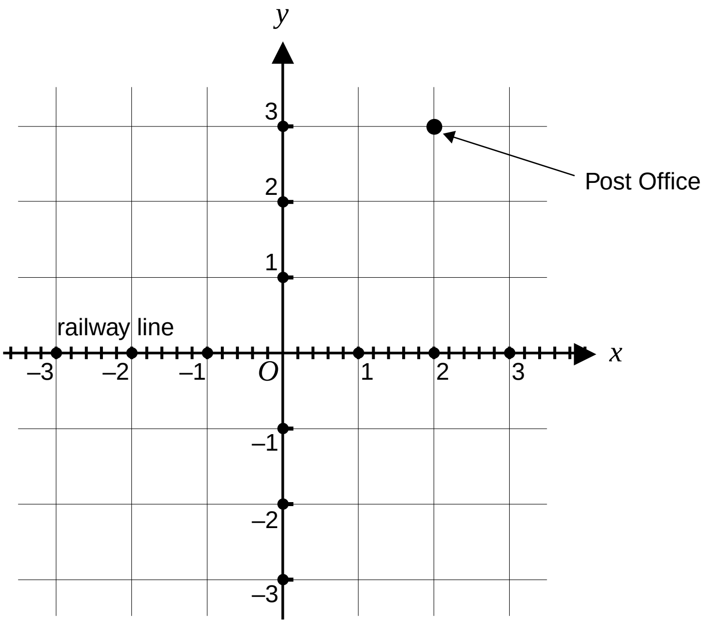
Total 13 marks

**Question 3**

In a country town, it is decided that a new road should be built.

The grid below shows the positions of the railway line and the Post Office.

In each direction, 1 unit represents 1 kilometre.



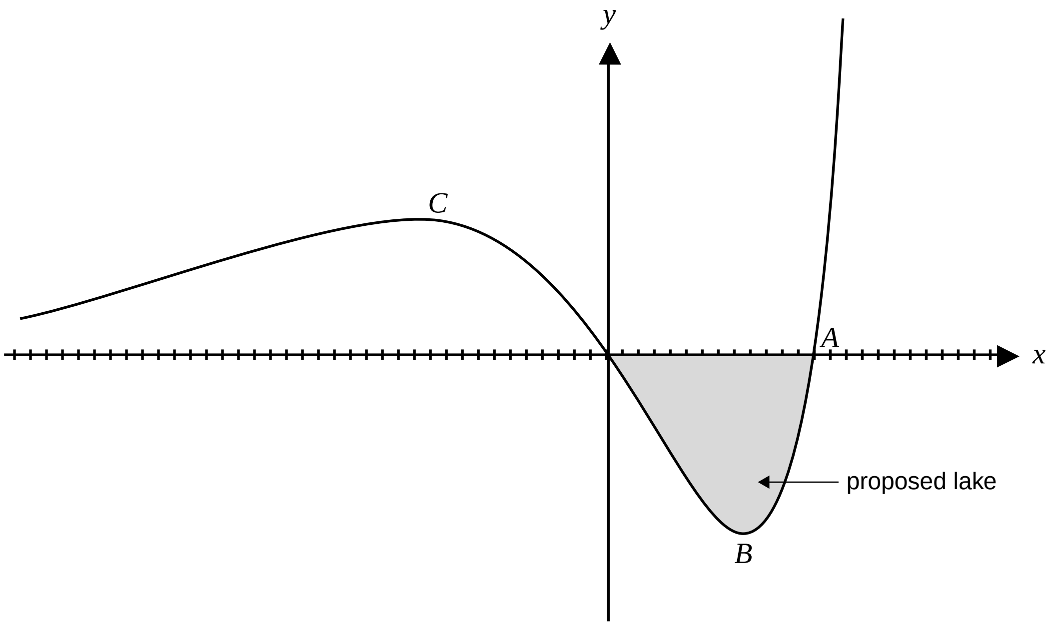

It is decided that the road should follow the path whose equation is

$$y = (2x^2 - 3x)\, e^{ax} \qquad \text{where } a > 0.$$

**a.** Find the value of $a$ for which the road will pass through the Post Office. Give your answer correct to three decimal places.

3 marks

In fact, they decide to build the road for which $a = 1$ as shown in the diagram below.




**b.** Find the $x$-coordinate of the point $A$ where the road crosses the railway line.

2 marks

**c.** Use calculus to find the coordinates of the turning point $B$. Give your answers correct to three decimal places.

4 marks

The town council wishes to develop the shaded area bounded by the road and the railway line as a lake for native water birds.

**d.** Find the values of $m$ and $n$ for which

$$\frac{d}{dx}\left\{\left(2x^2 + mx + n\right)e^x\right\} = \left(2x^2 - 3x\right)e^x$$

and hence find the exact area of the lake.

6 marks

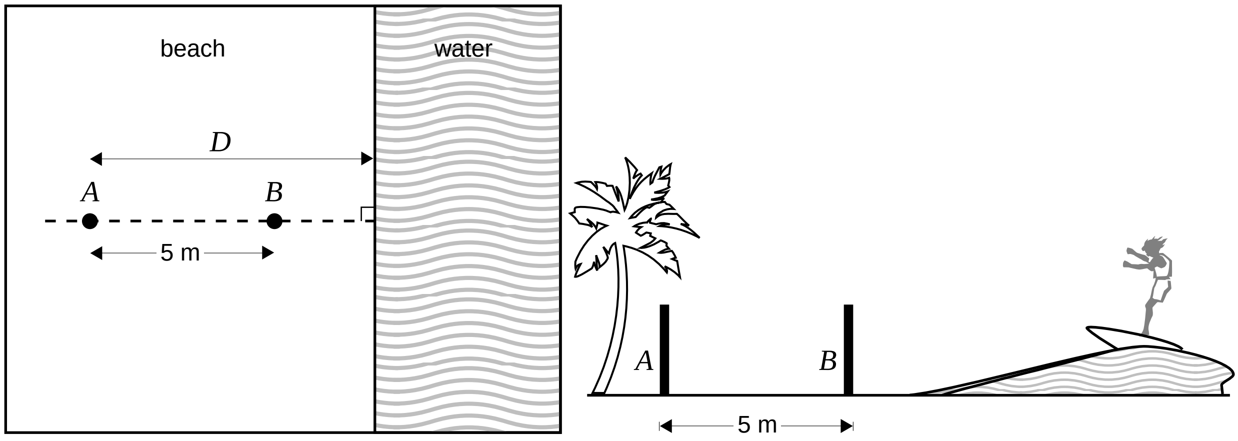
Total 15 marks

Working space

**Question 4**

When on holidays, Tasmania Jones heads for Paradise Beach where the waves roll on to the beach at regular intervals.

The diagrams below show the beach with 2 marker posts, *A* and *B*, where *A* is 5 metres further up the beach than *B*. The line *AB* is perpendicular to the water's edge.

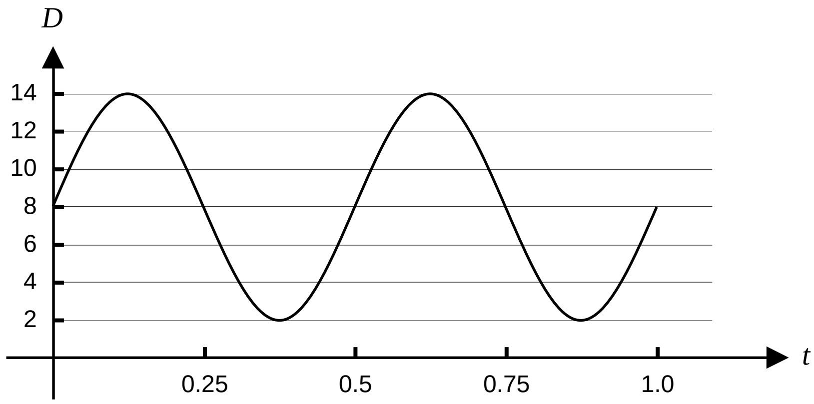



view from above side view

Tasmania Jones records the distance of the water's edge from the base of marker *A*. He discovers that, on one particular day, the distance (*D* metres) of the water's edge from the base of marker *A* is a function of *t* (the time in minutes from when he starts to observe the waves). It can be modelled exactly by the equation

$$D = a \sin(bt) + c$$

where *a*, *b* and *c* are positive constants. The graph of *D* as a function of *t* is shown below.




**a.** State the maximum and minimum distances of the water's edge from marker *A* on this day.

2 marks

**b.** Find the number of waves that hit the beach in one hour on this day.

2 marks

**Question 4** – continued

**c.** Find the values of $a$, $b$ and $c$.

3 marks

**d.** Write down an appropriate equation, solve it and hence find the exact percentage of time that marker $B$ is in the water on this day.

4 marks

Tasmania Jones also observes that on some days the waves come further up the beach and are closer together. He finds that, on such a day, the distance of the water's edge from marker $A$ can be modelled by the equation

$$D = (5 + R)\sin(8\pi t) + 8, \quad t \geq 0,$$

where $R$ is the roughness factor which varies with the roughness of the sea. $R$ is found to be normally distributed with a mean of 2 and a standard deviation of 0.6

**e.** On a particular day, when the above equation applies, the waves just reach marker $A$.

Find the value of $R$ on this day.

2 marks

**f.** Tasmania Jones likes to lie on the beach as close to the water as possible. On a particular day, when he does not know the value of $R$, how many metres up the beach from marker $B$ should he lie so that he has a 90% chance of **not** getting wet from the waves? Give your answer correct to one decimal place.

3 marks

Total 16 marks

**END OF QUESTION AND ANSWER BOOK**