THE HEFFERNAN GROUP

Student Name ....

MATHEMATICAL METHODS

TRIAL CAT 3

1998

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of 4 questions.
All questions should be attempted.
The marks allocated to each of the four questions are indicated throughout.
There is a total of 60 marks available
Students may bring up to two A4 pages of pre-written notes into the exam.
A formula sheet can be found on page 17 of this exam.

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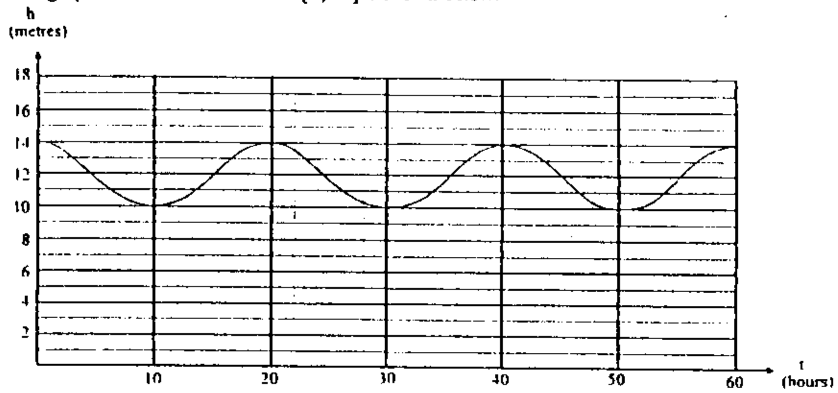
Question 1

A company runs a motor boat taxi service across a tidal river. The jetty, where the passengers embark and disembark, is fixed at a certain height above the river bed.
The height h, in metres, of the water above the river bed varies with the tide over time t, in hours, according to the function $h(t) = 2\cos\frac{\pi t}{10} + 12$.

The graph of the function for $t \in [0, 60]$ is shown below.



a. Write down the period of the graph.

1 mark

b. If $t = 0$ represents midnight one particular Sunday, write down the day and the time which $t = 60$ represents.

1 mark

The height of the jetty is fixed and it is 1 metre above the maximum height of the water.

c. Write down the height of the jetty above the river bed.

1 mark

Passengers access the motor boat by climbing down a vertical ladder. The length of the ladder is 2 metres.
If the height of the water is more than 2 metres below the jetty, then passengers cannot access the motor boat.

d. Write down the latest time on Monday morning when passengers are able to access the motor boats.

3 marks

e. Using part d., write down the day, and the starting and finishing times of the periods when the passengers can access the motor boats during the first 48 hours.

3 marks

f. If the company were to only run their motor boats between 6am and 6pm, write down the total time in the first 48 hours during which the motor boats could be accessed by passengers.

1 mark

g. Assuming that the company were to run their motor boats 24 hours a day, how long a ladder would the company require to allow access to the boats

i. at all times

ii. 50% of the time

1 + 2 = 3marks

Total 13 marks

Question 2

For many years Mrs Pearson and a group of her friends have been attending trivia nights at local schools. At these nights, each team has to answer a total of 30 questions. There are 5 questions on a particular subject and 6 subjects are covered in an evening.

The score X, represents the number of correctly answered questions out of a set of 5 questions on a particular subject.

For Mrs. Pearson and her team, the probability of obtaining a particular value of X in the subject of geography is shown in the table below.

X	0	1	2	3	4	5
Pr(X = x)	0.05	0.05	0.2	0.2	0.3	0.2

Table 1 - Probability of obtaining a particular score in the subject of geography.

a. Find the probability that at the next trivia night, Mrs. Pearson and her team will answer

i. all of their geography questions correctly.

ii. more than half of their geography questions correctly.

1 + 2 = 3 marks

For Mrs. Pearson and her group, the probability of obtaining a particular value of X in the subject of sport is shown in the table below

X	0	1	2	3	4	5
Pr(X = x)	a	b	2b	a	a	a

Table 2 - Probability of obtaining a particular score in sport.

b. If b = 2a, find the probability that at the next trivia night Mrs. Pearson and her team will answer 2 of their sport questions correctly.

2 marks

c. State whether or not Mrs. Pearson and her team are more successful in answering correctly geography or sport questions. Give reason for your answers.

2 marks

d. Find the probability that Mrs. Pearson and her team score a sum of 9 or better for their geography and sport questions at the next trivia night.

3 marks

e. The probability of obtaining correct answers in the subjects of history and science is the same for Mrs. Pearson and her team as that of obtaining correct answers in geography. The probability of obtaining correct answers in the subjects of current affairs and politics is the same as that of obtaining correct answers in sport.

Find the probability of Mrs Pearson and her team obtaining a perfect score of 30 at a trivia night.

2 marks

Mrs Pearson has a particular group of friends in what she calls her "regular" team who compete at these trivia nights. Over the years, the probability of this regular team playing is 0.8. On the other nights Mrs. Pearson invites other friends to fill in.

f. Find the probability that Mrs. Pearson's regular team will compete at the next 5 trivia nights.

1 mark

g. For how many successive trivia nights coming up can Mrs. Pearson have a better than even chance of having her regular team play?

2 marks

Total 15 marks

Question 3

A toy company produces jigsaws for very young children. The easiest of the puzzles has only 2 pieces and is shown in figure 1 below

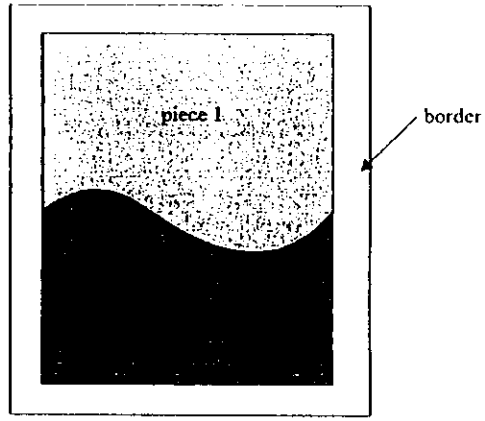


Figure 1

The puzzle consists of 2 pieces together with a border which is 2 cm wide. The curved edge of the puzzle pieces follows the curve with equation

$$y = \frac{1}{100}(x^3 + 2.5x^2 - 100x - 250)$$

as shown in figure 2 below

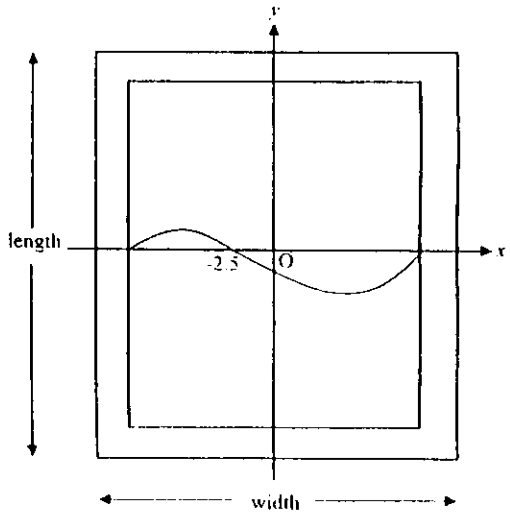


Figure 2

The origin $O(0,0)$ is at the centre of the jigsaw puzzle

a. i. Use figure 2 to write down one of the factors of the expression
$x^3 + 2.5x^2 - 100x - 250$.

ii. Use your result to part i. to factorise $x^3 + 2.5x^2 - 100x - 250$.

iii. Find the width of the puzzle (including the border).

1 + 2 + 1 = 4 marks

Point L is the lowest point on piece 1 of the puzzle and is 8.375 cm from the bottom edge of the puzzle as indicated in figure 3.
The curved edge of the puzzle pieces follows the curve with equation

$$y = \frac{1}{100}(x^3 + 2.5x^2 - 100x - 250)$$

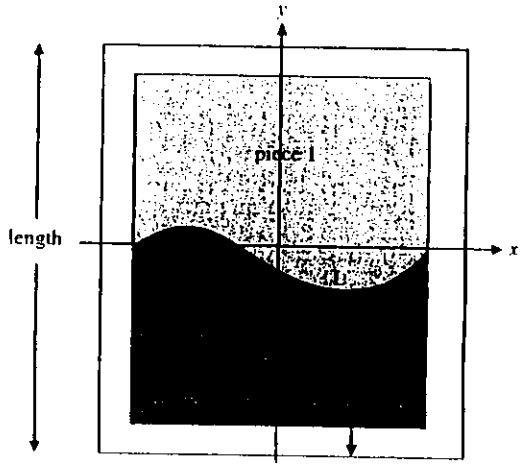


Figure 3

b. Find the length of the puzzle (including the border).

4 marks

c. Find the area of piece 1 of the puzzle. (Give your answer correct to 1 decimal place.)

6 marks

The next easiest puzzle produced by the toy company has 3 pieces as indicated in figure 4

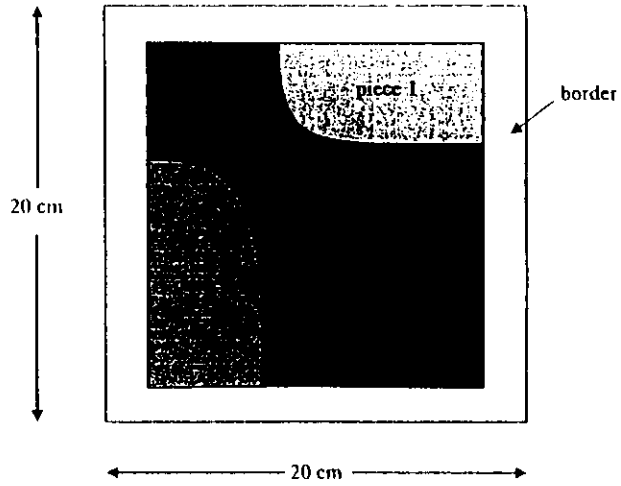


Figure 4

The puzzle is square with edge length 20 cm. The puzzle has a 2 cm border and the curved edges follow the curve defined by the equation $y = \frac{1}{x+2} + 3$, where the origin of a cartesian set of axes coincides with the centre of the puzzle as indicated in figure 5.

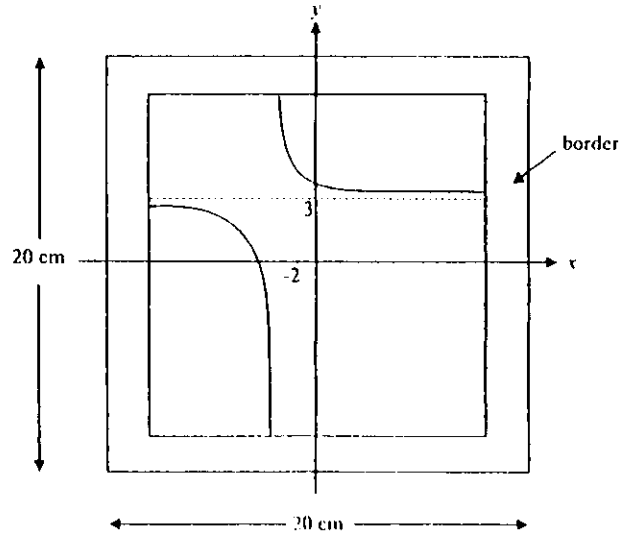


Figure 5

d. Write down the domain and range of the function shown in figure 5.

2 marks

e. Find the total length of the straight edges of piece 1 of the puzzle.

3 marks

Some of the 3-piece puzzles have the same shaped pieces but in a different configuration as shown in figure 6 below.

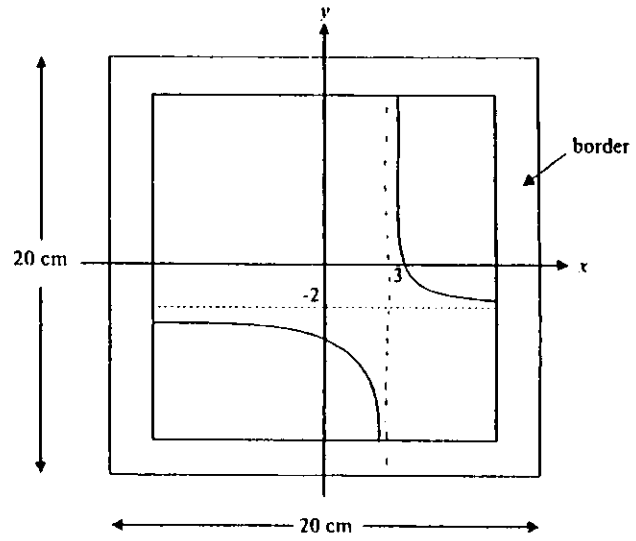


Figure 6

The edges of the 3-piece puzzles shown in figure 5 are reflected in the line $y = x$ to obtain the pieces shown in figure 6.

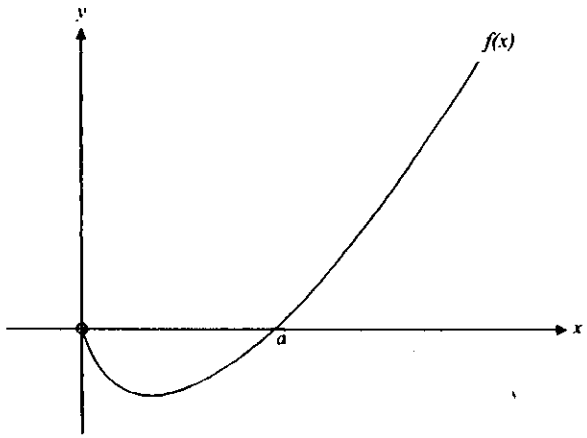
f. Write down the equation which defines the curves of the puzzle pieces shown in figure 6.

2 marks

Total 21 marks

Question 4

The graph of the function $f:(0,\infty) \to R, \quad f(x) = x\log_e(x) - x$ is shown below.



a. Find the exact value of a.

1 mark

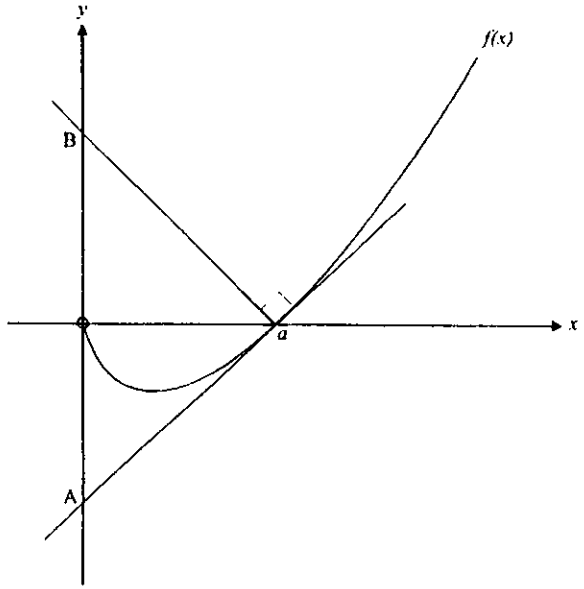
b. Show that $f'(x) = \log_e x$.

2 marks

c. Find the coordinates of the minimum point on the graph.

2 marks

At the point on the graph where the gradient is 1, the tangent cuts the y - axis at A and the normal cuts the y - axis at B.



d. Find the distance in units between point A and point B.

6 marks
Total 11 marks

END OF EXAM