Trial CAT 2 Answers & Solutions

’art I (Multiple-choice) Answers

.	D	2.	A	3.	E	4.	B	5.	A
.	C	7.	D	8.	E	9.	B	10.	C
1.	A	12.	E	13.	A	14.	B	15.	B
6.	D	17.	D	18.	B	19.	D	20.	E
1.	E	22.	D	23.	A	24.	E	25.	B
6.	C	27.	C	28.	E	29.	E	30.	C
1.	D	32.	C	33.	D				

’art I (Multiple-choice) Solutions

Question 1 [D]
$y = mx + c$
when $x = 0, y = -3$
$\therefore 0 + c = -3$
$\therefore c = -3$
when $y = 0, x = 5$
$\therefore mx - 3 = y$
$\therefore -5m - 3 = 0$
$\therefore m = \frac{-3}{5}$
$\therefore y = \frac{-3}{5}x - 3$
$\therefore 5y = -3x - 15$
$\therefore 5y + 3x = -15$

Question 2 [A]
All values from –3 to 3 are defined, except for $x = -1$

Question 3 [E]
- shape indicates -ve quartic
- -ve intercepts a and b indicates factors $(x - a)$ and $(x - b)$
- +ve intercept c indicates the factor $(x - c)$
- turning point at $(c, 0)$ indicates a squared factor

$\therefore y = -(x - a)(x - b)(x - c)^2$

Question 4 [B]
- $a > 0$ indicates no reflection about x-axis
- $b = 0$ indicates no horizontal shift
- $c > 0$ indicates asymptote shifted up
- when $x = 0, f(x) = a + c$

Question 5 [A]
A vertex of (a, b) indicates the equation $y = (x - a)^2 + b$ regardless of the sign of a or b.

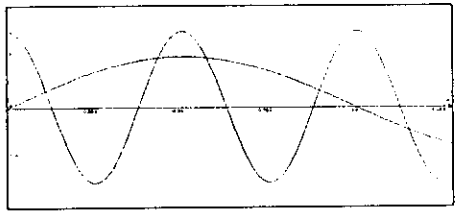
Question 6 [C]
At $x = -2, y = 3 + -7 = -4$
At $x = 2, y = 2 + 0 = 2$

Question 7 [D]
The amplitude is double. The period is π which is half the period of 2π. Since $(2x + \frac{\pi}{2}) = 2(x + \frac{\pi}{4})$, the graph moves left $\frac{\pi}{4}$ units

Question 8 [E]
Period = π, so $n = 2$
Amplitude = $\frac{2a - 0}{2} = a$
There is no horizontal translation.
There is a vertical translation of a units.
So, $y = a + a\sin(2x)$

Question 9 [B]
$$\cos 3x + \sqrt{3}\sin 3x = 0$$
$$\sqrt{3}\sin 3x = -\cos 3x$$
$$\tan 3x = \frac{-1}{\sqrt{3}}$$
$$3x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 3\pi - \frac{\pi}{6}$$
$$x = \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}$$

Question 10 [C]
The cosine graph has 2 cycles in the domain $[0, \pi]$, whereas the sine graph has ½ a cycle. The amplitude of the cosine graph is greater than the sine graph, so it crosses four times.



Question 11 [A]
$f(x) = e^x$
$\therefore f'(x) = e^x$
$\therefore f'(1) = e$
gradient of normal, $m = \frac{-1}{e}$
$y = mx + c$
$e = \frac{-1}{e} + c$
$c = e + \frac{1}{e}$
$y = \frac{-x}{e} + \frac{1}{e} + e$

Question 12 [E]
Let $m = 4e^{3x}$ and $n = \sin(2x)$
$\frac{dm}{dx} = 12e^{3x}$ $\quad \frac{dn}{dx} = 2\cos(2x)$
$$\frac{dy}{dx} = m\frac{dn}{dx} + n\frac{dm}{dx}$$
$$= 4e^{3x}[2\cos(2x)] + \sin(2x)12e^{3x}$$
$$= 8e^{3x}\cos(2x) + 12e^{3x}\sin(2x)$$
$$= 4e^{3x}[3\sin(2x) + 2\cos(2x)]$$

Question 13 [A]
The integral is –ve because it is below the graph

Question 14 [B]
$$\frac{3x^2 - x}{x} = 3x - 1$$
$$\frac{dy}{dx} = 3$$

Question 15 [B]
$$\int \frac{1}{(ax+b)^n}dx = \frac{-1}{a(n-1)(ax+b)^{n-1}}$$
$$\int \frac{1}{(3x+2)^5}dx = \frac{-1}{4 \times 3(3x+2)^4}$$
$$\int \frac{1}{(3x+2)^5}dx = \frac{-1}{12(3x+2)^4}$$

Question 16 [D]
- zero gradient at $x = -2$ and 1
- +ve decreasing gradient in the interval $[-3, -2]$
- -ve increasing then decreasing gradient in the interval $[-2, 0)$
- +ve increasing gradient in the interval $[0, 1]$
- $x = -2$ included, $x = 1$ not included

Question 17 [D]
$$y = \frac{\log_e(2x)}{2x}$$
$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$
$$= \frac{2x(\frac{2}{2x}) - 2\log_e(2x)}{4x^2}$$
$$= \frac{2 - 2\log_e(2x)}{4x^2}$$
$$= \frac{1 - \log_e(2x)}{2x^2}$$

Question 18 [B]
For the expression $(x + a)^n$
$T_{r+1} = {}^nC_r(x)^{n-r}(a)^r$
$T_{r+1} = {}^{12}C_r(3)^{12-r}(-2x)^r$
$T_{r+1} = {}^{12}C_r(-2)^r(3)^{12-r}(x)^r$
$x^r = x^{10}$
$r = 10$
Coefficient of $x^{10} = {}^{12}C_{10}(-2)^{10}(3)^2 = {}^{12}C_{10}(-2)^{10}(9)$

Question 19 [D]
$$3e^{2x} = 6$$
$$e^{2x} = 2$$
$$2x = \log_e 2$$
$$x = \frac{1}{2}\log_e 2$$

Question 20 [E]
The inverse function exists if h is one-to-one.
Intercepts are at
$2x - 2x^2 = 0$
$2x(1 - x) = 0$
$x = 0$ or $x = 1$
the smallest value of a would be at the turning point. Using symmetry this occurs when $x = ½$
$\therefore a = ½$

Question 21 [E]
Let $y = e^{2x-1}$
Interchanging x and y gives
$x = e^{2y-1}$
$\log_e x = 2y - 1$
$y = \frac{1}{2}(1 + \log_e x)$
$y = \frac{1}{2} + \frac{1}{2}\log_e x$

Question 22 [D]
$2 + \log_{10} 3x = \log_{10} y$
$2 = \log_{10} y - \log_{10} 3x$
$2 = \log_{10}(\frac{y}{3x})$
$10^2 = \frac{y}{3x}$
$y = 300x$

Question 23 [A]
$P(x) = x^3 + ax^2 - 6x + 8$
$P(-2) = -8 + 4a + 12 + 8$
$P(-2) = 24$
$4a = 12$
$a = 3$

Question 24 [E]

$$\frac{dy}{dx} = e^{2x}(2x+1)$$
$$\frac{dy}{dx} = 2xe^{2x} + e^{2x}$$
$$\int (2xe^{2x} + e^{2x})dx = xe^{2x}$$
$$\int 2xe^{2x}dx + \int e^{2x}dx = xe^{2x}$$
$$2\int xe^{2x}dx + \int e^{2x}dx = xe^{2x}$$
$$2\int xe^{2x}dx = xe^{2x} - \int e^{2x}dx$$
$$2\int xe^{2x}dx = xe^{2x} - \frac{1}{2}e^{2x} + c$$
$$\int xe^{2x}dx = \frac{e^{2x}}{2}(x - \frac{1}{2}) + c$$

Question 25 [B]
Each strip has a width of 1 unit. Area is simply the addition of all the lengths
$A = 1 + 2 + 5 + 10 + 17 = 35$ square units

Question 26 [C]
$\Pr(X < 5) = 1 - [\Pr(X = 5) - \Pr(X = 6)]$
$$= 1 - (\frac{13}{50} + \frac{8}{50})$$
$$= 1 - \frac{21}{50}$$
$$= \frac{29}{50}$$

Question 27 [C]
$$E(X) = \frac{1}{50}(0\times1 + 1\times2 + 2\times5 + 3\times6 + 4\times15 + 5\times13 + 6\times8)$$
$$= \frac{203}{50}$$
$$= 4.06$$

Question 28 [E]
Variance $= np(1-p) = \sigma^2 = 6$
$\mu = np = 10$
$$p = \frac{10}{n}$$
$$n(\frac{10}{n})(1 - \frac{10}{n}) = 6$$
$$10(1 - \frac{10}{n}) = 6$$
$$\frac{10}{n} = 1 - \frac{6}{10}$$
$$\frac{10}{n} = \frac{4}{10}$$
$$n = 25$$
$$p = \frac{10}{25}$$
$$p = 0.4$$

Question 29 [E]
Option E is the only random variable which is "measured". The others are examples of "counting".

Question 30 [C]
$$p = 0.3 \quad q = 0.7 \quad n = 10$$
$$\Pr(X > 1) = 1 - [\Pr(X = 0) + \Pr(X = 1)]$$
$$= 1 - [{}^{10}C_0(0.3)^0(0.7)^{10} + {}^{10}C_1(0.3)^1(0.7)^9]$$
$$= 1 - [(0.7)^{10} + 10(0.3)(0.7)^9]$$

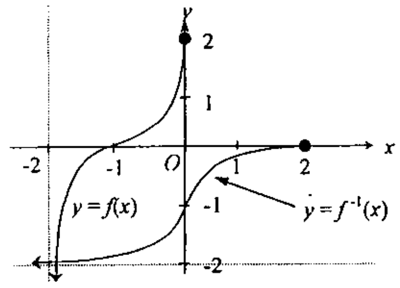
Question 31 [D]
The margin of error is given by
$$m = 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
$$(\frac{m}{2})^2 = \frac{\hat{p}(1-\hat{p})}{n}$$
$$n = (\frac{2}{m})^2\hat{p}(1-\hat{p})$$
The minimum value of $\hat{p}(1-\hat{p})$ is 0.25, so
$$n = (\frac{2}{0.05})^2 0.25 = 400$$

Question 32 [C]
$$\Pr(T > 27) = \Pr(Z > \frac{27-24}{2})$$
$$\Pr(Z > \frac{3}{2}) = 1 - \Pr(Z < \frac{3}{2}) \quad \text{[using symmetry]}$$

Question 33 [D]
$$\Pr(c_1 < T < c_2) = 0.95$$
$$\Pr(\frac{c_1 - 24}{2} < Z < \frac{c_2 - 24}{2}) = 0.95$$
$$\Pr(Z < \frac{c_2 - 24}{2}) - \Pr(Z < -\frac{(c_2 - 24)}{2}) = 0.95$$
$$\Pr(Z < \frac{c_2 - 24}{2}) - [1 - \Pr(Z < \frac{c_2 - 24}{2})] = 0.95$$
$$2\times\Pr(Z < \frac{c_2 - 24}{2}) - 1 = 0.95$$
$$\Pr(Z < \frac{c_2 - 24}{2}) = 0.975$$
By symmetry,
$$\frac{c_2 - 24}{2} = 1.9600 \qquad \frac{c_1 - 24}{2} = -1.9600$$
$$c_2 \approx 28 \qquad c_1 \approx 20$$

Part II (Short Answer Questions) Solutions

Question 1
a. correct shape [1A]
correct intercepts and asymptote [1A]



b. $\text{ran}(f^{-1}) = \text{dom}(f) = (-2, 0]$ [1A]

Question 2
a.
$$P = 2 + \sin\frac{2\pi}{28}\times7$$
$$= 2 + \sin\frac{\pi}{2}$$ [1A]
$$= 3$$

b.
$$2 + \sin\frac{2\pi}{28}t = 1$$ [1A]
$$\sin\frac{2\pi}{28}t = -1$$
$$\frac{2\pi}{28}t = \frac{3\pi}{2}$$
$t = 21$ days [1A]

Question 3
$f'(x) = 2\cos(2x)$ [1A]
$0.5 = \cos(2x)$
$\cos^{-1}(0.5) = 2x$
$$\frac{\pi}{3} = 2x$$
$$\frac{\pi}{6} = x$$ [1M]
Substitute x into $\sin(2x)$ to get:
$$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$$
So, the point is $(\frac{\pi}{6}, \frac{\sqrt{3}}{2})$ [1A]

Question 4.
a. $0.3(50)^2 + 20(50) + 200 = \$1950$ [1A]
b. Since the function is increasing for $x > 0$, the maximum cost will be at the domain maximum, i.e., $x = 100$. [1M]
$0.3(50)^2 + 20(50) + 200 = \$5200$ [1A]

Question 5
For the expression $(x + a)^n$
$$T_{r+1} = {}^nC_r(x)^{n-r}(a)^r$$
${}^4C_2(2x)^{4-2}(-1)^2 = 24x^2$ [1M]
${}^4C_0(2x)^{4-0}(-1)^0 = 16x^4$ [1M]
$$b = 24$$
$$a = 16$$
$$\frac{a}{b} = \frac{16}{24} = \frac{2}{3}$$ [1A]

Question 6
a. Let X = no. of bullseyes in 3 throws
$$p = \frac{136.5}{910.0} = 0.15 \qquad q = 0.85$$
$$\Pr(X \geq 1) = 1 - [\Pr(X = 0)]$$
$$= 1 - {}^3C_0(0.15)^0(0.85)^3$$
$$= 1 - (0.85)^3$$
$$= 0.385$$
$$\approx 0.39$$ [1A]

b.
$$\Pr(X = 1) = {}^3C_1(0.15)^1(0.85)^2$$
$$= 3\times0.15\times0.7225$$
$$= 0.325125$$
$$\approx 0.33$$ [1A]

Total 17 marks