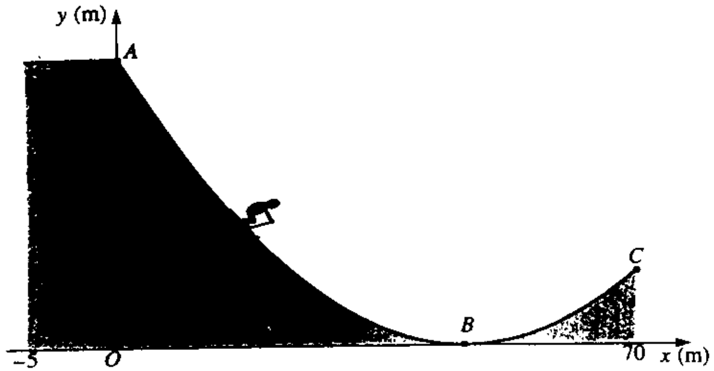
2000/10 NBAP-98

Question 1



The diagram above shows part of the design for an indoor ski ramp and platform. The curved surface from point A to point C can be modelled by the equation $y = \frac{1}{50}x^2 - 2x + 50$, where y is the height in metres above the ground and x is the horizontal distance in metres from the platform.
Point C is 70 metres horizontally from the y-axis. The platform is 5 m long.

a. State the coordinates of points A and C.

2 marks

b. Calculate the coordinates of B (assume B is at ground level).

2 marks

c. Write an expression in terms of x for the **gradient** of the ramp at any point.

1 mark

d. Find the gradient of the ramp when the skier is

i. 10 m horizontally from the y-axis.

1 mark

ii. at point C.

1 mark

e. At what angle (to the nearest degree) to the horizontal does the skier leave the ramp?

2 marks

f. Calculate the cross-sectional area enclosed

i. directly under the horizontal platform.

1 mark

ii. from the entire curved surface of the ramp to the ground.

3 marks

To project the skiers to a higher level, the designers of the ramp were able to change the path of the ski run from point B to point C by raising point C to 10 metres above the ground. This change reduced the horizontal length of the run by 2 metres, and created a new path from point C to point D which followed the equation

$$y = A(x - 50)^3$$

g. Determine the exact value of A (in fraction form).

2 marks

h. Determine the exact new area under the ramp from C to D.

3 marks
Total 18 marks

Question 2

A colony of foxes was established on a secluded island. Due to food constraints the island can only support a certain number of foxes.

A model for the number of foxes (N) at any time (t months) after their introduction is

$$N = \frac{2000}{1 + 24e^{-0.1t}}$$

a. What was the initial population of foxes?

1 mark

b. What was the theoretical maximum population of foxes according to this model?

1 mark

c. What was the fox poulation 10 months after the colony was established? (Round your answer to the nearest whole number.)

1 mark

d. Find an expression for the rate of change of population with respect to time.

4 marks

e. Calculate the rate of change (to 2 decimal places) of population after 10 months of colonization.

2 marks

After approximately the 49th month, an infectious disease begins to diminish the population. The model for the population becomes

$$N = 5.8t^2 + Bt + 24200$$

where B is a constant.

It is observed that the population continues to decrease until the 64th month at which time it begins to increase once again.

f. Determine the value of B.

2 marks

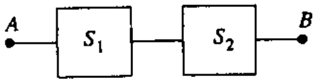
g. Determine the minimum population using the new model (to the nearest whole fox).

2 marks

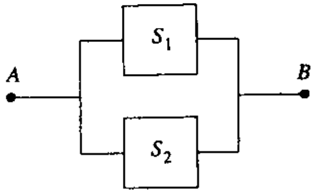
Total 13 marks

Question 3

An electrical signal is sent from A to B for each of the components shown below:



Component X



Component Y

For a signal to be received at B, the following conditions must apply:

Component X Both switches, S_1 and S_2, must work.

Component Y At least one of the switches, S_1 or S_2, must work.

The probability of any switch working is p. It should be assumed that S_1 and S_2 work independently of each other.

a. Show that the probability that a signal at B is received for

i. component X is p^2.

1 mark

ii. component Y is $2p - p^2$.

1 mark

b. If $p = 0.7$,

i. find the probability that a signal is received at B for component Y.

1 mark

ii. and a component is randomly selected, find the probability that component X was selected given that a signal is received at B.

2 marks

c. A batch of 10 component Y parts is randomly selected from the production line. If $p = 0.7$ and the random variable N denotes the number of components in this batch that receive a signal at B, find:

i. $Pr(N = 10)$

1 mark

ii. $Pr(N \geq 9)$

1 mark

iii. $Pr(N \geq 2)$

2 marks

d. **i.** Show that the smallest value of p for which there is at least a 50% chance that at least one component Y (from a batch of 10) will receive a signal at B, satisfies the equation

$$2(1 - 2p + p^2)^{10} \leq 1$$

3 marks

ii. Hence find the smallest value of p.

2 marks

It is found that the life span for component Y is normally distributed with a mean of 110 hours and standard deviation of 4 hours.

e. Find the probability that a randomly selected component Y

i. will last at least 115 hours.

2 marks

ii. will last at least 108 hours but no more than 114 hours.

2 marks

iii. will last at least 115 hours given that it has lasted 111 hours.

2 marks

f. Find an approximate 95% confidence interval for the mean life span of component Y parts.

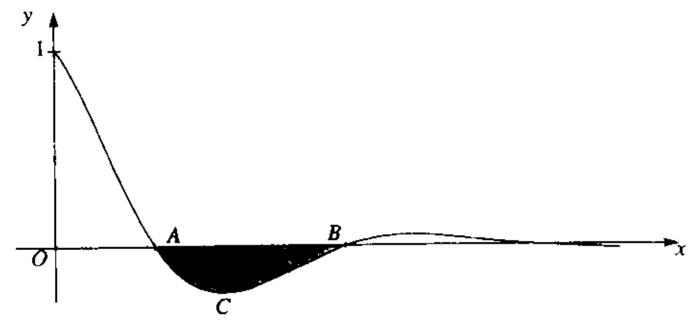
1 mark

Total 21 marks

Working Space

Question 4

Part of the graph of $f : [0, \infty) \rightarrow R$, where $f(x) = e^{-\frac{x}{2}} \cdot \cos x$, is shown below.



a. Find the coordinates of A and B.

2 marks

b. **i.** Show that a stationary point occurs where $\tan x = -\frac{1}{2}$.

4 marks

ii. Find, correct to three decimal places, the coordinates of the first local minimum of f.

1 mark

c. i. Show that if $\frac{d}{dx}\left[e^{-\frac{x}{2}}(a\cos x + b\sin x)\right] = e^{-\frac{x}{2}}\cos x$, then $a = -\frac{2}{5}$ and $b = \frac{4}{5}$.

4 marks

ii. Hence find the exact area of the shaded region in the diagram above.

3 marks
Total 14 marks

END OF QUESTION AND ANSWER BOOKLET