VCE 1999

Board of Studies
Report for Teachers

Mathematics

Cycle 3

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VCE REPORT FOR TEACHERS 1999

Further Mathematics CAT 2: Written examination (Facts, skills and applications task)

GENERAL COMMENTS

The number of students presenting for Further Mathematics CAT 2 in 1999 was 15 359, about a 7 per cent increase over the number who sat in 1998 (14 341). As previously, most students completed three modules in the multiple-choice section and clearly identified these modules. Students need to take care to ensure that this is done properly.

The multiple-choice paper format, as in previous years, generally moved from easier questions to more difficult questions. Two problems occurred in the core multiple-choice paper, a typographic error leading to the word ‘weights’ occurring instead of ‘heights’ in the stem of Question 2 and an ambiguity occurring in Question 7 where differing interpretations lead to two different solutions, both of which were accepted.

Overall, the results were better in multiple-choice responses this year, particularly on the core content.

Multiple-choice solutions

Core

The third column gives the percentage of correct responses.

1	**B**	**73**
2	**A**	**41**
3	**D**	**74**
4	**D**	**44**
5	**B**	**49**
6	**A**	**49**
7	**C, E**	**73**
8	**C**	**62**
9	**A**	**46**

Modules

The third column in each set gives the percentage of correct responses.

Arithmetic and applications

1	**B**	**67**
2	**C**	**78**
3	**C**	**41**
4	**A**	**61**
5	**E**	**43**
6	**D**	**45**
7	**B**	**45**

Probability and statistics

1	**D**	**44**
2	**B**	**60**
3	**C**	**84**
4	**B**	**51**
5	**E**	**45**
6	**C**	**42**
7	**C**	**38**

Geometry and trigonometry

1	**B**	**65**
2	**E**	**54**
3	**E**	**34**
4	**B**	**43**
5	**D**	**34**
6	**A**	**54**
7	**C**	**32**

Graphs and relations

1	**B**	**58**
2	**B**	**88**
3	**E**	**28**
4	**B**	**32**
5	**D**	**26**
6	**E**	**52**
7	**C**	**46**

Business-related mathematics

1	**B**	**42**
2	**D**	**37**
3	**B**	**51**
4	**A**	**48**
5	**C**	**46**
6	**B**	**40**
7	**B**	**39**

Networks and decision mathematics

1	**E**	**72**
2	**B**	**87**
3	**B**	**64**
4	**D**	**59**
5	**C**	**70**
6	**D**	**37**
7	**D**	**12**

As can be noted from the tables, the flow of easy to hard question was not always as might be anticipated; however, it was generally better than the previous year.

Short-answer section

Areas of weakness

As in the previous year, inefficient use of calculators was noticeable. Students should be able to use calculators efficiently to produce required univariate and bivariate statistics. They should also be able to specify results to a required accuracy. Student understanding of decimal place accuracy, clearly needs strengthening.

Marking policy

A marking policy of only 1 penalty mark per paper for decimal place errors was applied to avoid too many marks not being awarded on this account.

Question 1

a. (Average mark 1.62/Available marks 2)

Answer:

Categorical

Numerical

Usually this was answered correctly, yet despite the answers being supplied, some wanted to use others. A handful of responses used direct synonyms and these were expected.

b. (0.69/1)

Answer: 25%

The question called for a percentage so 1/4 was not accepted. It seems that many students still cannot calculate a percentage. A common error was 0.25.

c. (0.7/2)

Answer: 3.42

One mark was awarded in many cases if the population standard deviation was used instead of the sample one, if the sample variance was used or if the sample standard deviation of the rainfall data was given. These were common errors.

Question 2

a. (0.75/1)

Answer: Correct point is at (5, 26)

This was usually well done. Some were ‘off line’ in plotting the point and some reversed the coordinates.

b. (0.52/1)

Answer: 0.866

An answer of 86.6% was also accepted, as both are in common use.

c. (0.24/1)

Answer: Linear

The type of question on a major condition for regression modelling was poorly done again this year. Many students are still adding in extraneous information which detracts from a clear and correct response.

Question 3

a. (0.46/1)

Answer: 3.7

This is the only acceptable answer as it involves directly interpreting the given equation.

b. (0.61/1)

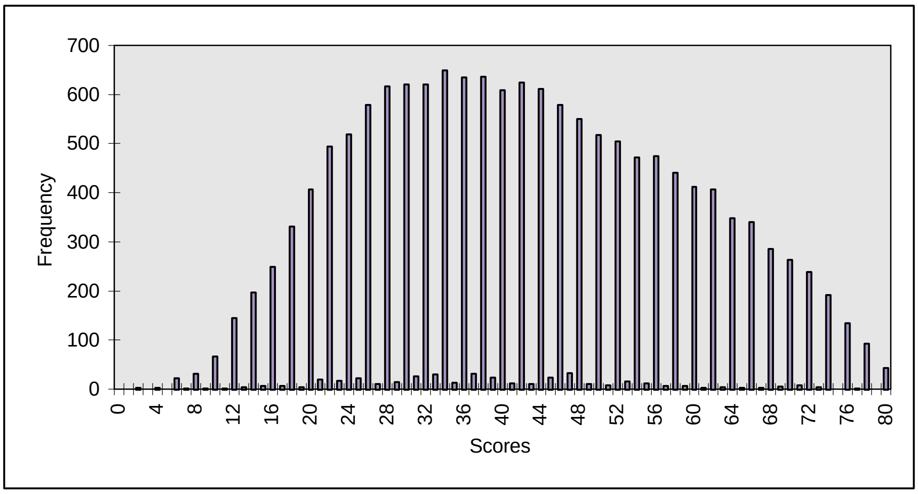
Answer: 52.2

The answer 52 was also accepted here as were consequential solutions based on part a. Most incorrect answers arose through errors in the order of operations. At this level students should be able to correctly carry out simple combinations of arithmetic operations in order.

CAT MA072 MATHEMATICS: Further Mathematics WRITTEN EXAMINATION

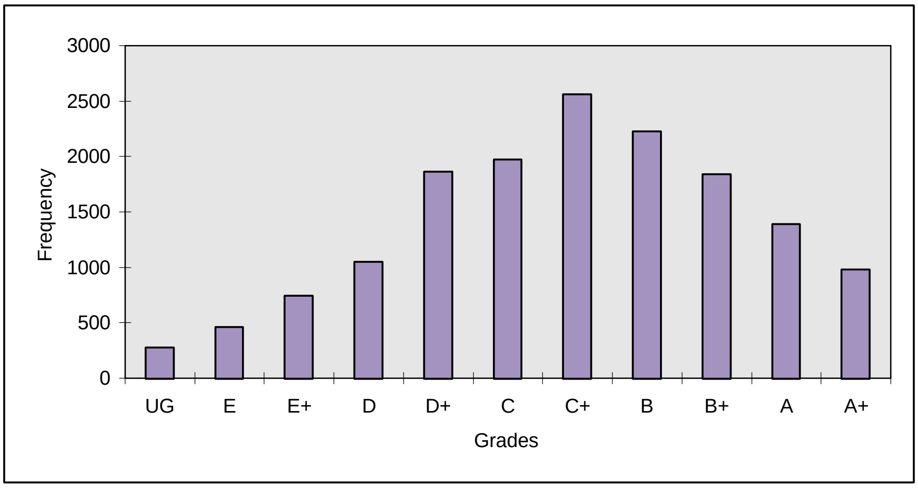
HISTOGRAM OF TOTAL SCORES 1999

Count **15357** Mean **42.14** Standard Deviation **16.38** NA Result **1587**



HISTOGRAM OF TOTAL GRADES 1999

Count **15357** Mean **5.88** Standard Deviation **2.42** NA Result **1587**



ENROLMENTS		**%**
Female	**8819**	**52.1**
Male	**8116**	**47.9**
Total	**16935**	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the 'average' score; that is all scores totalled then divided by the 'Count'.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Further Mathematics CAT 3: Written examination (Analysis task)

GENERAL COMMENTS

The number of students presenting for Further Mathematics CAT 3 in 1999 was 15 369, an increase of about 7 per cent over the 14 325 who sat in 1998. Most students (about 95 per cent) attempted exactly the required three modules. Where students attempted more, the results in all modules were very poor in almost every case.

Students need to understand that they can gain many marks through displaying all their working – this helps check answers and ensures access to consequential and method marks.

The paper format, as in previous years, generally moved from easier questions to more difficult questions. Overall performance in terms of distribution of grades matched that of previous years.

Areas of weakness

Raw results were a little down on those from the previous year with many areas of weakness common to all modules. The balance between modules in terms of difficulty, as reflected by student responses, was better than in previous years.

The inefficient use of calculators is of concern – either inappropriate use or poor use in generating the required accuracy. The number of decimal place errors required a standard rule, during marking, of no more than 1 mark per paper as a penalty.

Simple algebraic manipulations including substitutions generated many errors, although the handling of ratios showed improvement.

It is apparent that students apply methods poorly to anything unfamiliar and their understanding of basic concepts is often not well established. Most students are applying a recipe approach and hoping to get by on easier problems they have seen in similar contexts, without considering sufficiently which processes need to be applied.

Choice of modules

The pattern of module selection matched that of previous years as seen in this table:

Module	1996	1997	1998	1999
1 Arithmetic	7971	7715	8267	8033
2 Probability	1160	2026	2043	2460
3 Geometry & trigonometry	10 830	11 353	11 367	12 697
4 Graphs and relations	6054	5890	6492	7197
5 Business mathematics	9367	8967	9453	10 074
6 Networks	4562	4670	4899	5565

The level of performance tended to be down compared to previous year in Modules 3, 5 and 6 and this marginally affected overall results.

The most common errors in the modules were:

- Arithmetic and applications – difference equations and infinite series caused most problems
- Probability and statistics – results were improved overall but probabilities with values greater than 1 continued to appear and simulations were executed carelessly
- Geometry and trigonometry – many students had great difficulty with most of this work
- Graphs and relations – there was a significant lack of understanding about setting up and using linear programs and even those students able to obtain a feasible region often had little sense of how to use it in optimisation with an objective function
- Business-related mathematics – students often displayed very poor work on basic money problems similar to those that they are likely to meet in everyday life
- Networks and decision mathematics – the understanding of basic definitions and their application was often shaky with harder items causing major difficulties, although this is balanced somewhat by the easier work which often draws on common sense.

Note: the Probability module has been deleted from the Revised VCE Mathematics study.

SPECIFIC INFORMATION

Module 1: Arithmetic and applications

Question 1

ai.

Answer: 480

Usually well done.

aii.

Answer: 12 000

Most completed this satisfactorily.

bi.

Answer: 1.05

Straightforward conceptual work.

bii.

Answer: 12 252

Usually done satisfactorily.

Question 2

a.

Answer: $2n - 1$

Usually done satisfactorily.

b.

Answer: 400

A common error was to get the twentieth term as 39.

Question 3

Answer: 20

Better than in previous years with the most common error being 25, showing the mistake of not seeing 1 + 4 = 5 parts.

Question 4

a.

Answer: 18m.

Usually well done.

b.

Answer: $20 \times 0.9^{n-1}$

Satisfactorily done but this question still caused problems, even for those who could solve numerical parts.

c.

Answer: 82m

Usually answered correctly.

d.

Answer: 200m

Problems with infinite series still occurred.

Question 5

a.

Answer: 1.01

Often this was very badly done despite being a recurrent theme in examinations. Students find translating the written information into equation form very difficult and more practice on this type of question is required.

b.

Answer: 29 800

Quite often had to be sequentially marked due to an error being made in the previous part.

c.

Answer: 27 463.50

There was little comprehension of formula demonstrated in this part.

Module 2: Probability and statistics

Question 1

a.

Answer: 1.2

Generally satisfactorily done.

b.

Answer: 0.0081 or 0.3^4

Most students had only a limited sense of binomial probability.

c.

Answer: ${}^4C_2 0.3^2 0.7^2 = 0.2646$

Not well done, with recurring difficulties with binomial probability.

d.

Answer: 0.3483

As in parts b. and c. there was often little or no sense of binomial probability evident.

Question 2

Answer: 0.216

Too many students did not note that 17 minutes in a trip is one stop.

Question 3

a.

Answer: (for example)

0, 1, 2
3–9

These required any suitable complementary combination of digits and many students had trouble with this.

b.

Answer: (for example)

1	17

Usually satisfactorily done although a lot of consequential marks had to be awarded.

Question 4

a.

Answer: $\Pr(x = 3) + \Pr(x = 4) = 0.1 + 0.1 = 0.2$

Very poorly answered for such a question and it seems that students do not read questions carefully enough.

b.

Answer: 1.3

This was done better than part a., as usual with expected values.

ci.

Answer: 1 and 3

The numbers were easily supplied.

cii.

Answer: 1.25

Generally well done.

Module 3: Geometry and trigonometry

Question 1

ai.

Answer: 2 m

Units were a problem and were often omitted. It seems students may 'think' 200 cm by working in cm but write 200 or 200 m. Unit conversion is an important skill in this topic, and should be covered carefully.

aii.

Answer: 115 cm

Generally done satisfactorily.

b.

Answer: 1 100 cm^2

Poorly done, with a very common error being not to square the 200 and writing 4400/200 = 22 cm^2.

Question 2

a.

Answer: 10.625

Generally done satisfactorily.

b.

Answer: 2.87

Usually well done.

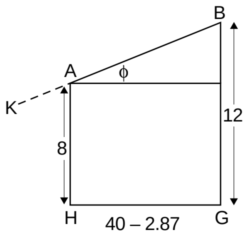
c.

Answer = 6 degrees

This was often not attempted.

$$\tan\phi = \frac{12-8}{40-2.87}$$

ϕ = 6°15' or 6° to the nearest degree



d.

Answer: 20 degrees

A failure to get the angle complement was a common error.

Question 3

ai.

Answer: 108 degrees

Usually well done.

aii.

Answer: 36 degrees

An easy question and usually correctly done.

bi.

Answer: 4.702

Most students used a correct area formula.

bii.

Answer = 34.026

This question was often not attempted. This was the challenge of the paper but proved too hard for many students – most could see that they could add the triangle areas but did not know how to proceed with their pentagon. Students who saw that triangles could be used were given some credit.

Choosing some appropriate triangles, the total surface area is found to be

$$\frac{3}{2}\times 16\times\sin 36° + \frac{1}{2}(x+4)^2\sin 108° \text{ where } \frac{x}{\sin 36} = \frac{4}{\sin 72}$$

giving x = 2.472 approx.

Module 4: Graphs and relations

Question 1

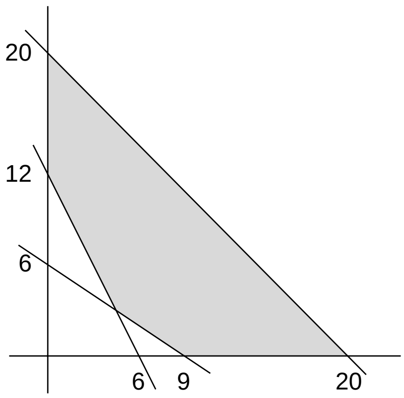
a.

Answer: $2x + 3y \geq 18$

Errors usually made on the coefficients but sometimes on the inequality.

b.

Answer:



A mark per constraint line and 1 for axes and 1 to indicate feasible region.

c.

Answer: 7.5

Often not well done, with a considerable number of consequential marks given. The sliding line technique or vertex testing was expected.

Solving $2x + y = 12$ and $2x + 3y = 18$ simultaneously gives $x = 4.5$ and $y = 3$ so $x + y = 7.5$.

d.

Answer: $2x + y \geq 14$; $2x + 3y \geq 22$

e.

Answer: 14 hours

Not done well as the combination of changes proved too hard for many students. The key issue was identifying that constraint and objective were parallel. All points on the border $x + y = 14$ were solutions.

Question 2

ai.

Answer: $120 + 13n$

Usually done satisfactorily.

aii.

Answer: $20n$

Most did this well.

b.

Answer: $n \geq 18$

Usually well done even allowing for the whole number solution. The result arises from solving $20n \geq 120 + 13n$, i.e. the requirement that revenue exceed costs.

Module 5: Business-related mathematics

Question 1

a.

Answer: 2723.25

An easy and well answered question.

b.

Answer: 7.71

Many students did not divide by 12.

c.

Answer: 2499.43

Not well done. The minimum monthly balance $y is found by solving the equation $y \times \frac{3.5}{12}\% = 7.29$. Note that the July interest dictates the answer not the incoming balance.

Question 2

a.

Answer:

11 700.72	1700.72
8775.72	1275.72

One mark for correct row or column. Some students confused total repayments with total number of repayments.

b.

Answer: 11 275.72

Generally well done although sometimes the $2500 was neglected.

c.

Answer: 385.80

Quite well done.

d.

Answer:

11 700.72	1 700.72	385.80	1 314.92

e.

Answer: Option 2

Generally well done.

f.

Answer: n = 36 and R = 1.0009

This was often badly executed. The n was usually not a problem but, the R needed students to think about both monthly adjustment and how to express as a multiple (not a percentage).

g.

Answer: $326.44

This needs inversion of the formula with $A = 0$. Students often do this even if they do not set $A = 0$ in part a. The link is not always immediately clear to students but the need to set $A = 0$ to save effort is more evident here.

Question 3

a.

Answer: $50 000

Straightforward.

b.

Answer: $46 948.23

Poorly done. The table had to be used to get a ratio and this ratio then correctly used.

c.

Answer: 9th year

This question was usually not done well. Most who did it managed by continuing the table using prediction rather than by the use of the equation.

Module 6: Networks and decision mathematics

Question 1

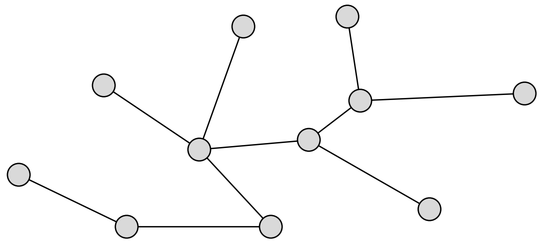
a.

Answer: 750

Generally satisfactory.

b.

Answer: The intended answer was the minimum length spanning tree.



Students usually could supply a spanning tree if not the minimal one.

c.

Answer: 2250.

Usually well done but only when based on part b.

Question 2

a.

Answer:

13
13 14
15

Many students made one or more errors.

b.

Answer: 19 weeks

Well done, but a common error was 18 weeks.

c.

Answer: BEFHJ

Not well done for this type of question.

d.

Answer:

C	6
D	1
G	1
I	1

Most students did this well if they could do part c.

Question 3

a.

Answer: 13 weeks

Usually done well and a table was a good way to identify the answer.

Activity	**EST**	**LST**
A	0	0
B	0	1
C	4	9
D	5	5
E	4	5
F	7	7
G	7	8
H	9	9
I	9	10
J	12	12

b.

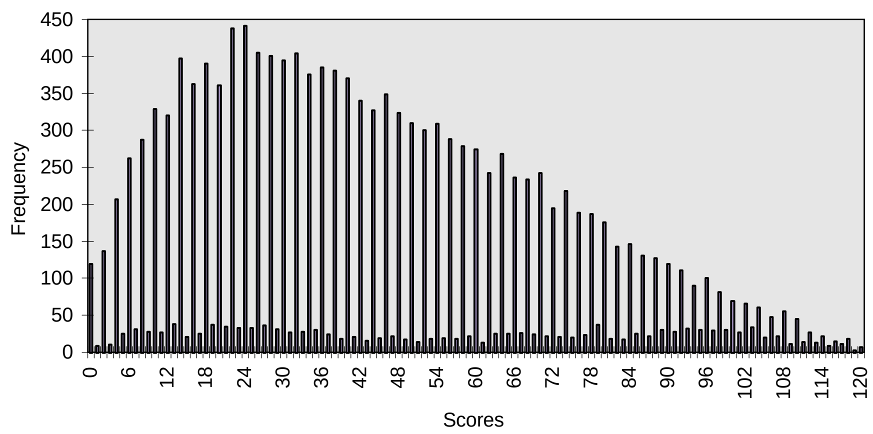
Answer: 80 000

Very few students gave this answer. It required a reworking of the problem and to notice where savings in time could reduce costs. This could be done by paying for 5 weeks for A at $10 000 a week and 6 weeks of B and E (in suitable combination) at $5000 each. It was important that expenditure only be made on activities where an overall advantage in time was to be gained.

CAT MA073 MATHEMATICS: Further Mathematics WRITTEN EXAMINATION

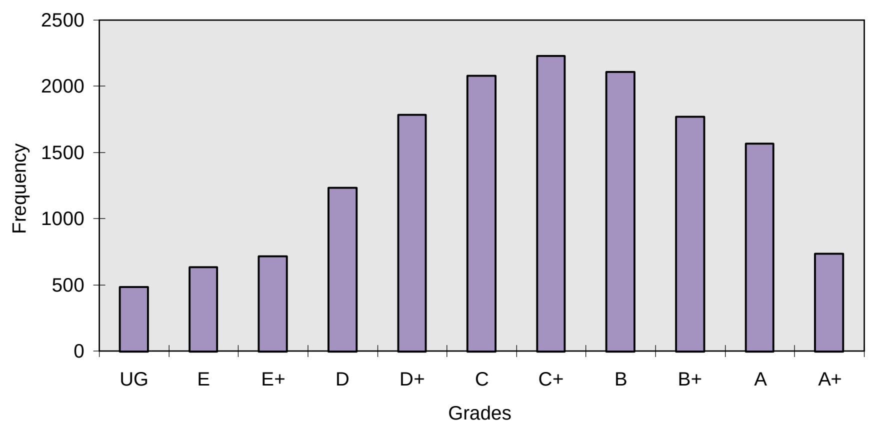
HISTOGRAM OF TOTAL SCORES 1999

Count **15329** Mean **44.97** Standard Deviation **27.59** NA Result **1615**



HISTOGRAM OF TOTAL GRADES 1999

Count **15329** Mean **5.68** Standard Deviation **2.53** NA Result **1615**



ENROLMENTS		**%**
Female	**8819**	**52.1**
Male	**8116**	**47.9**
Total	**16935**	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

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Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Mathematical Methods CAT 2: Written examination (Facts, skills and applications task)

GENERAL COMMENTS

The number of students who sat for the 1999 examination was 17 349. Just over 10 per cent scored 90 per cent or more of the available marks (almost twice as many as 1998), with 146 students scoring full marks on the paper (almost three times as many as 1998). This is a very pleasing improvement on overall performance in recent years. Just over 50 per cent of the students were able to obtain 50 per cent of the available marks on the paper.

As has become the pattern, students performed reasonably well on the multiple-choice questions. The short-answer section was designed to be accessible to all students and there was a clear improvement in students' abilities to answer these questions. The paper appeared to be of a good length as there was no clear evidence that students appeared to run out of time in completing the short-answer questions.

Poor algebraic skills were again evident, with far too many students unable to cope with expanding a simple polynomial or use the factor theorem. Many students were unable to recognise the difference between average and instantaneous rate of change – this important concept needs to be addressed more effectively.

Another concern is the presentation of mathematics in students' responses. Some very careless mathematics was seen in the definition of integrals and intervals. Students need to pay attention to the logical, clear and precise presentation of mathematics expected at this level.

It was evident in a number of student responses that graphics calculators were being employed to answer questions. The issue here is that if the answer given is incorrect, no marks can be awarded for the question unless some suitable supporting evidence is included. Students need to be aware of this, particularly if there is more than 1 mark available for the question.

SPECIFIC INFORMATION

Multiple-choice questions (Total 33 marks)

Question number	Correct answer	Percentage correct	Question number	Correct answer	Percentage correct
1	B	83	18	D	50
2	E	45	19	A	59
3	A	69	20	C	57
4	E	82	21	D	61
5	C	43	22	B	66
6	E	61	23	E	77
7	C	55	24	A	74
8	D	71	25	B	84
9	A	82	26	D	47
10	B	71	27	C	56
11	D	50	28	C *or* E	53
12	B	78	29	B	12
13	D	35	30	C	56
14	A	31	31	E	53
15	D	70	32	A	41
16	A	69	33	B	38
17	E	45			

Short-answer questions (Total 17 marks)

Question 1 (Average mark 1.44/Available marks 2)

Correct response:

Period = 8 (1 mark)

Amplitude = 1 (1 mark)

Many students were able to obtain the correct answers. However, some confused amplitude and period, with period frequently seen as $\frac{\pi}{4}$, and amplitude as 1.5. A common misconception was to confuse n with the period.

Question 2 (0.89/2)

Correct response:

$\Pr(X > d) = 0.25$

$\Pr\ (X > (d - 80)/3) = 0.75$

$(d - 80)/3 = 0.674$

$d = 82$

Many students were confused with issues such as whether d was a probability, using 0.25 as a value of X, and how to use the cumulative normal distribution. Nearly all students recognised the need to change to the standard normal distribution. A frequent incorrect response was $(d - 80)/3 = -0.6745$, $d = 78$ m. Many responses showed little appreciation of the practical situation. Although some students answered the question successfully using a graphics calculator, many did not and were unable to be awarded any marks as they showed no working at all.

Question 3 (1.01/2)

Correct response:

se = $\sqrt{(0.75 \times 0.25/300)} = 0.025$

95% confidence interval $\quad 0.75 \pm 2 \times 0.025$

$0.70 \le p \le 0.80$

Many students ignored, or did not understand, the term ‘proportion’. Answers were often given as ‘the number in favour’ instead of ‘proportion in favour’. A large number of students were unable to correctly use the formula for standard error. Common mistakes were to forget the square root, dividing by 300 after taking the square root or using $p = 225$. Understanding of 95 per cent confidence interval was reasonably good.

Question 4 (2.05/4)

a. Correct response:

$3x(2x - 5)^3 = 3x(8x^3 - 60x^2 + 150x - 125)$

$= 24x^4 - 180x^3 + 450x^2 - 375x$ (2 marks)

For a question that was based largely on material from Mathematical Methods Units 1 and 2 this was not handled well. The mistakes that were frequently seen indicated a poor understanding of basic algebraic processes, such as expansion. For part a., many students tried unsuccessfully to expand all the brackets instead of using the standard cubic expansion pattern. Often the $3x$ term was multiplied into the bracket and then the binomial theorem was applied, usually incorrectly. In using either the binomial theorem, or cubic expansion it was common to see the coefficient ignored and all terms become positive.

b. Correct response is:

$P(2) = 8 + 8 + 2a - 2 = 0$

$14 + 2a = 0$

$a = -7$ (2 marks)

For part b., many students attempted to use long division without success but for a very small number of cases. It appeared that many had no idea of the remainder theorem. Those who did usually obtained the correct answer quickly and neatly. Some used P(-2) rather than P(2).

Question 5 (1.27/3)

It was not uncommon to see the same response to both parts a. and b. of the question, with no recognition by the students that two different things were being asked for.

a. Correct response: (1 mark)

$$\text{average rate of change } = \frac{V(10) - V(0)}{10 - 0} = \frac{1000 - 250 - 1 - 1000}{10} = -25.1$$

For part a., quite a few students obtained the correct answer by incorrect means; usually to average $V'(t)$ every minute, or averaging $V'(10)$ and $V'(0)$. The most frequent incorrect response was to give the instantaneous rate of change. Some students also averaged using $V'(0) - V'(10)$ and therefore found the negative of the required answer, while some simply ignored the negative. Many good students recognised the negative as indicating that the volume was decreasing.

b. Correct response:

$V'(t) = -25 - 0.02t$

when $t = 10$, $V'(10) = -25.2$ (2 marks)

For part b., many students obtained the correct derivative and the correct answer. Some students, however, were unable to correctly substitute into the correct derivative.

Question 6 (part a. 0.71/2 and part b. 0.76/2)

Good students were able to complete this question in a precise and efficient manner, but those who could not integrate correctly involved themselves in complicated and time consuming calculations.

a. Correct response:

$$\text{Area } = \int_0^p \frac{12}{(x+1)^2}dx = \left[-\frac{12}{(x+1)}\right]_0^p = 12 - \frac{12}{(p+1)} = \frac{12p}{(p+1)}$$

In part a., many students could not correctly write down an expression for a definite integral. Many integrals without terminals were seen, or integrals with terminals such as $p > 0$. There were many poor attempts at finding the integral involving logarithmic functions, or the evaluation of the derivative rather than anti-derivative.

b. Correct response:

$$\frac{12p}{(p+1)} = 6$$

$$\frac{p}{(p+1)} = 0.5$$

$$\frac{(p+1)}{p} = 2$$

$$1 + \frac{1}{p} = 2$$

$$\frac{1}{p} = 1$$

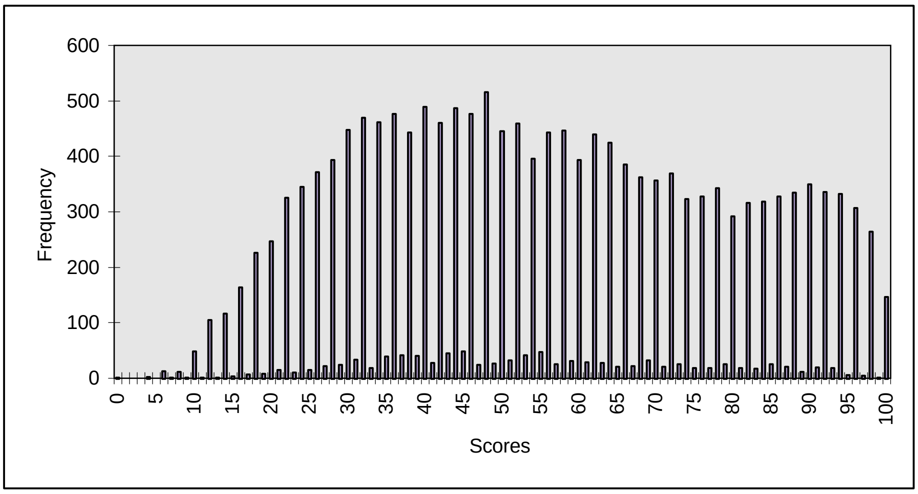
$$p = 1$$

For part b., many students obtained the correct answer. Some students who had failed to obtain any sensible response to part a. still managed to obtain this result. Quite frequently a negative answer was obtained, but this did not concern most students who simply stated $p > 0$, therefore ignoring the negative sign. Most students knew to equate their expression from part a., but were unable to solve the resulting equation. Some students clearly used a graphics calculator to obtain the response and showed no working, but were still able to be awarded 2 marks.

CAT MA082 MATHEMATICS: Mathematical Methods WRITTEN EXAMINATION

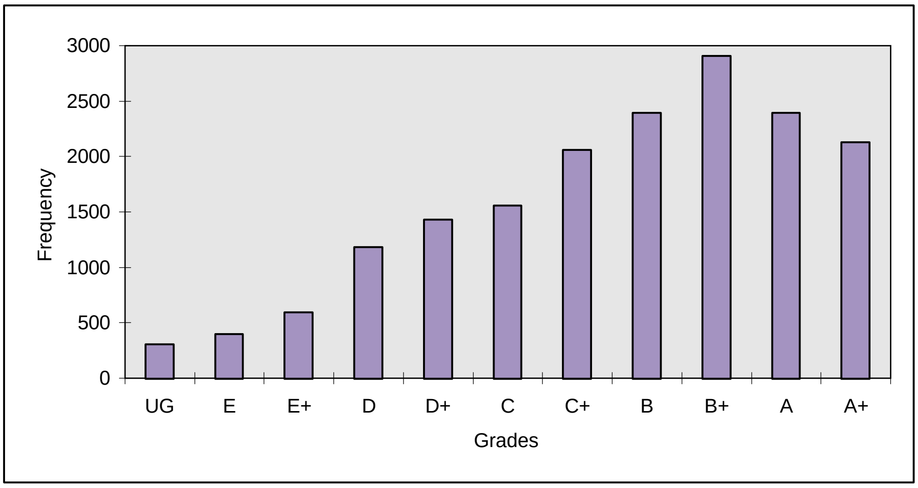
HISTOGRAM OF TOTAL SCORES 1999

Count **17349** Mean **55.47** Standard Deviation **23.19** NA Result **583**



HISTOGRAM OF TOTAL GRADES 1999

Count **17349** Mean **6.56** Standard Deviation **2.55** NA Result **583**



ENROLMENTS		**%**
Female	**8278**	**46.2**
Male	**9640**	**53.8**
Total	**17918**	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the 'average' score; that is all scores totalled then divided by the 'Count'.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Mathematics Methods CAT 3: Written examination (Analysis task)

GENERAL COMMENTS

The entire range of marks from 0 to 55 was present. There continued to be a significant proportion of students who could gain only a very small number of marks, despite there being marks available which required only elementary level work. At the other end of the scale, there were highly competent students who showed their abilities by submitting perfect or near-perfect papers.

Student ability to use algebra effectively continues to be a major weakness. Many students lost marks through carelessness in their algebra and through a lack of understanding of basic algebra and probability. General use of notation, including set notation and use of parentheses in particular, was poor in many scripts.

Students frequently missed out on marks because they failed to answer the specific question, or because they did not answer the question in the way required. This was a particular problem where exact answers or answers correct to a given number of decimal places were required (but approximate answers were given). Students also lost marks because they did not read questions carefully enough, such as in Question 2c, some students did not find both intercepts; in Question 2g, the first instruction was often ignored and in Question 3ci, the term **net loss** was interpreted incorrectly.

The use of graphics calculators by students was generally sensible. Some students attempted to transfer a graph from their calculator to answer Question 2f, frequently omitting scales, labels and asymptotes, and there was evidence of degree mode calculations in Question 4.

SPECIFIC COMMENTS

Question 1

	Average mark	Available marks
a.	0.66	1
b.	0.62	1
c.	0.60	1
d.	0.70	2
e.	0.61	1
f.	0.42	1
g.	0.91	3

This question involved application of mathematics based mainly on work from the Coordinate geometry and Algebra areas of study. However, more than one third of students failed to gain the mark in each of the first three parts of the question due to poor algebra. A common incorrect answer to part b., from the correct equation $50 = \frac{A}{10} + 2$, was 498, demonstrating that some students struggled to solve basic linear equations. Many students failed to interpret ‘adjacent sides’ correctly in part d. In part g., the first mark was awarded if the student demonstrated that they could differentiate the correct function, or one of similar difficulty, using the product rule. Less than half the students were awarded this mark.

Question 2

	Average mark	Available marks
a.	0.47	1
b.	1.07	2
c.	1.23	2
d.	1.20	2
e.	1.70	3
f.	0.67	2
g.	0.85	3

The early parts of this question were similar in style to Question 2 on the 1998 paper and it was apparent that the better students were familiar with this and handled it fairly well. However, it is also clear that some students had not mastered the skills involved in a question of this kind, particularly the transformation work.

While part a. appeared to have been answered better than the equivalent question in 1998, the correct use of notation is still an area in need of significant improvement. Teachers should be aware that correct set notation is expected in stating the domain of a function. Despite comments in the *1998 Report to Teachers*, little improvement was noted in description of transformations in part b. This should be a standard question, similar to questions practised during the year. Many students did not write coordinates for the intercepts in part c.

It was clear that most students were aware of instructions to use calculus where the question required this. Little was seen of students using calculators to find the gradient of a curve at a given point in part d. and over half the students were able to obtain full marks. Most others found difficulty in differentiation of a logarithmic function.

The first mark of part e. was awarded for a demonstration of a knowledge to interchange the *x* and *y* in order to find the rule for the inverse function. This mark was the most frequently awarded mark on the whole paper.

About half the cohort were unable to get any marks for part f. mostly because they did not have a rule for the inverse function from part e. to graph (even the wrong rule graphed correctly gained marks). Many students lost marks because they did not include the asymptote or indicate its equation, either specifically or by an appropriate scale on the *y*-axis. Others lost marks because there was no indication of scale on each axis, because too little of the graph was drawn or because the asymptotic behaviour of the graph was not indicated correctly.

In part g. conceptual analysis was required. It was necessary to realise that the area enclosed by the graph of a function and the *x*-axis is the same as the area enclosed by the graph of the inverse function and the *y*-axis, as a direct result of one graph being the reflection of the other in the line $y = x$. Many students realised this but there were few who carried the integration to completion.

Question 3

	Average mark	Available marks
a.	1.88	5
b.	1.21	2
c.	1.13	3
d.	0.84	3

Part ai. was handled competently by most students, but part aii. was poorly done with a majority of students scoring zero. Many answers of less than 1.4 were given and a common error was a misreading of the cdf table supplied, giving 1.36 instead of 1.036. The probability was handled reasonably well in part aiii., if at all and the need for binomial distribution was generally recognised in part b., but poor arithmetic and a poor reading of the question (four decimal places, exactly four computers) frequently contributed to wrong answers and a subsequent loss of marks. There was evidence of a larger number of students using graphic calculators to answer questions about normal and binomial distributions and usually doing this well.

In part c., some students failed to interpret a profit as a negative loss, but most handled i. well. In ii., there were few students who recognised the conditional probability condition, despite the question saying ‘given that …’. There was a large number of ‘no attempts’ to part d. Those students who knew what to do handled the question well and those who provided themselves with a formula (for example, $n = \frac{4pq}{m^2}$) for this fairly standard question gave themselves an advantage.

Question 4

	Average mark	Available marks
a.	0.51	1
b.	0.35	1
c.	0.99	5
d.	1.41	4
e.	0.68	4
f.	0.27	2

Despite the fact that most students attempted part a., the standard of explanation was generally poor. A number of ingenious but incorrect explanations were seen, as some students seized the opportunity to gain a mark for which the answer was given ($W = 20$), e.g. ‘width = length/2 = 40/2 = 20.’

Lack of use of brackets in part b. cost many students the mark here; $\sin\left(\frac{\pi x}{10}\right) - \cos\left(\frac{\pi x}{10}\right) - 3$ is **not** the correct answer, even if a student uses a correct expression in later parts of the question. Communication is an important part of mathematics, for which there are a number of standard conventions, of which the use of brackets is one.

In part c., differentiation of the correct expression (or the one given above) leads to an equation, the solution of which is included in the Study Design. Some students chose to solve this with a graphics calculator but then failed to select the solution which was required. Students who found the minimum using a graphics calculator without writing down the correct derivative received no marks. Many students, having found $x = 17.5$ correctly, failed to answer the question by finding the minimum distance.

Part d. was well done, but many students lost marks through careless algebra and antidifferentiation, and use of brackets was poor. A large number of students failed to get a successful start on part e., with the correct integrand less common than an incorrect one. The suggested volumes of gold ranged from very small (a negative sometimes appeared) to very large (over 4000 m^3 – a mass of over 80 000 tonnes). Part f. was only attempted by students who had some success with part e. and the correct answer was seen on about 10 per cent of scripts.

4a. Width, W, is given by the period of $\sin\left(\frac{\pi x}{10}\right)$

(or $\cos\left(\frac{\pi x}{10}\right)$)

$= 2\pi \div \left(\frac{\pi}{10}\right) = 20$

b. $D = \sin\left(\frac{\pi x}{10}\right) - \cos\left(\frac{\pi x}{10}\right) + 3$

c. $\dfrac{dD}{dx} = \dfrac{\pi}{10}\cos\dfrac{\pi x}{10} + \dfrac{\pi}{10}\sin\dfrac{\pi x}{10}$

= 0 for minimum

$\Rightarrow \tan\left(\dfrac{\pi x}{10}\right) = -1$

$\Rightarrow \dfrac{\pi x}{10} = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, \ldots$

$\Rightarrow$ x= –2.5, 7.5, 17.5, …

From graph, minimum is for x ε (10, 20), so minimum is when x = 17.5

When x = 17.5, D = 3 – √2 (exact value).

d. $A = \displaystyle\int_0^{20} \left(\sin\left(\frac{\pi x}{10}\right) - \cos\left(\frac{\pi x}{10}\right) + 3 \right) dx$

$= \left[-\dfrac{10}{\pi}\cos\left(\dfrac{\pi x}{10}\right) - \dfrac{10}{\pi}\sin\left(\dfrac{\pi x}{10}\right) + 3x \right]_0^{20}$

$= -\dfrac{10}{\pi}\cos(2\pi) - \dfrac{10}{\pi}\sin(2\pi) + 60 + \dfrac{10}{\pi}\cos(0) + \dfrac{10}{\pi}\sin(0)$

$= 60.$

e. $V_{seam} = 40\displaystyle\int_0^{20} \left(0.2 - 0.002(20 - x)^{1.5}\right) dx$

$V_{gold} = \dfrac{40 \times 0.2}{100}\left[0.2x + \dfrac{0.002(20 - x)^{2.5}}{2.5} \right]_0^{20}$

$= 0.08\left(4 - \dfrac{0.002 \times 20^{2.5}}{2.5} \right)$

≈ 0.08 × 2.568916

= 0.206m^3, correct to 3 decimal places.

f. Area of gold

$= \displaystyle\int_{15}^{20} \left(0.2 - 0.002(20 - x)^{1.5}\right) dx$

$= \left[0.2x + \dfrac{0.002(20 - x)^{2.5}}{2.5} \right]_{15}^{20}$

≈ 0.9553

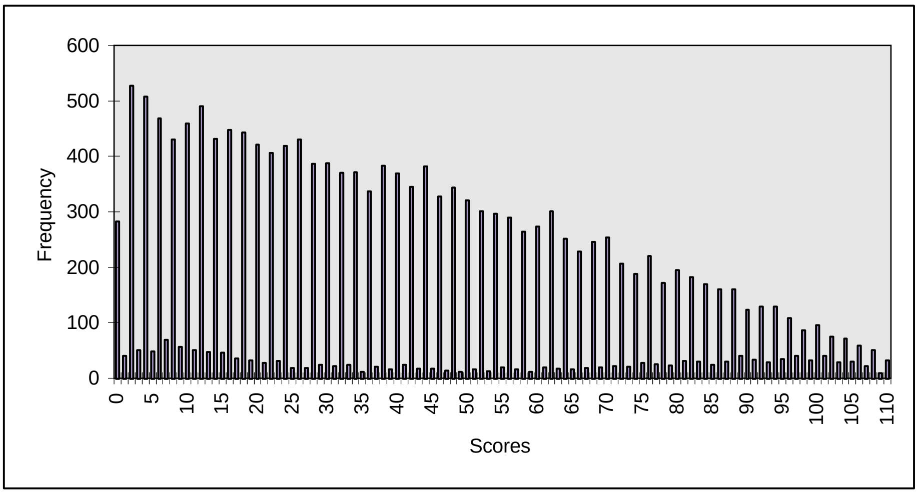
As a percentage of the total this is $\dfrac{0.9553 \times 100}{2.568916} = 37\%$

correct to the nearest per cent.

CAT MA083 MATHEMATICS: Mathematical Methods WRITTEN EXAMINATION

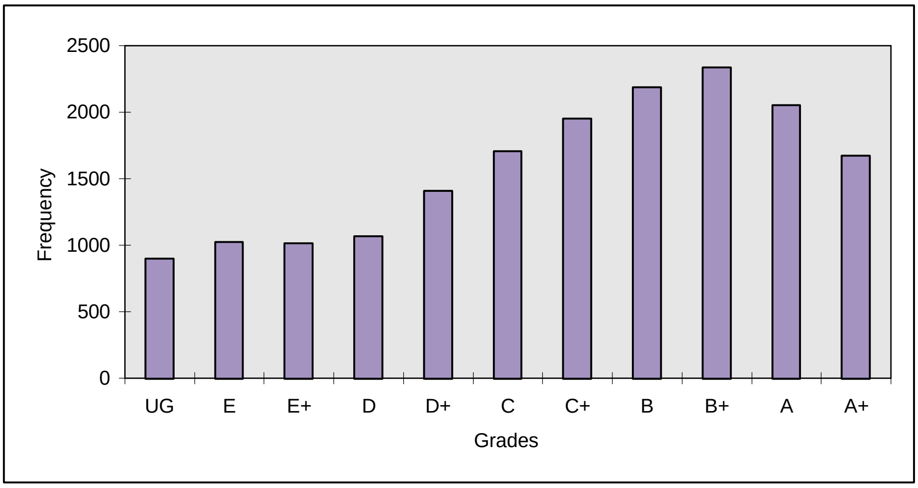
HISTOGRAM OF TOTAL SCORES 1999

Count **17327** Mean **41.13** Standard Deviation **28.72** NA Result **605**



HISTOGRAM OF TOTAL GRADES 1999

Count **17327** Mean **5.85** Standard Deviation **2.92** NA Result **605**



ENROLMENTS		%
Female	8278	46.2
Male	9640	53.8
Total	17918	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

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Specialist Mathematics CAT 2: Written examination (Facts, skills and applications task)

GENERAL COMMENTS

The number of students who sat for the 1999 examination was 6032, with only 145 students (2.4 per cent) choosing Geometry as their optional module. Just over 3 per cent of students scored more than 90 per cent of the available marks with eight students scoring full marks. It appeared that Part I (Multiple-choice questions) was a little too time-consuming.

Note: the number of multiple-choice questions to be attempted on the Specialist Mathematics Examination 1 from year 2000 will be reduced to 30.

As continues to be the case, the standard of responses to Part II (Short-answer questions) covered a wide spectrum. For example, there were scripts that were:

- concise, legible and accurate, earning full, or nearly full, marks
- marred by use of clumsy methods, careless slips, poor handwriting and untidy setting out
- littered with serious errors, or such fragmentary answers, that few (if any) marks were scored.

Some scripts were outstanding; however, many students produced very 'sloppy' work.

Some students attempted very little, if any, of Part II and teachers should ensure that students are given instruction on the manner in which they should manage their time during the examination. As a rough guide, time should be distributed between Parts I and II in direct proportion to the total marks available for each.

The gender breakdown of the 6032 students was 2222 females (36.8 per cent) and 3810 males (63.2 per cent). Analysis of scores for each question by gender showed that girls generally performed better than boys on the core sections (A and C) and Geometry items, whereas performance on the Mechanics questions was about the same. This is shown in the following table:

Part I

Section		No. of questions	Female mean > Male mean	Male mean > Female mean
A	Core	22	15	6
B	Mechanics	11	5	6
	Geometry	11	7	4

Part II

Section		No. of questions	Female mean > Male mean	Male mean > Female mean
C	Core	4	4	0
D	Mechanics	2	1	1
	Geometry	2	2	0

The most significant difference between the performance of girls and boys on any question occurred for Question 22 in Section A. This was answered correctly by 42 per cent of boys, but only 27 per cent of girls.

SPECIFIC INFORMATION

Part I: Multiple-choice questions

Section A

Question number	Correct answer	Percentage correct	Question number	Correct answer	Percentage correct
1	E	74	12	D	68
2	B	66	13	A	40
3	E	25	14	E	43
4	C	82	15	A	65
5	E	70	16	B	85
6	D	73	17	A	49
7	C	50	18	C	75
8	C	92	19	D	46
9	C	72	20	B	27
10	A	65	21	A	15
11	B	63	22	D	37

Section B

Module 1: Mechanics			***Module 2: Geometry***		
Question number	Correct answer	Percentage correct	Question number	Correct answer	Percentage correct
1	B	66	1	D	44
2	C	66	2	C	50
3	B	78	3	B	68
4	C	54	4	B	65
5	D	52	5	D	58
6	A	54	6	E	23
7	A	60	7	C	35
8	D	37	8	E	17
9	E	83	9	A	52
10	E	55	10	C	46
11	C	37	11	A	28

Questions 3, 20, 21 and 22 in Section A were each answered correctly by less than 40 per cent of students. Question 3 was answered correctly (E) by only 25 per cent of students with alternative D being the most popular choice (35 per cent) – most students ignored the advice that x was in the fourth quadrant.

Only 27 per cent of students answered Question 20 correctly (B), with just over half (51 per cent) apparently either omitting the integration constant altogether, or evaluating it as 0, and hence selecting C as the answer. Question 21 had the lowest success rate (A: 15 per cent), with just under half (46 per cent) choosing B and just under one quarter (23 per cent) choosing E. Many students expected that 6 (the number of grams of chemical initially undissolved) should appear in the answer and both B and E satisfied this criterion as well as having the correct sign for the x term. Teachers should occasionally include redundant information in practice questions of this type.

Question 22 was expected to be relatively difficult, yet 37 per cent of students selected the correct (discontinuous) graph (D). This was also the most popular choice, with E (28 per cent) being the only other 'popular' choice. Students should be able to describe for any given velocity-time graph, a situation that could produce that graph.

Questions 8 and 11 were the only Mechanics module questions answered correctly by less than 50 per cent of students. In both cases, the correct answer was the most popular choice and was chosen by 37 per cent of students. For Question 8, 31 per cent of students gave the length in metres (A) as the answer, apparently not noticing that the length in centimetres (D) was required. The most popular incorrect answer for Question 11 was B (24 per cent).

Note: although Mechanics is a compulsory area of study in 2000, these two questions, and Question 7, could not be asked on Examination 1 because simple harmonic motion is no longer a designated topic. Similarly, Question 2 concerns a topic (conservation of momentum) which has been omitted in the revised course.

Six of the 11 questions in the Geometry module had success rates less than 50 per cent including four that had success rates less than 40 per cent. Two of these, Questions 6 and 7, concern 'Representation of relations and regions in the complex plane' which is a compulsory part of the course in 2000. The only other question that could be asked in 2000 is Question 5, another question on this topic.

For Question 6, responses were fairly evenly spread between the correct answer (E) and the other four choices. Students need to be familiar with the various forms in which the equation of a circle in the complex plane can be written. The correct answer (C) was clearly the most popular choice (35 per cent) for Question 7, with the remaining responses divided fairly evenly between the other four choices. Students should be able to answer such a question without needing to find the cartesian equation of the locus.

Part II: Short-answer questions

Question 1 (Average mark 0.90/Available marks 2)

Answer: $\log_e(2 + \log_e x)$

Generally well done by those students who recognised that if $u = (2 + \log_e x)$ or $\log_e x$, then $\frac{du}{dx} = \frac{1}{x}$. The most common error was to try and express $\frac{1}{\log_e(2+\log_e x)}$ in partial fraction form.

Question 2 (1.23/ 3)

Answer: $\log_e 3$

Reasonably well done by those students who knew to use partial fractions. The most common error made by these students was to write $\int \frac{1}{1-x}dx = \log_e(1-x)$ instead of $-\log_e(1-x)$, whilst some left the answer as $-\log_e \frac{1}{3}$. A large percentage of students did not use partial fractions, generally either opting for $\int \frac{1}{x(1-x)}dx = \log_e(x(1-x))$ or getting nowhere with the substitution $u = 1 - x$. This was the first time that the use of partial fractions had been required in an antidifferentiation without a prior 'prompt', and students need to practise this type of question.

Question 3 (1.19/ 3)

Answer: $2\,\text{cis}\frac{\pi}{6}, 2\,\text{cis}\frac{5\pi}{6}, 2\,\text{cis}\frac{3\pi}{2}$

The fact that the cube roots were required in polar form should have prompted the use of de Moivre's theorem. However, many students tried to solve $x^3 - 8i = x^3 + 8i^3 = 0$ algebraically and then convert their answers to polar form. Nearly all of these students either had trouble with their factorising or obtained two conjugate zeros for $x^2 - 2ix - 4$. A very high number of students seemed to be confused or unsure about the meaning of 'polar form' and, after using de Moivre's theorem to calculate the roots, converted them to cartesian form as their final answer.

Question 4 (0.74/ 3)

Answer: $c=-\frac{1}{2}$

Many students had difficulty differentiating $x\,\mathrm{Tan}^{-1}\frac{c}{x}$, either because they did not recognise that the product rule was needed or because they did not use the chain rule to differentiate $\mathrm{Tan}^{-1}\frac{c}{x}$ and omitted the $-\frac{c}{x^2}$ term. Many of the students who did obtain the correct identity, $-\frac{cx}{x^2+c^2}=\frac{2x}{4x^2+1}$, got bogged down trying to solve for *c* (this was best done by inspection or by equating coefficients).

Question 5 (1.30/ 2)

Answer: $a=\frac{g}{4}(2.45)$

This question was well done, with many students gaining most or all of their marks on this question. Most students treated the two particles separately, though many considered the system as a whole. The most common error was being inconsistent with the sign of *a* when treating the particles separately – taking 'up' as positive for one particle but 'down' as positive for the other particle.

Question 6 (0.83/ 4)

Answer: 18 N/m

This question was done poorly, with many students even regarding the spring as hanging vertically, complete with diagram. Most students incorrectly used constant acceleration formulas to solve for the stiffness of the spring.

Note: due to the changes in the course, this question could not be asked in 2000.

Question 7 (0.57/ 2)

Answer: a. Line with equation Im (z) = 1

b. Region specified by {z: (Im (z) ≥ Re (z)) and (Im (z) ≥ 0)}, but excluding *O*.

Part a. was done reasonably well by letting $z = x + yi$ and obtaining $y = 1$. Attempts at part b. generally either stopped at drawing the ray $\mathrm{Arg}(z)=\frac{\pi}{4}$ or proceeded to shade the region where $0\le \mathrm{Arg}(z)\le\frac{\pi}{4}$.

Note: the origin is not included in the correct region since Arg 0 is not defined.

Question 8 (0.57/ 4)

Answer: a. $\left|\overrightarrow{OC}\right|^2=\left(\underset{\sim}{a}+\underset{\sim}{b}\right)\cdot\left(\underset{\sim}{a}+\underset{\sim}{b}\right)=a^2+2\underset{\sim}{a}\cdot\underset{\sim}{b}+b^2$;

$\left|\overrightarrow{AB}\right|^2=\left(\underset{\sim}{b}-\underset{\sim}{a}\right)\cdot\left(\underset{\sim}{b}-\underset{\sim}{a}\right)=a^2-2\underset{\sim}{a}\cdot\underset{\sim}{b}+b^2$

b. $OC = AB \Rightarrow OC^2 = AB^2$, i.e. $\left|\overrightarrow{OC}\right|^2=\left|\overrightarrow{AB}\right|^2$. Now show that this means $\underset{\sim}{a}\cdot\underset{\sim}{b}=0$.

Hence *OACB* is a rectangle because it is a parallelogram with right angles.

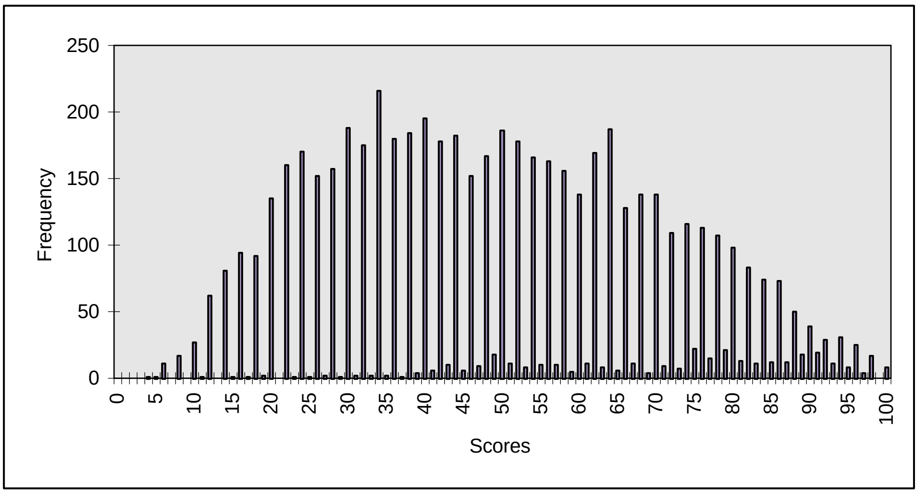
Most students obtained the correct expressions for $\overrightarrow{OC}$ and $\overrightarrow{AB}$ in part a., but did not recognise that they then had to use the scalar product to find $\left|\overrightarrow{OC}\right|^2$ and $\left|\overrightarrow{AB}\right|^2$. Consequently, few students were able to make a sensible attempt at part b.

Note: this question could be asked in 2000 as 'vector proofs of simple geometric results' is part of the revised course.

CAT MA092 MATHEMATICS: Specialist Mathematics WRITTEN EXAMINATION

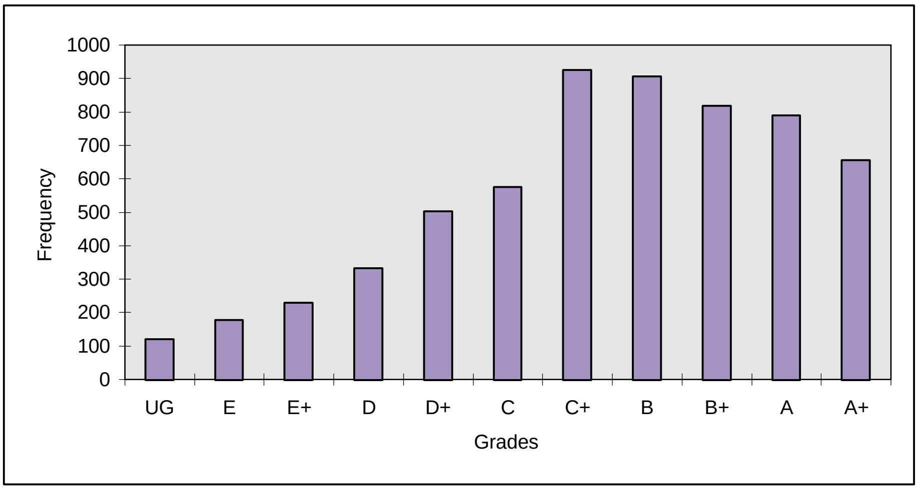
HISTOGRAM OF TOTAL SCORES **1999**

Count **6032** Mean **49.78** Standard Deviation **21.40** NA Result **142**



HISTOGRAM OF TOTAL GRADES **1999**

Count **6032** Mean **6.40** Standard Deviation **2.54** NA Result **142**



ENROLMENTS		**%**
Female	**2269**	**36.8**
Male	**3904**	**63.2**
Total	**6173**	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the 'average' score; that is all scores totalled then divided by the 'Count'.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Specialist Mathematics CAT 3: Written examination (Analysis task)

GENERAL COMMENTS

The number of students who sat for the 1999 examination was 6028 with only 2 per cent of students attempting Question 6 (Geometry) as their optional question. Just over 2 per cent of students scored 90 per cent or more of the marks, with seven students scoring full marks. There were no questions on which students generally got 'bogged down', but parts of questions – especially 2d and 4cii – were only answered correctly by a few students.

About 36 per cent of the students who sat for the examination were female and 64 per cent male. The gender breakdown of the students who attempted optional Question 5 was similar, whereas the breakdown for the students who attempted Question 6 was about 30 per cent female and 70 per cent male. Analysis of scores for each question by gender showed that girls performed better than boys on all questions in Section A and on all parts of Questions 5 and 6 except part e. of Question 5. The most significant difference between the performance of boys and girls was on Question 6 (Geometry), where the female mean was 6.85 (38.0 per cent) out of 18 and the male average was 4.22 (23.4 per cent).

Many students would benefit from improving their 'examination technique', e.g. completing several past papers under examination conditions, getting teacher feedback on their attempted responses. The following general points should be emphasised:

- taking care in writing and setting out to minimise errors caused by students copying down their own work incorrectly from one line to another
- ensuring that calculators are in the correct angle mode each time. For example, for Question 5, degree mode is needed for parts c. and di., but radian mode is needed for part dii.
- showing clearly and logically every mathematical step that leads to a given result when answering 'show' questions, such as Questions 3a, 3b, 4bi, 5di and 6aii
- recognising the need to use results established earlier in the question when answering 'hence' questions such as Questions 1cii and 4cii
- recognising that a 'verify' question involves using a **different** (usually stipulated) method to check a previous answer (e.g. Question 1d), and that the answer must be obtained **in the same form**
- recognising the need to give specific explanations in 'explain' questions such as Questions 4cii and 5e, not general comments like 'Phil will remain stationary if the resultant force down the incline is greater than that up the incline' (Question 5e)
- being aware that solving later parts of questions may require the use of results from earlier parts of that question. A good example is Question 3c where the results given in parts a. and b. are needed in part c.
- noting that given results may be used later in the same question even if you have not been able to prove them yourself.

Although examiners could assume, for the first time, that each student had access to a graphics calculator, no question on the 1999 paper required a graphics calculator to answer it. Nevertheless, graphics calculators could be used to advantage in a number of question parts, most notably 1a, 2c, 4biii, 5dii and 6aiii. Since there was a direction to 'use calculus', no marks were awarded for using a graphics calculator's numerical integration capability to obtain the value of the definite integral in Question 3c – although it would have been sensible for students to check their answers this way.

It is likely that the Specialist Mathematics Examination 2 for the revised study in 2000 will be 'graphics calculator active' and students' attention should be drawn to the sample Examination 2 question published in Supplement 1 to the February 2000 *VCE Bulletin* (p. 92).

Note: students invariably tackle questions involving differential equations, such as $\frac{dv}{dt} = -\frac{v^2+144}{150}$ which arises in

Question 5dii, by first finding a general solution and then substituting in the initial conditions to find the value of the arbitrary constant. This gives them a particular solution, e.g. $t = 1.25\left(\text{Tan}^{-1}(0.575) - \text{Tan}^{-1}\left(\frac{v}{12}\right)\right)$ to use in the remainder of the question. Teachers should ensure that students are aware that it is not always necessary to obtain a general or particular solution. For example, in this question the answer can be obtained directly from the original differential equation by writing $\int_0^t dt = -\int_{6.9}^{0} \frac{150}{v^2+144}dv$. The advantage of this latter method is that the definite integral on the right side can be evaluated using the numerical integration capability of a graphics calculator even if the antiderivative is unknown (in which case solution via the general solution would be impossible).

SPECIFIC INFORMATION

Question 1

a–b. (Average mark 1.54/Available marks 2)

Answers: a. $\text{cis}\left(\frac{-\pi}{4}\right)$; b. $\text{cis}\left(\frac{\pi}{12}\right)$

a. Well done, though many students gave $\text{cis}\left(\frac{\pi}{4}\right)$ as their answer. Such errors can be avoided by employing a rough Argand diagram plot to identify the correct quadrant.

b. Very well done – most students knew that the arguments had to be added.

c–d. (2.40/5)

Answers: ci. $\frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)+\frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right)i$; cii. $\frac{\sqrt{3}-1}{2\sqrt{2}}$

ci. Many students did not answer this part correctly. Most either omitted the cross-product terms in their expansions or proceeded as though they were finding the scalar product of two vectors whereas others had trouble simplifying i^2 and collecting like terms.

cii. Many students ignored ‘hence’ and got in some practice for part d.

d. Reasonably well done, perhaps indicating that most students do attempt past examination papers as part of their preparation. Some students expanded $\sin\left(\frac{\pi}{3}-\left(\frac{-\pi}{4}\right)\right)$ and hence found $\sin\left(\frac{7\pi}{12}\right)$; others showed a lack of conceptual understanding by writing $\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\sin\left(\frac{\sqrt{3}}{2}\right)\cos\left(\frac{1}{\sqrt{2}}\right)-\sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{\sqrt{2}}\right)$. If the exact value of $\sin\left(\frac{\pi}{12}\right)$ was found in a different form to that obtained in part cii, for example $\frac{\sqrt{6}-\sqrt{2}}{4}$ compared to $\frac{\sqrt{3}-1}{2\sqrt{2}}$, then it was necessary to show their equivalence to obtain full marks.

Question 2

a–b. (1.19/4)

Answers: a. $\sqrt{16t^4-8t+5}$;

b. $t = 0.5, \underset{\sim}{r} = -\underset{\sim}{i} - \underset{\sim}{j}$ (i.e. $(-1,-1)$)

a. The most common error was to express the distance as $\sqrt{\left(4t^2-2\right)^2-\left(4t-1\right)^2}$. However, most students had the correct sign and so obtained the answer mark, though many made mistakes when simplifying the expression (thus making it impossible to gain full marks in part b).

b. Poorly done. Many students did not recognise this as a minimisation problem or, if they did, differentiated the $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components of $\underset{\sim}{r}$ and set them equal. Note that the easiest way to find where the distance is a minimum is to find where its square, $16t^4 - 8t + 5$, is a minimum. The most common error was to find the distance from the origin at $t = 0$ instead of the minimum distance from the origin, whilst some students correctly obtained $t = 0.5$ but then failed to substitute to find the minimum distance.

c. (2.08/5)

Answer: $x = \frac{1}{4}(y-1)^2 - 2$, where $x \geq -2$ and $y \leq 1$; lower part (only) of the parabola, but including the vertex $(-2, 1)$

Most students made a reasonable attempt at eliminating t to obtain the cartesian equation of the path, though many commenced incorrectly by writing y = 4t –1 instead of $y = -(4t-1)$. Many students drew something like part or all of a horizontal parabola, but only about 10 per cent drew a path that was sufficiently correct to obtain full marks. A common error was to correctly obtain $x = \frac{1}{4}(1-y)^2 - 2$, but to ignore the implication of $t \geq 0$ for the **range** of the path and so draw a full parabola. Some students just did not draw the path sufficiently accurately – a reasonably accurate graph is required when a grid is given.

Many students failed to recognise that they had obtained the equation of a parabola and tried to draw the path either by plotting a few points and joining them up, or by using their graphics calculator (with $y = 1-2\sqrt{x+2}$ or $y = 1+2\sqrt{x+2}$ or both) and copying the display without further thought. This latter approach usually meant that the vertex and an adjacent section were missing from the path due to limitations of the calculator display. The best way to obtain the path with a graphics calculator is to draw it in parametric mode, with the minimum value of the parameter for t set at 0, and teachers should ensure that students are familiar with this capability of their graphics calculator.

2d. (0.63/3)

Answer: $\left(\frac{3}{2},\ 1\right)$ or $\left(-\frac{3}{2},\ -1\right)$

Most students got no further than finding $\underset{\sim}{r}(1)$. Some students were able to find a perpendicular vector, but few were able to incorporate the distance requirement into their solution and even then often only finding the one solution $\left(\frac{3}{2},\ 1\right)$. Most students who obtained both correct answers did so by employing sound algebraic skills to solve two simultaneous equations: $2a - 3b = 0$ and $a^2 + b^2 = \frac{13}{4}$, where the position of the second particle is given by $a\underset{\sim}{i} + b\underset{\sim}{j}$. However, the easiest way to solve the question is to note, by inspection, that a vector perpendicular to $\underset{\sim}{r}(1)$ is $\pm k\left(3\underset{\sim}{i} + 2\underset{\sim}{j}\right)$, $k>0$ and that a particle with this position vector will be half the distance from the origin if we let $k = \frac{1}{2}$.

Question 3

a–b. (2.65/5)

a. Well done with most students using $\cot x = \frac{\cos x}{\sin x}$ rather than $\cot x = \frac{1}{\tan x}$.

b. Many students were able to apply the chain rule and get $\frac{d}{dx}\left(\log_e\left(\tan\frac{x}{2}\right)\right) = \frac{\frac{1}{2}\sec^2\frac{x}{2}}{\tan\frac{x}{2}}$, but only a minority were able to show this is equal to cosec x by using $2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$.

c. (2.03/5)

Answer: 2m^3

Most students obtained the correct definite integral for the volume. Common errors were expanding $(2\text{cosec}x - 1)^2$ incorrectly as $4\text{cosec}^2 x + 1$ and not recognising the connection with parts a. and b., although many did, when attempting the integration. Of those students who obtained the correct antiderivative, many encountered difficulties after substituting the terminals and less than 10 per cent of students obtained full marks. This was another question that rewarded good revision as there have been a number of similar questions on past papers.

Question 4

a–bi. (2.62/5)

Answer: a. $\frac{1}{6\pi}$ m/h (0.053 m/h)

a. Reasonably well done, though perhaps not as well as should be expected given its routine nature. Some students just substituted $h = 0.5$ in the expression for V. Another common error was to use an incorrect chain rule, particularly $\frac{dh}{dt} = \frac{dV}{dt}\frac{dV}{dh}$.

bi. This was answered better than part a., with most students recognising the need for a chain rule even if they had not used one in part a.

bii–ci. (2.08/6)

Answers: bii. $4\pi\left(\frac{2}{5}h^{\frac{5}{2}} - \frac{4}{3}h^{\frac{3}{2}} + \frac{14}{15}\right)$; biii. 11.7 h;

ci. $\frac{dh}{dt} = \frac{1 - 2\sqrt{h}}{8\pi h(2-h)}$

bii. Reasonably well done with most students getting at least as far as $t = -\int 4\pi\sqrt{h}(2-h)dh$. Common errors when integrating included attempting the substitution $u = 2 - h$, taking out $\sqrt{h}$ as a constant factor, 'losing' π and adding fractions incorrectly.

biii. Students who got 4bii correct usually also got this correct, though some left their answer as $\frac{56\pi}{15}$.

ci. The most common error was to write

$$\frac{dh}{dt} = \text{(rate in)} - \text{(rate out)} = \text{(rate in)} - \left(-\frac{1}{4\pi\sqrt{h}(2-h)}\right)$$

$$= \text{rate in} + \frac{1}{4\pi\sqrt{h}(2-h)}$$

4cii. (0.09/ 2)

Answer: the water level drops, approaching (but never quite reaching) a height of 0.25 m

Not well done. Many students appeared to guess and wrote 'it spills over the top' or something similar. Few students tried to argue from their previous work.

Question 5

a. (1.45/3)

Answer: $\underset{\sim}{\dot{x}}(t) = 5\underset{\sim}{i} + \left(\frac{3\pi}{4}\cos\left(\frac{\pi}{2}t\right)\right)\underset{\sim}{j}$; maximum speed is 5.5 m/s

Most students found the velocity correctly, though a common mistake was to write $\underset{\sim}{\dot{x}}(t) = 5\underset{\sim}{i} + \left(\frac{3}{4}\cos\left(\frac{\pi}{2}t\right)\right)\underset{\sim}{j}$. Many students did not recognise that the maximum value of $\sqrt{25 + \frac{9\pi^2}{16}\cos^2\left(\frac{\pi}{2}t\right)}$ occurs when $\cos\left(\frac{\pi}{2}t\right) = 1$ and wasted time trying to find the maximum using calculus.

b. (1.51/ 2)

Answer: 2.4 m/s

Very well done.

Note: this question could not be asked in this form in 2000 because the topic (conservation of momentum) has been omitted from the revised course.

c. (2.7/5)

Answer: 73.7 N

Reasonably well done. The most common errors concerned '*ma*' (=75(0.1)) when resolving parallel to the incline: it was often taken to be 0 or written as $75g(0.1)$. Students should be encouraged to draw a force diagram as the first step in answering such questions.

di–dii. (1.08/5)

Answers: di. $a = -\dfrac{v^2 + 144.03}{150} \approx -\dfrac{v^2 + 144}{150}$; dii. 6.5 s

di. Not well done and often omitted. Many students did not have a clear idea of the forces involved and included tension or had sign errors. Some students treated the resistance as a negative acceleration (i.e. as a retardation), rather than as a force.

dii. Most students who got as far as $t = -\int \dfrac{150}{v^2 + 144} dv$ obtained the correct antiderivative, though some then came to grief because they had their calculator in degree mode instead of radian mode. Many students, however, used the constant acceleration formula $v = u + at$, with $a = \dfrac{v^2 + 144}{150}$.

e. (0.74/3)
Answer: Phil remains stationary since his weight component down the incline (29.5N) is less than the maximum opposing friction force (36.7N).

Reasonably well done. To obtain full marks, students needed to make it clear that 36.7N was the **maximum** friction force, not the actual friction force in this case.

Note: no comments are included for Question 6 because of the small number of students who attempted it and because no part of this question could be asked in 2000 since Analytic geometry and Deductive two-dimensional geometry are not part of the revised Specialist Mathematics course.

Question 6

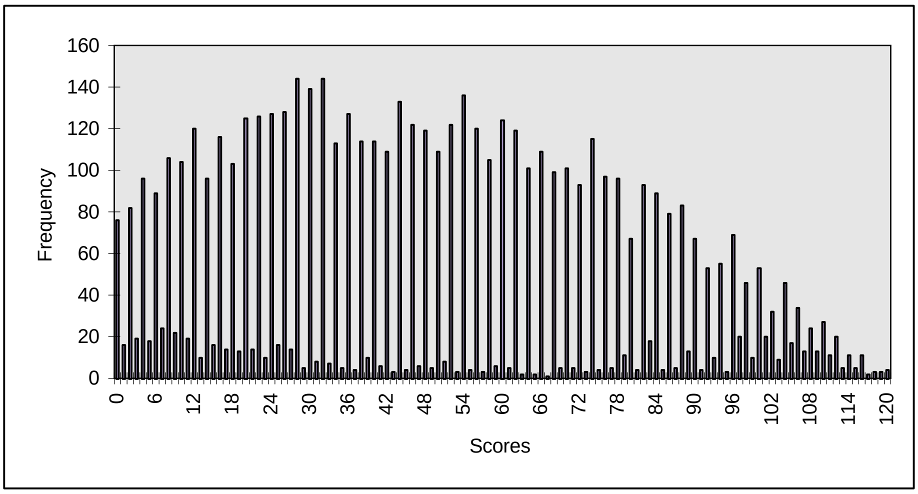
ai. (1.35/3)
Answer: $y = (-\cot\theta)x + 3\operatorname{cosec}\theta$
aii–aiii (1.45/7)

Answer: aiii. $\dfrac{y^2}{9} - \dfrac{x^2}{9} = 1$, $x \geq 0$ and $y \geq 3$; the quarter (only) of the hyperbola lying in the first quadrant, but including the vertex (0, 3)
bi–bii (2.2/8)

CAT MA093 MATHEMATICS: Specialist Mathematics WRITTEN EXAMINATION

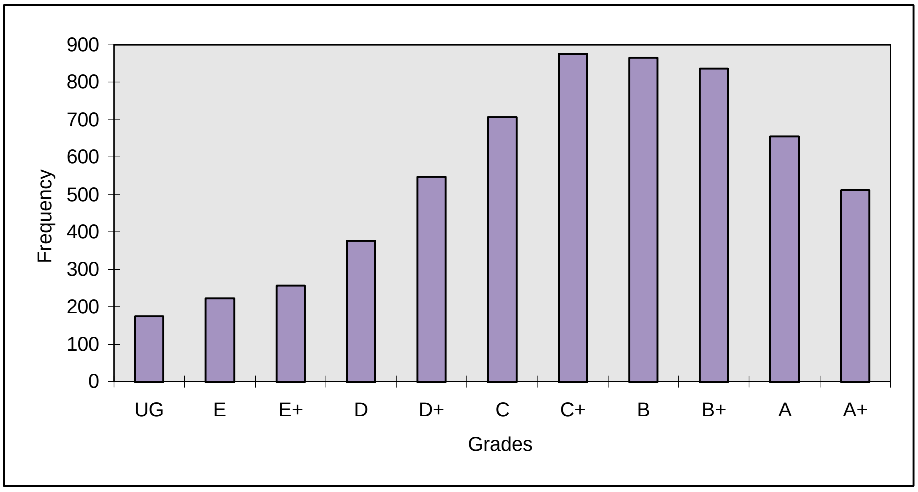
HISTOGRAM OF TOTAL SCORES **1999**

Count **6028** Mean **49.04** Standard Deviation **29.96** NA Result **146**



HISTOGRAM OF TOTAL GRADES **1999**

Count **6028** Mean **6.07** Standard Deviation **2.59** NA Result **146**



ENROLMENTS		**%**
Female	**2269**	**36.8**
Male	**3904**	**63.2**
Total	**6173**	

GLOSSARY OF TERMS

Count	Number of students undertaking the CAT. This excludes those for whom NA was the result.
Mean	This is the 'average' score; that is all scores totalled then divided by the 'Count'.
Standard Deviation	This is a measure of how widely values are dispersed from the average value (the mean).

Notes

‘Report for Teachers’ series of booklets contain reports from Chief Assessors and State Reviewers for the common assessment tasks undertaken in 1999. Each report contains an overview of student performance on individual CATs. Chief Assessors and State Reviewers have commented on such matters as the assessment criteria and student performance on the CATs.

Users of these reports should be aware that these reports are for 1999 CATs. Changes may have been made to study designs, CATs and assessment criteria since the completion of the reports.

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