Victorian Certificate of Education
1999

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination (Facts, skills and applications task)

Thursday 4 November 1999: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this book.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
You may retain this question book.

Specific instructions to students

This part consists of 33 questions.

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

Question 1

The probability distribution for the random variable X is given by

x	1	2	3	4
$\Pr(X = x)$	k	$4k$	$9k$	$16k$

The value of k is

A. $\frac{1}{100}$

B. $\frac{1}{30}$

C. 1

D. 30

E. 100

Question 2

Research has shown that Hurtz Pain Killing Capsules relieve the pain of toothache in 90 per cent of patients. Six patients with toothache were randomly selected and were treated with Hurtz Pain Killing Capsules. The probability that at least three patients obtained relief from the pain of toothache is

A. $1 - [(0.9)^6 + {}^6C_1\,(0.9)^5\,(0.1) + {}^6C_2\,(0.9)^4\,(0.1)^2]$

B. $(0.1)^6 + {}^6C_1\,(0.1)^5\,(0.9) + {}^6C_2\,(0.1)^4\,(0.9)^2$

C. $1 - [(0.1)^6 + {}^6C_1\,(0.1)^5\,(0.9) + {}^6C_2\,(0.1)^4\,(0.9)^2 + {}^6C_3\,(0.1)^3\,(0.9)^3]$

D. $1 - [(0.1)^6 + (0.1)^5\,(0.9) + (0.1)^4\,(0.9)^2]$

E. $1 - [(0.1)^6 + {}^6C_1\,(0.1)^5\,(0.9) + {}^6C_2\,(0.1)^4\,(0.9)^2]$

Question 3

Anastasia takes the bus to and from school each day, making a total of ten trips per week. The probability that the bus is running late on exactly three occasions is given by

$$^{10}C_3\,(0.2)^3\,(0.8)^7.$$

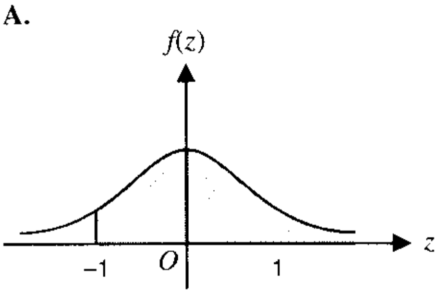
The mean and variance of the number of occasions that Anastasia finds that the bus is running late is

	mean	variance
A.	2	1.6
B.	2	0.4
C.	8	1.6
D.	8	0.4
E.	0.2	0.8

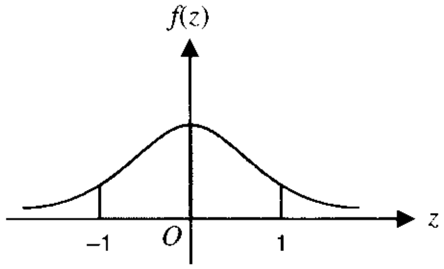
Question 4

Z is a standard normal random variable with a mean of 0 and a variance of 1. Which one of the following diagrams illustrates the probability that Z is greater than 1 or less than –1 as the shaded region?

A.



B.



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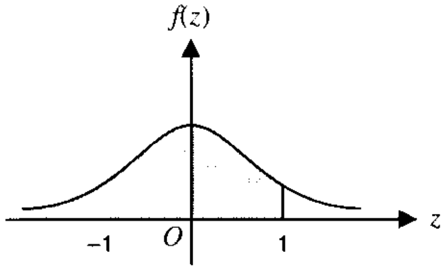

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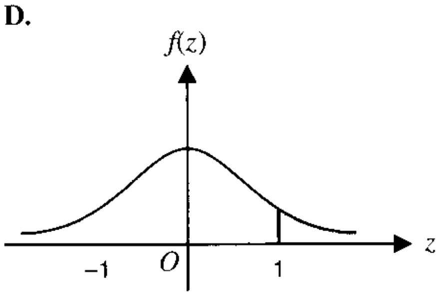
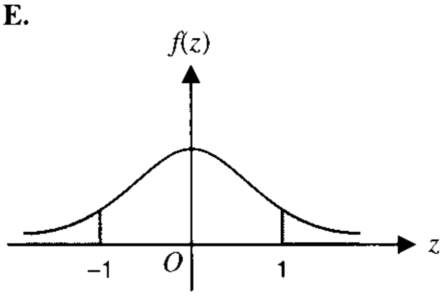


E.



Question 5

If Z has a standard normal distribution, which one of the following is **not** true?

A. The mean, median and mode of Z are all equal.

B. The mean of Z equals 0 and the standard deviation of Z equals 1.

C. $\Pr(Z < -1) = 1 - \Pr(Z > 1)$

D. $\Pr(Z < 0) = 0.5$

E. $\Pr(-2 < Z < 2) \approx 0.95$

Question 6

Volunteers for a weight loss program have weights which are normally distributed with a mean of 100 kg and a standard deviation of 8 kg. One of the volunteers is selected at random. The probability that this person's weight is over 110 kg is approximately

A. 0.8944

B. 0.8849

C. 0.5

D. 0.1151

E. 0.1056

~~Question 7~~

In a particular electorate of several thousand voters, 60% of the voters favour the Politically Correct Party. From one of many random samples of 100 voters, the proportion of voters in the sample who favour the Politically Correct Party is recorded. The variance for such a sample proportion is

A. 0.24

B. 0.024

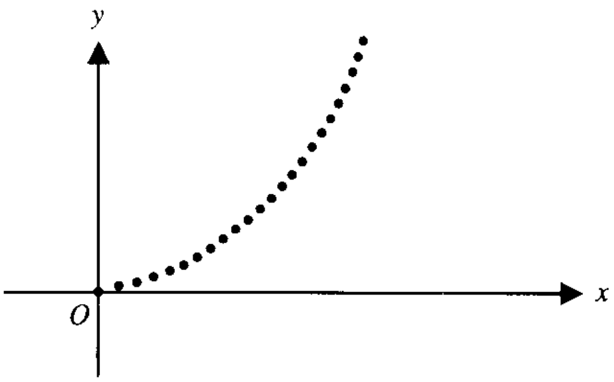
C. 0.0024

D. $\sqrt{0.024}$

E. $\sqrt{0.0024}$

Question 8

The relationship between two quantities x and y is represented graphically as shown below.



If a is a positive constant, the equation relating x and y is most likely to be of the form

A. $y = \frac{a}{x}$

B. $y = \frac{a}{x^2}$

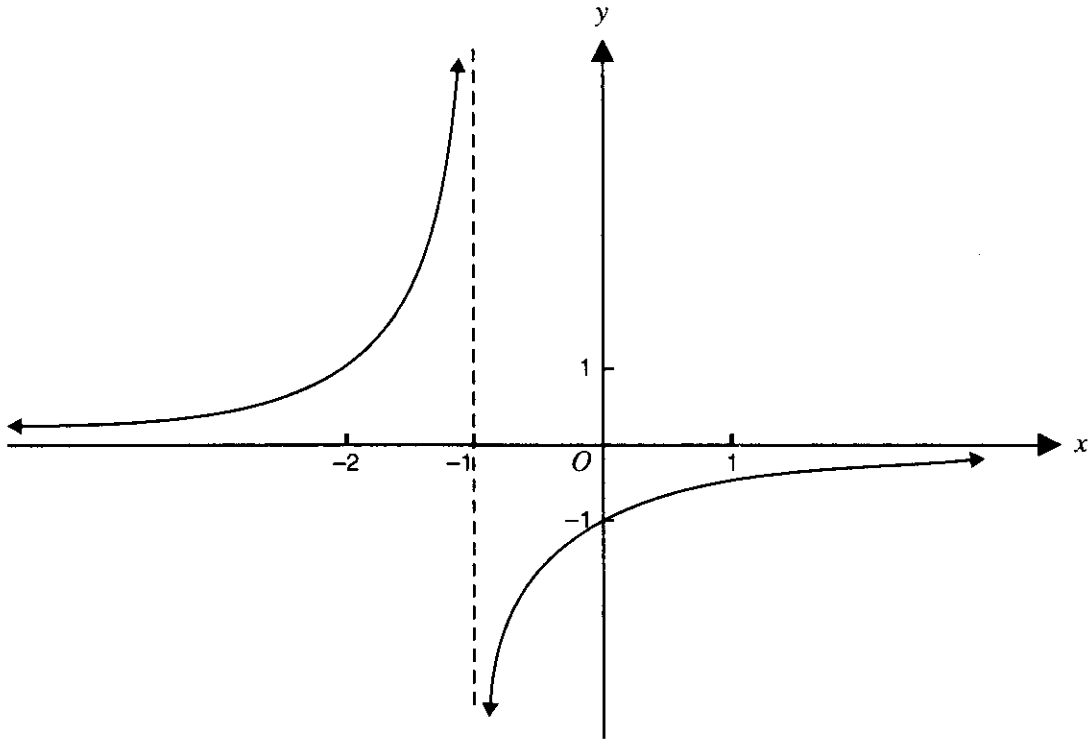
C. $y = ae^x$

D. $y = ax^2$

E. $y = ax^{\frac{1}{2}}$

Question 9

The following is the graph of a function with equation $y = \dfrac{A}{(x+B)}$.

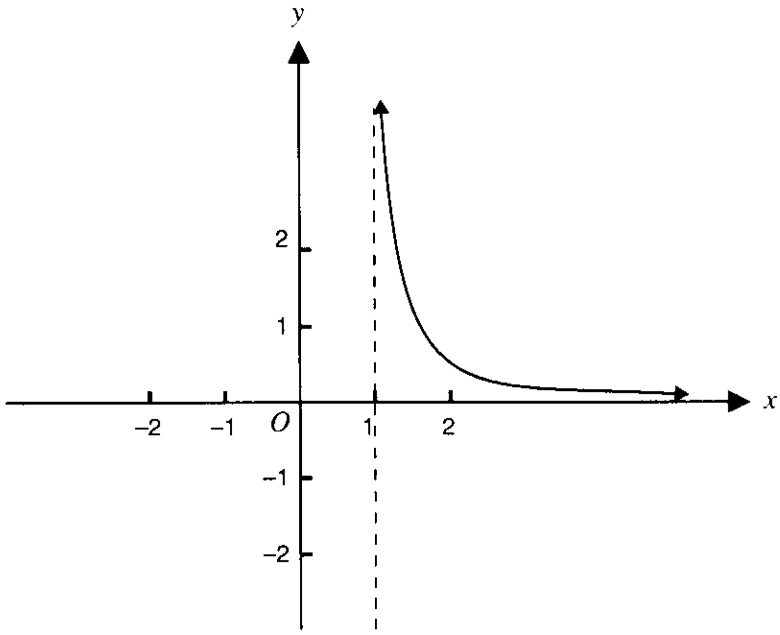


The values of A and B respectively are

	A	B
A.	−1	1
B.	−1	−1
C.	1	−1
D.	1	1
E.	−2	2

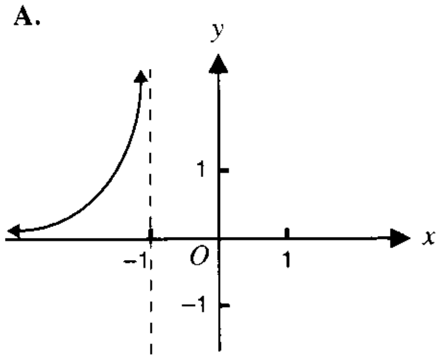
Question 10

The graph of the function with equation $y = f(x)$ is shown.

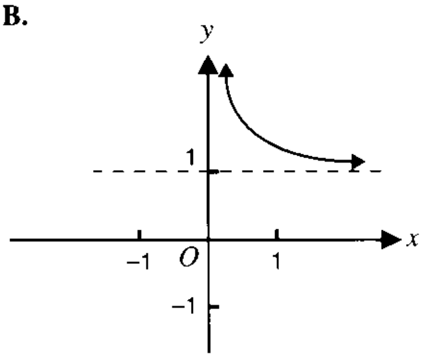


Which one of the following is most likely to be the graph of the inverse function?

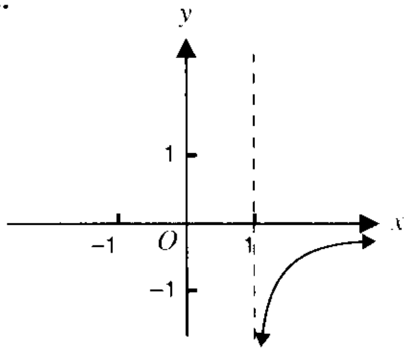
A.



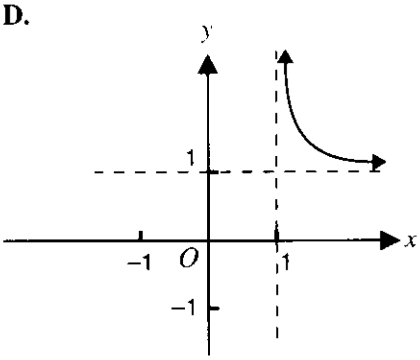
B.



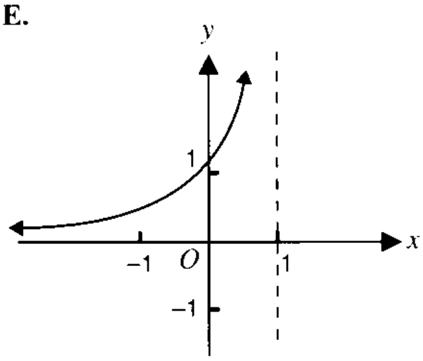
C.



D.

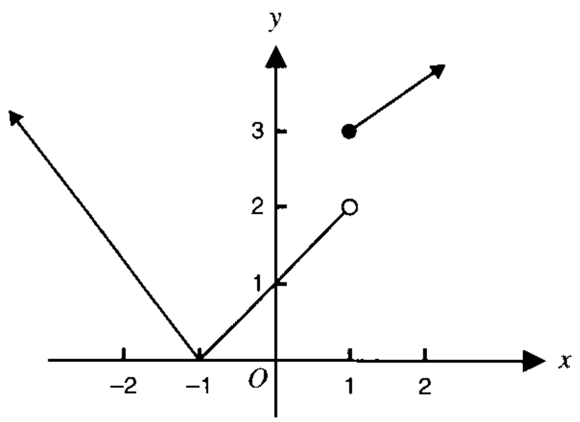


E.



Question 11

For the graph of the function shown, which one of the following statements is **not** true?



A. The range is $[0,\infty)$.

B. The domain is R.

C. The gradient of the graph is positive for all $x > 1$.

D. The derived function exists for all values of x except $x = 1$.

E. The value of y is positive for all values of x, except $x = -1$.

Question 12

The function $f: R \to R$, where $f(x) = 2^{(0.1x + 0.1)} - 3$ has range

A. $(3,\infty)$

B. $(-3,\infty)$

C. $(0.1,\infty)$

D. $(-0.1,\infty)$

E. R^+

Question 13

A curve with equation $y = \sin x$ is transformed by a dilation from the y-axis by a scale factor of 2, a translation of $\frac{\pi}{2}$ to the left in the x-direction and a translation of 2 units downwards in the y-direction. The equation of the transformed curve is

A. $y = \sin\left(2\left(x - \frac{\pi}{2}\right)\right) - 2$

B. $y = 2\sin\left(x + \frac{\pi}{2}\right) + 2$

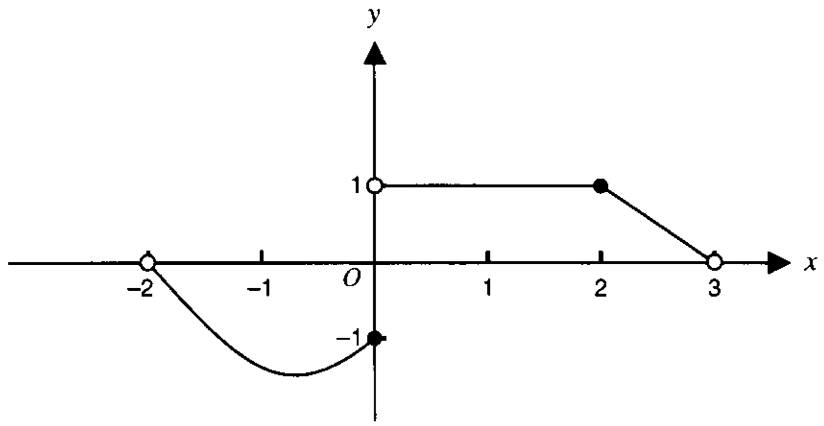
C. $y = \sin\left(2\left(x + \frac{\pi}{2}\right)\right) - 2$

D. $y = \sin\left(0.5\left(x + \frac{\pi}{2}\right)\right) - 2$

E. $y = \sin\left(0.5\left(x - \frac{\pi}{2}\right)\right) + 2$

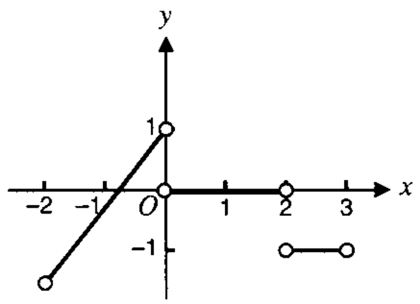
Question 14

The graph of the function f is shown below.

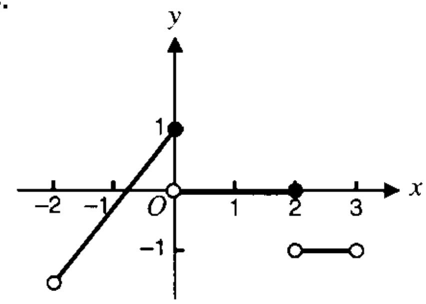


The graph of the derived function, f', is best represented by

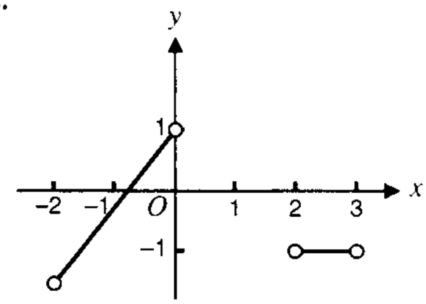
A.



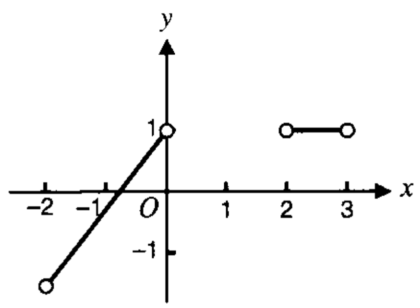
B.



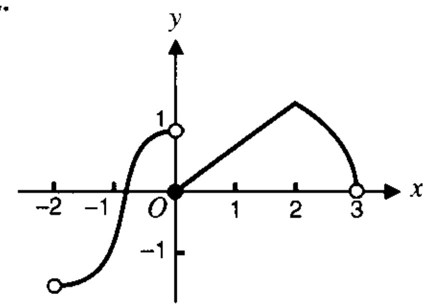
C.



D.



E.



Question 15

If $f(x) = x(x^2 - 3x - 9)$ then $f'(x)$ is

A. $2x - 3$

B. $x(2x - 3)$

C. $x^2 - 3x - 9$

D. $x(2x - 3) + (x^2 - 3x - 9)$

E. $x(2x - 3) + (x^2 - 3)$

Question 16

The derivative of $\dfrac{\cos(3t)}{t^2}$ with respect to t is equal to

A. $\dfrac{-3t^2\sin(3t)-2t\cos(3t)}{t^4}$

B. $\dfrac{-3\sin(3t)-2\cos(3t)}{t^2}$

C. $\dfrac{3t^2\sin(3t)-2t\cos(3t)}{t^4}$

D. $\dfrac{2t\cos(3t)+3t^2\sin(3t)}{t^4}$

E. $\dfrac{-t^2\sin(3t)-2t\cos(3t)}{t^4}$

Question 17

If $y=\log_e\left(\dfrac{1}{x}\right)$, then $\dfrac{dy}{dx}$ is equal to

A. x

B. $\dfrac{1}{x}$

C. $\log_e\left(\dfrac{1}{x}\right)$

D. $\log_e(x)$

E. $-\dfrac{1}{x}$

Question 18

If $y = x - e^{-x}$, the rate of change of y with respect to x when $x = 0$ is

A. -1

B. 0

C. 1

D. 2

E. $1 + e$

Question 19

If $y = \frac{x^3}{3} - x^2 - 15x$, the value of x which gives the local maximum of y is

A. -3

B. 0

C. 1

D. 3

E. 5

Question 20

Given that $f'(x) = 2\sin(2x) - 4e^{-2x}$ and $f(0) = 2$, then $f(x)$ is equal to

A. $\cos(2x) - 2e^{-2x} + 3$

B. $\cos(2x) + 2e^{-2x} - 1$

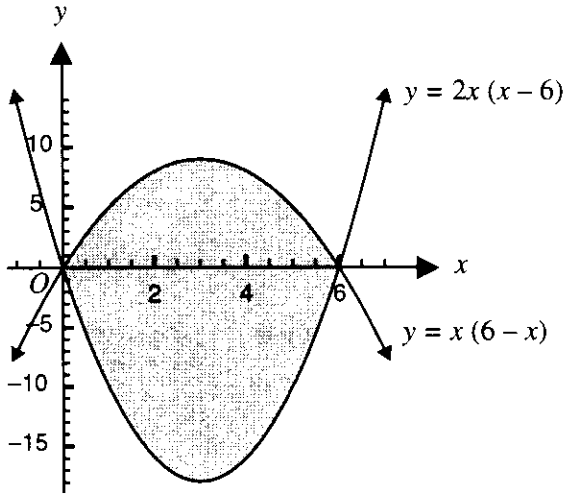
C. $-\cos(2x) + 2e^{-2x} + 1$

D. $-\cos(2x) + 2e^{-2x} + 2$

E. $\sin(2x) + 2e^{-2x}$

Question 21

The diagram below shows portions of the graphs of the functions with equations $y = 2x(x-6)$ and $y = x(6-x)$.



The area of the shaded region in square units is

A. -36

B. 36

C. 72

D. 108

E. 324

Question 22

The graph of the function $f: R \to R, f(x) = x^4 - 4x$ has

A. no stationary points.

B. exactly one stationary point, which is a minimum point.

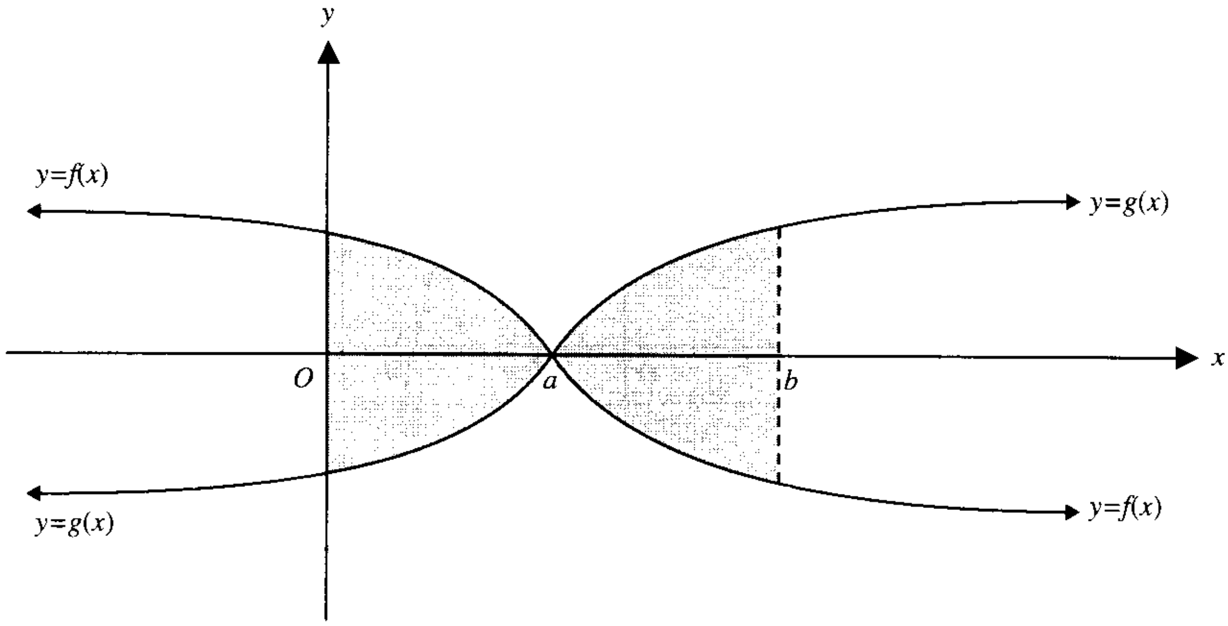
C. exactly two stationary points, which are a minimum and a point of inflexion.

D. exactly three stationary points, which are two minimums and a maximum.

E. exactly four stationary points.

Question 23

The diagram below shows portions of the graphs of two functions f and g.



The area of the shaded region is equal to

A. $\int_0^b \left(f(x) - g(x)\right) dx$

B. $\int_0^a \left(f(x) - g(x)\right) dx + \int_a^b \left(f(x) - g(x)\right) dx$

C. $\int_0^a f(x)dx + \int_a^b g(x)dx$

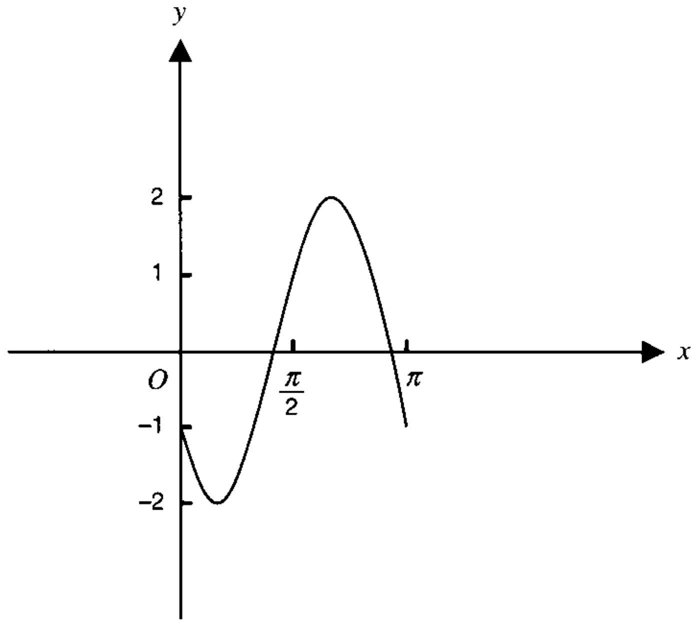
D. $\int_0^b f(x)dx + \int_0^b g(x)dx$

E. $\int_0^a \left(f(x) - g(x)\right) dx + \int_a^b \left(g(x) - f(x)\right) dx$

Question 24

One complete cycle of the graph with equation $y = a\sin(bx + c)$ is shown below. The values of a, b and c could be

	a	b	c
A.	-2	2	$\frac{\pi}{6}$
B.	-2	$\frac{1}{2}$	$\frac{\pi}{6}$
C.	-1	2	$\frac{-\pi}{6}$
D.	-1	2	$\frac{-\pi}{3}$
E.	-2	2	$\frac{-\pi}{6}$



Question 25

A function f is given by $f: R \to R, f(x) = 2\cos(3x + \pi) - 1$.
The amplitude, period and range of f are

	Amplitude	Period	Range
A.	2	π	R
B.	2	$\frac{2\pi}{3}$	$[-3, 1]$
C.	2	$\frac{2\pi}{3}$	$[-2, 2]$
D.	π	3	$[-3, 1]$
E.	3	π	$[-2, 2]$

Question 26

For the equation $2\cos(2x) = \sqrt{2}$, the **sum** of the solutions in the interval $[0, 2\pi]$ is

A. $\frac{\pi}{8}$

B. π

C. 3π

D. 4π

E. 8π

Question 27

A solution of the equation $b \sin(ax) = \sqrt{2} \cos(ax)$ is $\frac{\pi}{4}$.

Possible values of a and b are

	a	b
A.	1	$-\sqrt{2}$
B.	1	$\frac{1}{\sqrt{2}}$
C.	1	$\sqrt{2}$
D.	3	$\sqrt{2}$
E.	3	$\frac{1}{\sqrt{2}}$

Question 28

Let $f(x) = p\sin(2x) + q$, where $p > 0$. Then $f(x) > 0$ for all values of x if

A. $q > 0$

B. $p > -q$

C. $p < q$

D. $-p < q < p$

E. $p < \frac{1}{2}q$

Question 29

$P(x) = (x^2 + a)(x - b)(x - c)^2$, where a, b and c are three different positive real numbers.

The equation $P(x) = 0$ has exactly

A. 1 real solution.

B. 2 distinct real solutions.

C. 3 distinct real solutions.

D. 4 distinct real solutions.

E. 5 distinct real solutions.

Question 30

In the expansion of $(px + 4)^5$, the coefficient of x^3 is 4320. The value of p is

A. -3

B. $\dfrac{\sqrt{27}}{2}$

C. 3

D. 9

E. 27

Question 31

The function $f: R\backslash\{-2\} \to R$, where $f(x) = \dfrac{1}{x+2} - 1$ has an inverse f^{-1}.

The rule and domain of f^{-1} are

	Rule	Domain
A.	$f^{-1}(x) = \dfrac{1}{x+1} - 2$	$R\backslash\{-2\}$
B.	$f^{-1}(x) = \dfrac{1}{x-1} + 2$	$R\backslash\{-1\}$
C.	$f^{-1}(x) = \dfrac{-1}{x-1} + 2$	$R\backslash\{-2\}$
D.	$f^{-1}(x) = \dfrac{1}{x+1} - 2$	$R\backslash\{2\}$
E.	$f^{-1}(x) = \dfrac{1}{x+1} - 2$	$R\backslash\{-1\}$

Question 32

If $\log_a(x^2) - 2 = 2\log_a 5$, where $a > 0$ and $x > 0$, then x is equal to

A. $5a$

B. $25a$

C. $\sqrt{10a}$

D. $\sqrt{(a^2 + 25)}$

E. $\sqrt{(25 - 2a)}$

Question 33

The exact solution of the equation $4 \times 10^{2x} = 9$ is

A. $x = \log_{10}\left(\dfrac{9}{4}\right)$

B. $x = \log_{10}\left(\dfrac{3}{2}\right)$

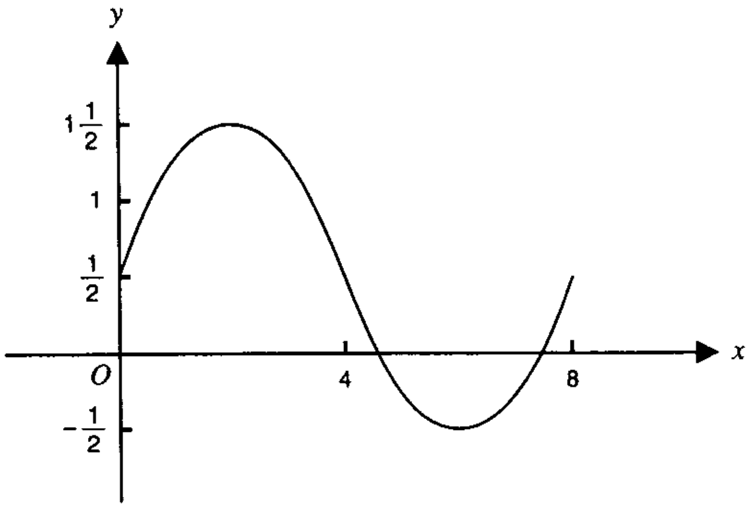
C. $x = \dfrac{\log_{10} 3}{\log_{10} 2}$

D. $x = \dfrac{\log_{10} 9}{\log_{10} 4}$

E. $x = \dfrac{\log_{10} 9}{2\log_{10} 4}$

Question 1

The diagram below shows one cycle of the graph of a trigonometric function.



a. State the period of the function. (1)

b. State the amplitude of the function. (1)

Question 2

The distance that Antonio can throw a cricket ball is approximately normally distributed with a mean of 80 metres and a standard deviation of 3 metres. On twenty-five per cent of a large number of occasions, Antonio throws the ball past a line d metres away. Find the value of d, to the nearest metre. (2)

Question 3

In this question, p represents the population proportion and $\hat{p}$ represents the sample proportion.

A random sample of 300 is selected from the population of Australia. From this sample, 225 expressed concern about global warming.

Find an approximate 95% confidence interval estimate for the proportion of the population p that expressed concern about global warming. (2)

Question 4

a. Expand $3x\,(2x-5)^3$ fully. (2)

b. x^3+2x^2+ax-2 is exactly divisible by $x-2$. Find the value of a. (2)

Question 5

A block of ice is melting so that its volume V cm^3 at any time t minutes is given by

$$V = 1000 - 25t - 0.01t^2, \quad t \in (0, 35).$$

a. Find the average rate of change of the volume over the first ten minutes, correct to one decimal place. (1)

b. Find the instantaneous rate of change of the volume when $t = 10$, correct to one decimal place. (2)

Question 6

A region is bounded by the coordinate axes, the line $x = p$, $p > 0$, and the curve whose equation is

$$y = \frac{12}{(x+1)^2}.$$

a. Use calculus to write down an expression for the area of the region **algebraically in terms of** p. (2)

b. The area of the region is 6. Find the exact value of p. (2)