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YEAR 12
IARTV TEST — OCTOBER 2000
MATHEMATICAL METHODS — EXAMINATION 2 (ANALYSIS TASK)
ANSWERS & SOLUTIONS

Question 1.

a) $c'(t) = \cos t - \sqrt{3}\sin t$

b) Turning points when $c'(t) = 0$,

$\cos t = \sqrt{3}\sin t$, $\tan t = \frac{1}{\sqrt{3}}$, $t = \frac{\pi}{6}, \frac{7\pi}{6}$

Coordinates $(\frac{\pi}{6}, 2)$, $(\frac{7\pi}{6}, -2)$

c) $(0, \sqrt{3})$

d) t – intercepts occur when $c(t) = 0$,

$\tan t = -\sqrt{3}$, $t = \frac{2\pi}{3}, \frac{5\pi}{3}$.

e) see over page. below

f) A= amplitude = 2
B= 1
C= $\pi/6$

g) *solving* $1 = 2\cos(t - \frac{\pi}{6})$,

$t = \frac{\pi}{2}, \frac{11\pi}{6}$.

h) $\frac{2}{3}$

i) $\frac{4}{\pi}$

Question 2

a) $t = 2$, $P = 4.84$

b) $t = 10^{\frac{1}{15}(12-P)} - 1$

c) $t = 10^{0.6} - 1 = 2.98 \approx 3$ months

d) $10^y = x$, $\log_e(10^y) = \log_e x$

$y \log_e 10 = \log_e x \Rightarrow$ *result*

$\frac{dy}{dx} = \frac{1}{x \log_e 10}$

e) $\frac{dP}{dt} = \frac{-15}{(1+t)\log_e 10}$

f) when $t = 2$, $\frac{dP}{dt} = -2.17$

P is reducing by \$2170 per month after 2 months.

g) $t = 1.17$

h) $t = 5.31$ *months*

i) *Total bprofit* $= \int_0^7 P dt = 21.23$, ie. \$21,230.

Question 3

a) i) \$533.33
ii) \$300

b) $C = \begin{cases} \frac{8000}{n}, & n \le 20 \\ 400, & 20 \le n \le 30 \\ 700 - 10n, & 30 \le n \le 50 \end{cases} \} \; n \in N$

c) $30 + x$

d) $R = n(700 - 10n) = (x + 30)(400 - 10x)$

$R = 12000 + 100x - 10x^2$

e) $x \in [0,20]$ and $x \in N$

f) $\frac{dR}{dx} = 0 = 100 - 20x \Rightarrow x = 5$

Thus 35 passengers maximises R, the receipts.

g) R contains points on a negative quadratic function and so R achieves a maximum.

Question 4

a) $\frac{28}{55}$

b) $\frac{63}{64}$

c)

i) 0.067

ii) 0.061

iii) 0.988

d) 108.42

e)

i) 0.871

ii) 0.129