Trial Examination 1 Answers & Solutions

Part I (Multiple-choice) Answers

1. **A**	2. **C**	3. **C**	4. **E**	5. **A**
6. **D**	7. **B**	8. **B**	9. **E**	10. **C**
11. **B**	12. **A**	13. **D**	14. **D**	15. **D**
16. **C**	17. **E**	18. **A**	19. **D**	20. **A**
21. **D**	22. **D**	23. **B**	24. **B**	25. **D**
26. **A**	27. **D**	28. **C**	29. **B**	30. **B**

Solutions

Multiple Choice

Question 1 [A]

For points A(–3, 4) and B(1, –3) the gradient is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-3)}{-3-1} = -\frac{7}{4}$$

Question 2 [C]

The n-value is obtained by expanding.

$2(\frac{3}{2}x+\frac{\pi}{4}) = 3x+\frac{\pi}{2}$ n = 3 and the period is $\frac{2\pi}{n} = \frac{2\pi}{3}$

Question 3 [C]

$$f(x) = \frac{4}{3}x^3 - 2x^2 - kx + 5$$

$$f'(x) = 4x^2 - 4x - k$$

$$2 = 4(3)^2 - 4(3) - k$$

$$2 = 24 - k$$

$$k = 22$$

Question 4 [E]

Amplitude = 2 ∴ Not A or B

Period = $\frac{2\pi}{3}$ ∴ $n = 3$ Must be D or E

Moved to the right $\frac{\pi}{6}$ therefore, $x - c = x - \frac{\pi}{6}$

And $c = \frac{\pi}{6}$ ∴ E is the only possible answer from the selections

Question 5 [A]

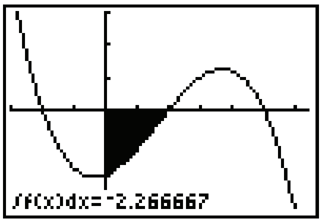
$${}^5C_4(3x^2)^4(-4)^1 = 5\times 3^4 x^8 \times -4$$

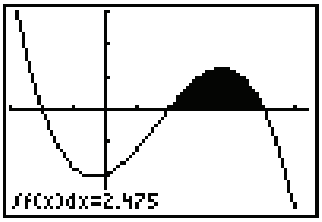
$$= -1620x^8$$

$$\text{coefficient} = -1620$$

Question 6 [D]

The most efficient way of solving this question is by using a graphics calculator.
Type the equation into the calculator and use it to find the area between x = 0 and x = 2 which is = 2.267 units squared and the area between x = 2 and x = 5 which is = 2.475 units squared.
Total 4.741 to 3 dps.





Question 7 [B]

At t = 3 the curve $x = 2t^2 + 3t$ has the value
x = 18 + 9 = 27
At t = 7 the curve $x = 2t^2 + 3t$ has the value
x = 98 + 21 = 119
The average rate of change equals

$$\frac{119-27}{7-3} = \frac{92}{4} = 23 \text{ km / hr}$$

Question 8 [B]

A reflection in the line $y = x$ will change the x-intercept (x = 1) into a y-intercept (y = 1). The translation of +3 parallel to the y-axis will move the whole graph up the y-axis to a y-intercept of +4.

Question 9 [E]

X = no. of defective ovens in sample.

$$\Pr(X=0) = \frac{{}^{4}C_0 \, {}^{8}C_4}{{}^{12}C_4} = \frac{70}{495} = \frac{14}{99}$$

Question 10 [C]

This question can be completed faster using the normal probability function on a graphics calculator.

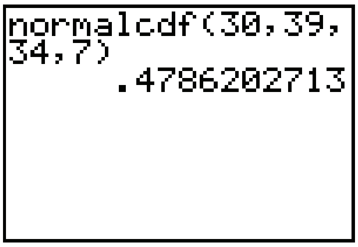
$$\Pr(30 < X < 39) = \Pr\left(\frac{30-34}{7} < Z < \frac{39-34}{7}\right)$$

$$= \Pr\left(-\frac{4}{7} < Z < \frac{5}{7}\right)$$

$$= \Pr\left(Z < \frac{5}{7}\right) - \left(1 - \left(Z < \frac{4}{7}\right)\right)$$

$$= \Pr\left(Z < \frac{5}{7}\right) + \Pr\left(Z < \frac{4}{7}\right) - 1$$

$$= 0.76247 + 0.71614 - 1 = 0.47861$$

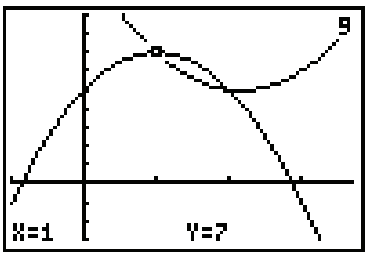


Question 11 [B]

This question can be done more efficiently using the graphing function on a graphics calculator to determine the intersection points of the graphs and which graph is the upper one.

At $x = 1$ the curve $f(x) = 7$ and $g(x) = 7$
At $x = 2$ the curve $f(x) = 5$ and $g(x) = 5$
Between $x = 1$ and $x = 2$, $f(x) > g(x)$

$$\therefore \int_1^2 \left(f(x) - g(x)\right) x = B$$



Question 12 [A]

$$\therefore \int_1^4 x^2 - \frac{1}{x}\,dx = \left[\frac{x^3}{3} - \log_e x\right]_1^4$$

$$= \left[\frac{64}{3} - \log_e 4\right] - \left[\frac{1}{3} - \log_e 1\right]$$

$$= 21 - \log_e 4$$

Question 13 [D]

$$5e^{3x} = 6 \quad \Rightarrow \quad e^{3x} = \frac{6}{5}$$

$$3x = \log_e \frac{6}{5} \quad \Rightarrow \quad x = \frac{1}{3}\log_e \frac{6}{5}$$

Question 14 [D]

$$\frac{1}{2}\log_{10} x + \frac{2}{3}\log_{10} 125 - \frac{1}{3}\log_{10} 27 = 2$$

$$\log_{10} x^{\frac{1}{2}} + \log_{10} 25 - \log_{10} 3 = 2$$

$$\log_{10} x^{\frac{1}{2}} = \log_{10} 100 - \log_{10} 25 + \log_{10} 3$$

$$\log_{10} x^{\frac{1}{2}} = \log_{10} 12$$

$$x^{\frac{1}{2}} = 12$$

$$x = 144$$

Question 15 [D]

Using the product rule

$$f(x) = e^{2x}\cos x$$

$$f'(x) = 2e^{2x}\cos x - e^{2x}\sin x$$

$$f'(x) = e^{2x}(2\cos x - \sin x)$$

Question 16 [C]

The rate of change of the graph is the gradient of the graph. The gradient is only positive when the graph is sloping upwards from the left to the right.

This happens only in the interval $\{-0.4 < x < 3.8\}$

Question 17 [E]

$$\cos x = \frac{1}{2} \quad \Rightarrow \quad x = \cos^{-1}\frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad \Rightarrow \quad \text{The sum is } 2\pi$$

for $0 \le x \le \pi$

Question 18 [A]

x-intercepts are –2, –1, 0, and 1
Factors are x, $(x + 2)$, $(x + 1)$, $(x – 1)$
Therefore only A and B are possible.
At x = -0.5 $f(x) \approx 1$ from the graph.
Substituting into each equation:

A $f(x) = 2x(x + 2)(x + 1)(x - 1)$, $f(x) = \dfrac{18}{16}$

B $f(x) = x(x + 2)(x + 1)(x - 1)$, $f(x) = \dfrac{9}{16}$

Question 19 [D]

1st asymptote occurs when $x - 2 = 0 \;\therefore\; x = 2$

2nd asymptote occurs when $\dfrac{3}{x-2} \to 0 \;\therefore\; y = 1$

Question 20 [A]

$$\sin(2x) = -\cos(2x)$$

$$\frac{\sin(2x)}{\cos(2x)} = -1,\ \tan(2x) = -1,\ 2x = \tan^{-1}(-1)$$

$$2x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4} \text{ for } 0 \le x \le 2\pi$$

$$x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$$

Question 21 [D]

By a process of elimination the only domain that produces a one-to-one function that enables an inverse is:

$x \in [-3, 0) \cup [3, \infty)$

Question 22 [D]

Expand $y = A(x + b)^2 + B$

$$y = Ax^2 + 2Abx + Ab^2 + B$$

$$\Delta > 0$$

$$\Delta = (2Ab)^2 - 4A(Ab^2 + B)$$

$$\Delta = 4A^2b^2 - 4A^2b^2 - 4AB$$

$$\Delta = -4AB$$

$$-4AB > 0$$

Question 23 [B]

The y-values are symmetrical around $x = -2.5$, increasing by the same rate on each side of this x-value. A quadratic function fits this data, and a quadratic is a polynomial.

Question 24 [B]

$p = 0.38, q = 0.62, X =$ no. of hits

$$\Pr(X \ge 3) = \Pr(X = 3) + \Pr(X = 4)$$

$$\Pr(X = 3) = {}^4C_3(0.38)^3(0.62)^1$$

$$\Pr(X = 4) = {}^4C_4(0.38)^4$$

$$\Pr(X \ge 3) = {}^4C_3(0.38)^3(0.62)^1 + {}^4C_4(0.38)^4$$

Therefore B

Question 25 [D]

$$y = A\sin(Bx)$$

$$\frac{2\pi}{B} = \pi,\ \therefore B = 2$$

Using $\left(\dfrac{\pi}{12}, \dfrac{3}{2}\right)$ and $B = 2$

$$\frac{3}{2} = A\sin\left(2 \times \frac{\pi}{12}\right),\ \frac{3}{2} = A\sin\left(\frac{\pi}{6}\right)$$

$$\frac{3}{2} = \frac{A}{2} \quad \therefore A = 3,\ y = 3\sin(2x)$$

Question 26 [A]

Even numbers are 2, 4 and 6
$\therefore$ $0.5 - 3k + 4k + 2k = 3k + 0.5$

Therefore A

Question 27 [D]

Eleven occurs when 5 then 6 are rolled or 6 the 5 are rolled. Twelve only can occur when 6 then 6 are rolled.
Pr(5 then 6) = $(k + 0.1) \times 2k = 2k^2 + 0.2k$
Pr(6 then 5) = $2k \times (k + 0.1) = 2k^2 + 0.2k$
Pr(6 then 6) = $2k \times 2k = 4k^2$
Find $\Pr(X \ge 11) = \Pr(X = 11) + \Pr(X = 12)$
$\Pr(X \ge 11) = 2k^2 + 0.2k + 2k^2 + 0.2k + 4k^2$
$\Pr(X \ge 11) = 8k^2 + 0.4k = k(8k + 0.4)$

Question 28 [C]

$f(x) = -\frac{2}{4}\sin(4x) + c = -\frac{1}{2}\sin(4x) + c$

Question 29 [B]

$\Pr(X < 1.65) = \Pr(Z < \frac{1.65 - 1.67}{0.04})$

$= \Pr(Z < -\frac{0.02}{0.04}) = \Pr(Z < -\frac{1}{2}) = 1 - \Pr(Z < \frac{1}{2})$

From tables $= 1 - (0.6915) = 0.3085$

Number of women $= 0.3085 \times 305 = 94.09 = 94$ to the nearest person.

Question 30 [B]

Mean = np

Standard Deviation = $\sqrt{npq}$

Mean = $60 \times 0.65 = 39$

Standard Deviation =

$\sqrt{60 \times 0.65 \times 0.35} = \sqrt{13.65} \approx 3.69$

Answer is correct to 2 decimal places

Part II (Short answer questions)

Question 1

${}^nC_r(ax)^r(b)^{n-r}$ since we are finding the coefficient of the x^3 term $r = 3$

${}^5C_3(ax)^3(b)^2$

$10a^3x^3b^2 = 1080x^3$

$a^3x^3b^2 = 108x^3$

$a^3b^2 = 108$ Eqn 1 **[M1]**

${}^nC_r(ax)^r(b)^{n-r}$ since we are finding the coefficient of the x^1 term $r = 1$

${}^5C_4(ax)^1(b)^4$

$5a^1x^1b^4 = 240x^1$

$a^1x^1b^4 = 48x^1$

$a^1b^4 = 48$ Eqn 2 **[A1]**

Rearrange eqn 1 and sub into eqn 2

$a^3b^2 = 108 \Rightarrow b^2 = \frac{108}{a^3} \Rightarrow b^4 = \frac{108^2}{a^6}$

Eqn 2 $a(\frac{108^2}{a^6}) = 48$

$\frac{108^2}{a^5} = 48 \Rightarrow \frac{108^2}{48} = a^5$

$\sqrt[5]{\frac{108^2}{48}} = a \Rightarrow a = 3$ **[M1]**

Question 2

$f(x) = 2\tan(x)\sin(x)$

Let $u = 2\tan(x)$

Let $v = \sin(x)$

$\frac{du}{dx} = 2\sec^2 x$

$\frac{dv}{dx} = \cos x$

Product Rule $f'(x) = u.\frac{dv}{dx} + v.\frac{du}{dx}$ **[M1]**

$f'(x) = 2\tan x.\cos x + \sin x.2\sec^2 x$

$f'(x) = 2\left(\tan x.\cos x + \sin x.\sec^2 x\right)$ **[A1]**

$f'(\frac{\pi}{3}) = 2\tan(\frac{\pi}{3}).\cos(\frac{\pi}{3}) + \sin(\frac{\pi}{3}).2\sec^2(\frac{\pi}{3})$

$f'(\frac{\pi}{3}) = 2\sqrt{3} \times 0.5 + \frac{\sqrt{3}}{2} \times 2 \times 4$

$f'(\frac{\pi}{3}) = \sqrt{3} + 4\sqrt{3} = 5\sqrt{3}$ **[A1]**

Question 3

a. Hypergeometric distribution

$$\Pr(X=0) = \frac{{}^3C_0\,{}^9C_3}{{}^{12}C_3} = 0.3818$$

$$\Pr(X=1) = \frac{{}^3C_1\,{}^9C_2}{{}^{12}C_3} = 0.4909$$

$$\Pr(X=2) = \frac{{}^3C_2\,{}^9C_1}{{}^{12}C_3} = 0.1227$$

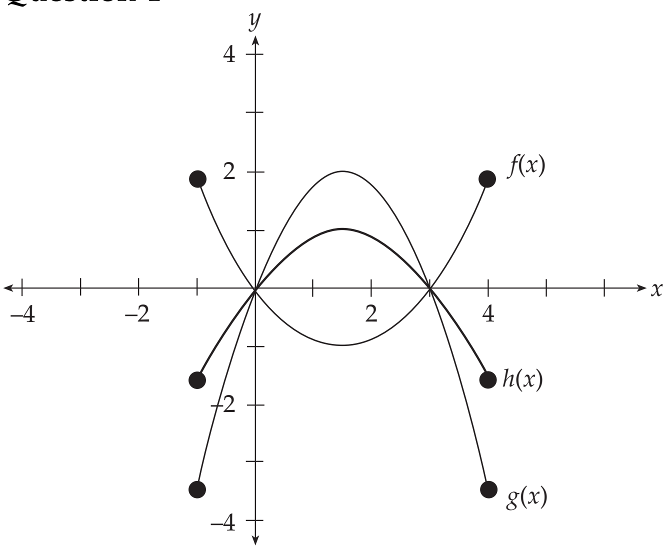
$$\Pr(X=3) = \frac{{}^3C_3\,{}^9C_0}{{}^{12}C_3} = 0.0045$$

X	0	1	2	3
Pr(X = x)	.3818	.4909	.1227	.0045

[A2]

b. $\Pr(x \geq 1) = 0.4909 + 0.1227 + 0.0045 = 0.6181$
or 0.6181 **[A2]**

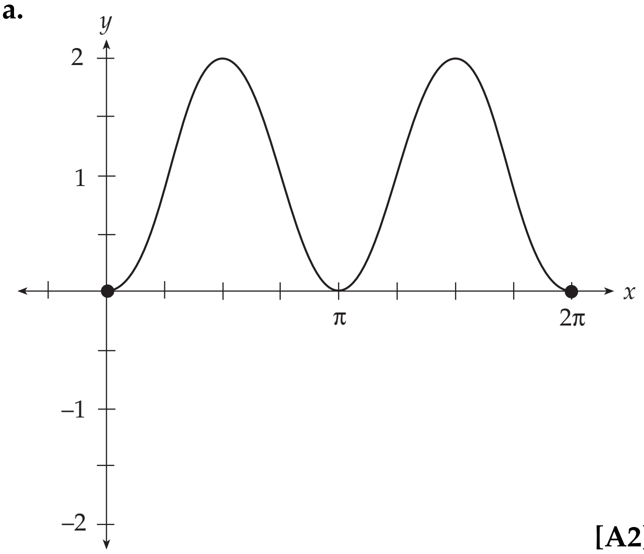
Question 4



Important:
Must have the same x-intercepts **[A1]**
Must have both end points approximately 1.5 above g(x)'s end points and h(x)'s turning point must be between g(x)'s turning point and the x-axis **[A1]**

Question 5

a.



[A2]

b. y = 1 – cos(2x) **[A1]**

Question 6

a.

$y - y_1 = m(x - x_1)$ where $m = 4, x_1 = 2, y_1 = 4$

$y - 4 = 4(x - 2)$

$y - 4 = 4x - 8$

$y = 4x - 4$ **[A1]**

b.

$$\text{Area} = \int_0^2 x^2\,dx - \int_0^2 (4x-4)\,dx = \int_0^2 \left(x^2 - 4x + 4\right)dx$$

$$\left[\frac{x^3}{3} - 2x^2 + 4x\right]_0^2$$

$$\left[\frac{2^3}{3} - 2\times 2^2 + 4\times 2\right] - \left[\frac{0^3}{3} - 2\times 0^2 + 4\times 0\right]$$

$$\left[\frac{8}{3} - 8 + 8\right] - [0] = \frac{8}{3}\text{sq units}$$

[A1]

Question 7

Determine the points P and Q via either of two methods:

1. Equating the line and the curve.
2. The graphics calculator

$$\frac{7}{5}-\frac{6}{5}x=\frac{7}{5}x^2+\frac{1}{5}x-7$$

$$0=\frac{7}{5}x^2+\frac{7}{5}x-\frac{42}{5}$$

$$0=\frac{7}{5}(x^2+x-6)$$

$$0=(x+3)(x-2)$$

$$x=-3,+2$$

Substitute into the line $5y+6x=7$

$x=-3$	$x=2$
$5y-18=7$	$5y+12=7$
$5y=25$	$5y=-5$
$y=5\Rightarrow(-3,5)$	$y=-1\Rightarrow(2,-1)$

[A2]

$$\overline{PQ}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\overline{PQ}=\sqrt{(2--3)^2+(-1-5)^2}$$

$$\overline{PQ}=\sqrt{(5)^2+(-6)^2}$$

$$\overline{PQ}=\sqrt{25+36}$$

$$\overline{PQ}=\sqrt{61}$$ **[A1]**

or using the graphics calculator to determine the intersection points.