

Trial Examination 2 Solutions

Question 1

a. i. At 10am $t = 1$, $\therefore r = \frac{1}{2}$ km [A1]

ii. Area = πr^2 , Area = $\pi(\frac{1}{2})^2 = \frac{\pi}{4}$ km² [A1]

b. $r = \frac{t}{2}$, $\frac{dr}{dt} = \frac{1}{2}$ km/hr [A1]

c. $r = 10$ km, $\therefore 10 = \frac{t}{2}$, $t = 20$ hrs [M1]

9 am + 20 hrs = 5 am on December 26 [A1]

d. Let x = distance from oil slick centre to Swino.

$$x^2 = 10^2 + 5^2$$

$$x^2 = 125$$

$$x \approx 11.18 \text{ kms } \therefore x \approx 11.2 \text{ kms}$$

Swino is 11.2 km from the centre to 1 dp. [A1]

e. $r = 11.2$ km, $\therefore 11.2 = \frac{t}{2}$, $t = 22.4$ [M1]

$$9 \text{ am} + 22.4 \text{ hrs} = 7 \text{ am and } 24 \text{ minutes}$$

$$\therefore 7:24 \text{ am on December } 26 \quad [A1]$$

f. Circumference = $2\pi r$

$$\therefore 2\pi r = \frac{2\pi}{5} \Rightarrow r = \frac{1}{5} \quad [A1]$$

$$\therefore k(2) = \frac{1}{5} \Rightarrow k = \frac{1}{10} \quad [A1]$$

g. Distance travelled = Circumference [M1]

$$\therefore \frac{t^2}{10} = 2\pi kt \Rightarrow \frac{t^2}{10} = \frac{2\pi t}{10} \Rightarrow t = 2\pi \text{ hrs} \quad [A1]$$

$$\therefore r = kt \Rightarrow r = \frac{2\pi}{10} \Rightarrow r = \frac{\pi}{5} \text{ kms} \quad [A1]$$

h. Time to encircle the slick = 2π hrs

$$\text{Distance boat travels } \therefore x = \frac{4\pi^2}{10} \Rightarrow x = \frac{2\pi^2}{5} \quad [A1]$$

$$\begin{aligned} \therefore \text{Length of boom unused} &= 4 - \frac{2\pi^2}{5} \\ &= \frac{2(10 - \pi^2)}{5} \text{ km} \end{aligned} \quad [A1]$$

Question 2

a. i. $B = 15$ [A1]

ii. $A = -3$ [A1]

iii. $P = 365$

$$\therefore P = \frac{2\pi}{n}$$

$$365 = \frac{2\pi}{n} \Rightarrow n = \frac{2\pi}{365} \quad [A1]$$

$$T(t) = -3 \sin\left(\frac{2\pi}{365}\right)t + 15 \quad [A1]$$

b. Max temperature = 18°C [A1]

$$18 = -3 \sin\left(\frac{2\pi}{365}\right)t + 15$$

$$-1 = \sin\left(\frac{2\pi}{365}\right)t$$

$$\sin^{-1}(-1) = \frac{2\pi}{365}t \Rightarrow \frac{3\pi}{2} = \frac{2\pi}{365}t \text{ for } 0 \leq t \leq 365$$

$$1095\pi = 4\pi t \Rightarrow t = 273.75 \Rightarrow t = 274 \quad [A1]$$

c. 16.5°C or above occurs in the domain $\{t: 215 \leq t \leq 335\}$ to nearest day [A2]

d. $t = 182.5$

$$P = k \cos(\pi) \Rightarrow P = -k \Rightarrow -3 = -k$$

Population decreased by 3%

$$\therefore P = -3$$

$$\therefore -3 = -k$$

$$\therefore k = 3 \quad [A1]$$

e. $3 \cos\left(\frac{2\pi t}{365}\right) = \sin\left(\frac{2\pi t}{365}\right)$

$$3 = \tan\left(\frac{2\pi t}{365}\right) \quad \cos\frac{2\pi t}{365} \neq 0 \quad [M1]$$

$$3 = \tan\left(\frac{2\pi t}{365}\right) \Rightarrow \tan^{-1}(3) = \frac{2\pi t}{365} \Rightarrow \frac{2\pi t}{365} = 1.1071$$

$$t = 72.559 \Rightarrow t = 73 \text{ to the nearest day} \quad [A1]$$

f. $t = 255$ using a graph as calculator. [A1]

- g. $t = 73$ is the 13th of July 1998
 $t = 255$ is the 11th of January 1999 [A2]

Question 3

- a. $3k^2 + \frac{7k}{6} + \frac{7k}{12} + \frac{k}{4} = 1$ [M1]
 $\frac{36k^2}{12} + \frac{14k}{12} + \frac{7k}{12} + \frac{3k}{12} = 1$
 $36k^2 + 24k - 12 = 0$ [A1]
 $12(3k - 1)(k + 1) = 0$
 $k = -1$ or $\frac{1}{3}$ $\therefore k = \frac{1}{3}$ as $k \geq 0$
 is due to $0 \leq Pr(X = x) \leq 1$ or must be +ve [A1]

- b. $Pr(x \leq 2) = Pr(x = 1) + Pr(x = 2)$
 $Pr(x \leq 2) = 3 \times \frac{1}{9} + \frac{7}{6} \times \frac{1}{3}$ [M1]
 $Pr(x \leq 2) = \frac{1}{3} + \frac{7}{18} = \frac{13}{18}$ [A1]

- c. $\frac{dC}{dt} = \frac{1}{2}(-3t^2 + \frac{16t}{3} + 11) = 0$, for stationary point [M1]
 $-3t^2 + \frac{16t}{3} + 11 = 0$ [A1]
 $-9t^2 + 16t + 33 = 0$
 $9t^2 - 16t - 33 = 0$
 $(9t + 11)(t - 3) = 0$
 $t = -\frac{11}{9}$, or 3 $\therefore t = 3$ hrs as $0 \leq t \leq 4.9$ [A1]

Sub into C(t)

- $\frac{1}{2}(-3^3 + \frac{72}{3} + 33) = \frac{1}{2}(-27 + \frac{72}{3} + 33)$
 $\frac{1}{2}(\frac{90}{3}) = 15$
 \therefore maximum concentration = 15% [A1]
- d. $\frac{dC}{dt} = \frac{1}{2}(-3t^2 + \frac{16t}{3} + 11)$
 At $t = \frac{8}{9}$ hrs, $\frac{dC}{dt} \approx 6.82\%$
 $\therefore 7\%$ to the nearest per cent [A1]

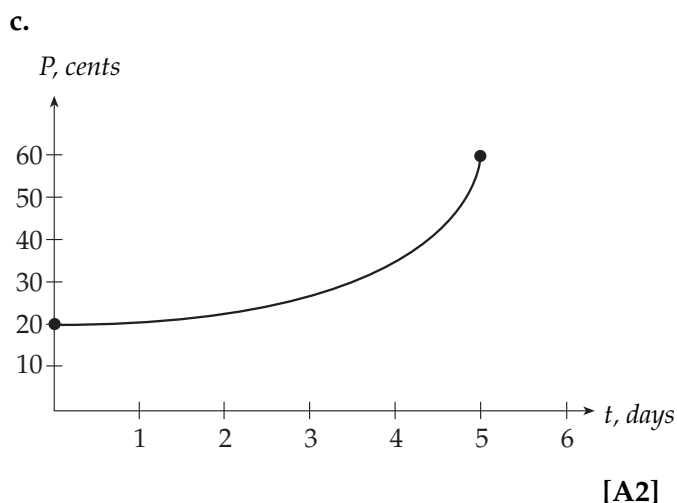
- e. Let $X =$ conc. of petalene
 $Pr(X < (p - 4)) = 0.1587$
 $Pr(X < (p - 4)) = Pr(Z < \frac{p - 4 - p}{d})$
 $Pr(Z < \frac{-4}{d}) = 0.1587$
 $1 - Pr(Z > \frac{-4}{d}) = 0.1587$ [M2]
 $\therefore Pr(Z > \frac{4}{d}) = 0.1587$ using symmetry
 $\therefore Pr(Z < \frac{4}{d}) = 0.8413$
 \therefore from tables $\frac{4}{d} = 1$
 $d = 4$ [A1]

- f. $Pr\left(X < \left(\frac{3p}{2} - 10\right)\right) = 0.5987$
 $Pr\left(X < \left(\frac{3p}{2} - 10\right)\right) = 0.5987 = Pr\left(Z < \frac{\frac{3p}{2} - 10 - p}{4}\right)$
 $Pr\left(Z < \frac{\frac{p}{2} - 10}{4}\right) = 0.5987$
 $\frac{\frac{p}{2} - 10}{4} = 0.25$ [M2]
 $\therefore p = 22$ [A1]

Question 4

- a. i. $t = 0, P = 20 \therefore 20 = Ae^0 + B$
 $20 = A + B$ [A1]
- ii. $t = 3, P = 25 \therefore 25 = Ae^3 + B$ [A1]
- iii. Using $25 = Ae^3 + B$ and $20 = Ae^0 + B$ [M1]
 $25 = Ae^3 + B$
 Subtract $20 = Ae^0 + B$
 $5 = A(e^3 - 1)$
 $A = \frac{5}{(e^3 - 1)} \approx 0.262$ [A1]
 Substitute into $20 = Ae^0 + B$
 $20 = 0.262 + B$
 $B = 19.738$ to three d.p.s.
 $A = 0.262$ [A1]

b. $t = 5, \therefore P = 0.262e^5 + 19.738 = 58.6$ cents
to one d.p. [A1]



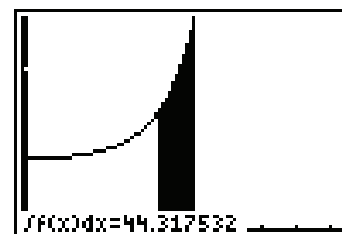
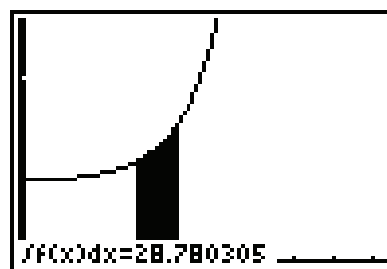
It would be an advantage to use a graphics calculator for the questions from here onwards.

d. i. $\int_1^2 (0.262e^t + 19.738) dt$
 $= [0.262e^t + 19.738t]_1^2$ [A1]
 $= [0.262e^2 + 19.738(2)] - [0.262e^1 + 19.738(1)]$
 ≈ 20.96 sq units to 2 dps. [A1]

ii. $\int_2^3 (0.262e^t + 19.738) dt$
 $[0.262e^t + 19.738t]_2^3$ [A1]
 $[0.262e^3 + 19.738(3)] - [0.262e^2 + 19.738(2)]$
 $= 23.06$ sq units to 2 dps. [A1]

iii. $23.06 - 20.96 = 2.1$
 The percentage increase
 $= \frac{2.1}{20.96} \times \frac{100\%}{1} = 10\%$
 to the nearest per cent. [A1]

e. i. Using the graphic calculator



Thursday to Friday = 28.78 sq units
 Friday to Monday = 44.32 sq units [A1]
 (to 2 dps)
 Percentage Increase
 Wednesday to Thursday area and
 Thursday to Friday area increase is 24.8%
 Thursday to Friday area and Friday to
 Monday area increase is 54% to nearest %.
 \therefore At the end of Friday 18th August or
 at the start of Monday 21st August [A1]

ii. 54% increase in one day [A1]