Mathematical Methods GA 2: Written examination 1

GENERAL COMMENTS

The number of students who sat for the 2000 examination was 16 898, which was 2.6% less than the 17 349 who sat in 1999. Almost 7% scored 90% or more of the available marks (compared with 10% in 1999) with only sixteen students receiving full marks (substantially less than the 146 in 1999). Over 42% of students received over 50% of the available marks, a decrease of about 8% from 1999.

The overall quality of the responses to the examination was similar to that of recent years, although not as strong as last year. This may be due to the increase in the weighting of short-answer questions for 2000 and the inclusion of 'graphics-calculator-active questions'. As in recent years, there were some excellent responses, receiving full or almost full marks. There was also a significant percentage who obtained less than 5 marks on the short-answer part of the examination. The length of the paper appeared to be appropriate, with no clear evidence that students did not have sufficient time to complete the paper.

As has been noted in the *Report for Teachers* over several years, many students displayed poor algebraic skills, poor setting out and poor use of mathematical notation. Although it was evident in student responses that graphics calculators were being employed, they were often used inappropriately or incorrectly. The one question in the short-answer part that required students to use the graphing calculator to obtain the correct response was often attempted without its use.

Students need to be able to use **analytical**, **numerical** or **graphical** approaches to tackle questions, consistent with the area of study and outcomes for Mathematical Methods in the Study Design. Where a numerical answer to a question, or part of a question, is required, this may be obtained using any of these three approaches unless instructed otherwise. Students will need sufficient opportunity to become conversant with these approaches to identify which approach is likely to be more effective or efficient in a particular context.

There may well be some questions where no readily accessible analytical approach or no analytical approach, is available, in other instances it may be the case that only an analytical approach will be suitable. Where a numerical answer is required, students are expected to provide the answers correct to the specified accuracy.

SPECIFIC INFORMATION

Multiple-choice questions (Total 27 marks)

The third column gives the percentage of correct responses.

1	A	7
2	B	86
3	E	87
4	D	44
5	A	69
6	D	37
7	D	64
8	C	64
9	E	72
10	E	50
11	B	75
12	B	52
13	A	52
14	E	40
15	B	69
16	B	49
17	D	46
18	A	64
19	D	30
20	A	36
21	C	64
22	E	39
23	C	88
24	B	66
25	E	55
26	B	62
27	A	59

Short-answer questions (Total 23 marks)

Question 1 (Average mark 1/Available marks 2)

Correct response:

$$
\begin{aligned}
\Pr(X > 55) &= \Pr\left(Z > \frac{5}{3}\right) \\
&= 1 - \Pr\left(Z < \frac{5}{3}\right) \\
&= 1 - 0.9522 \\
&= 0.0478
\end{aligned}
$$

$$
\begin{aligned}
\text{Required probability} &= \Pr(X < 45) + \Pr(X > 55) \\
&= 2 \times 0.0478 \\
&= 0.096
\end{aligned}
$$

Most students recognised this as a normal distribution question. However, some students found the satisfactory rather than unsatisfactory probability, or forgot to double their answer after finding correctly Pr ($X > 55$).

Question 2 (1.56/3)

Correct response:

$X \sim \text{Hg}\ (4, 12, 15)$

$$
\begin{aligned}
\Pr(X \geq 3) &= \Pr(X = 3) + \Pr(X = 4) \\
&= \frac{{}^{12}C_3 \cdot {}^{3}C_1}{{}^{15}C_4} + \frac{{}^{12}C_4 \cdot {}^{3}C_0}{{}^{15}C_4} \\
&= 0.846
\end{aligned}
$$

Most students attempted this question, but in general, this question was not done well. While a large number of students recognised it involved a hypergeometric distribution, quite a few tried to employ the binomial distribution and some even tried the normal distribution. Setting out was very poor and many students tried to find Pr ($X > 3$) or Pr ($X \leq 3$). Often students used 1 – Pr ($X \leq 3$), including the probability for $X = 3$ and therefore obtaining an incorrect solution. Although substitution into a formula was generally well done, the calculation of the combinations was quite poor.

Question 3

a. (0.33/1)

Correct response:

$$
\begin{aligned}
f_1(x) &= (x - 1)(x - 3)(x + 2) \\
f_2(x) &= 4
\end{aligned}
$$

This is only one of several possible correct responses. For those students who knew what was required this was the easiest and most obvious partitioning of the given function. Many students expanded, often incorrectly, the function and then attempted to partition. Some expanded the expression and gave that as their response. Too many students had no idea what was required by the question.

A common response was $(x – 1)(x – 3)$ and $(x + 2) + 4$.

b. (0.78/3)

Correct response:

If $f(x) = 0$

Then $x^3 - 2x^2 - 5x + 10 = 0$

$(x - 2)(x^2 - 5) = 0$

so $x = 2, \sqrt{5}, -\sqrt{5}$

The expansion and factorisation required in answering this question proved to be beyond many students. It was also disappointing to see the number of students who did not know what the term 'roots' means. A number of students knew how to use their graphics calculator to obtain a response, but this did not give exact values as required. Others probably used the graph on their calculator but missed $x = 2, \sqrt{5}, -\sqrt{5}$, as it was so close to 2, a common response to the question was 1, 3, –2.

Question 4 (0.77/2)

Correct response:

- dilated by a factor of 0.5 from the y-axis
- dilated by a factor of 5 from the x-axis.

There were a number of acceptable responses to this question. Many students obtained a mark by simply stating 'dilation' (with various different spellings). It was common for students to state amplitude doubled and period halved and to give the new equation but not to state the transformations involved. A small but significant number of students also gave the answer as a dilation of factor 2 parallel to both axes.

Question 5 (0.89/2)

Correct response:

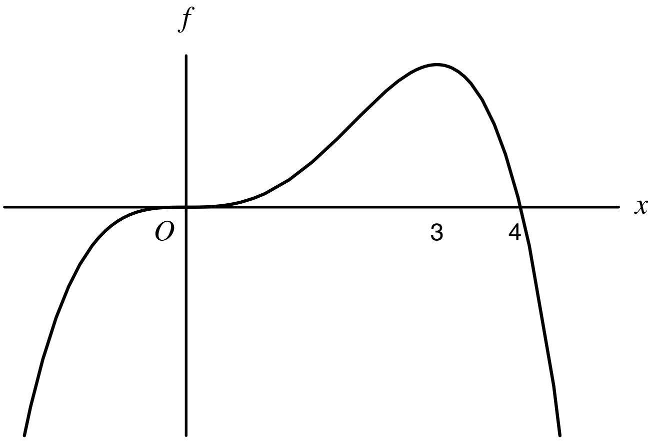
$$
\begin{aligned}
\tan(2x) &= \sqrt{3} \\
2x &= \frac{-5\pi}{3}, \frac{-2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3} \\
x &= \frac{-5\pi}{6}, \frac{-\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}
\end{aligned}
$$

This question was generally quite well done, although simple algebraic errors were often seen.

Far too many students simply cancelled out the 2 in sin ($2x$) and cos ($2x$) to obtain $\tan x = \sqrt{3}$. It was also common to see tan ($2x$) or $\tan x = \frac{1}{\sqrt{3}}$. Many students obtained the correct four responses. Sometimes the negative responses were given as the negative of the correct positive responses or simply missed out altogether.

Question 6 (1.21/3)

Correct response:



This question was generally handled better than other questions requiring a graphical response in recent years. Many students showed the turning point and x-axis intercepts correctly, but often missed the stationary point of inflexion at the origin. The most common mistakes were:

- a cubic with negative x^3 term with x-intercepts of 0 and 4 and a minimum turning point at $x = 3$;
- a parabola with positive x^2 term with x-intercepts at 0 and 4, or some variation on this idea;
- a cubic with an additional x-intercept rather than a stationary point of inflexion at the origin.

Students who obtained the correct stationary point of inflexion, and maximum turning point also usually had the correct x-intercepts.

Question 7

a. **(0.34 /1)**
b. **(0.29/1)**
c. **(0.21/1)**

Correct responses:

a. 104.896
b. 1 100
c. –2 when $x = 100$

This question required students to use their graphics calculator and several of its functions. Those students who were able to do this efficiently generally obtained full marks. However, while many students used their calculator for part a. they happily differentiated $\log_{10}(x)$ 'by hand' to get $\frac{1}{x}$ without any apparent dissatisfaction. The most common mistake for part a. was stating the value of x for which the maximum occurred rather than the maximum value. In part b. most students found $x = 100$, but neglected $x = 1$. In part c. quite a lot of students tried unsuccessfully to differentiate $\log_{10}(x)$ by hand. It was interesting to note that students who otherwise obtained few marks overall on the paper, often obtained full marks for this question because they were able to use their graphics calculator.

Question 8

a. (0.3/1)

Correct response:

$$\frac{dy}{dx} = \log_e(2x) + 1 - 1 = \log_e(2x)$$

This question was done quite poorly. Students were unable to use the product rule correctly in a number of cases, and also found it difficult to differentiate the logarithm part. Quite a number of students treated the function as $x$ln and $(2x - x)$, resulting in meaningless and incorrect work.

b. (1.02/3)

Correct response:

$$\int_{0.5}^{\frac{e}{2}} \log_e(2x)dx = \left[x\log_e(2x) - x\right]_{0.5}^{\frac{e}{2}}$$

$$= \frac{e}{2}\log_e(e) - \frac{e}{2} - 0.5\log_e(1) + 0.5$$

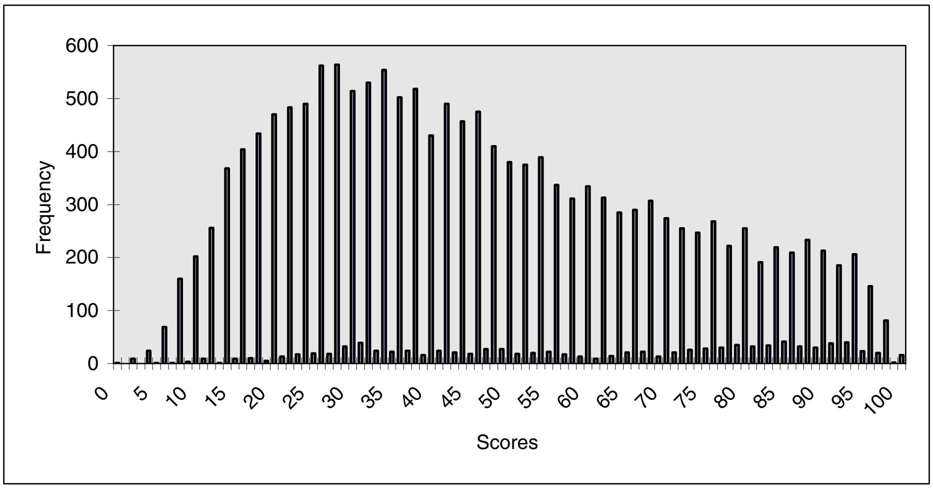
$$= 0.5$$

Some students obtained 0.5 from incorrect working. This was probably due to an ability to use their calculators in some cases, and in others recognition that no matter what they had written for part a. the original function needed to be used in part b. Both of these approaches resulted in either 1 mark or no marks being awarded, as the instructions **hence** required students to use their result for part a. Many students were unable to obtain even the first mark, as they could not write down the definite integral correctly with the correct function and terminals.

GA MA082 MATHEMATICS: Mathematical Methods WRITTEN EXAMINATION 1

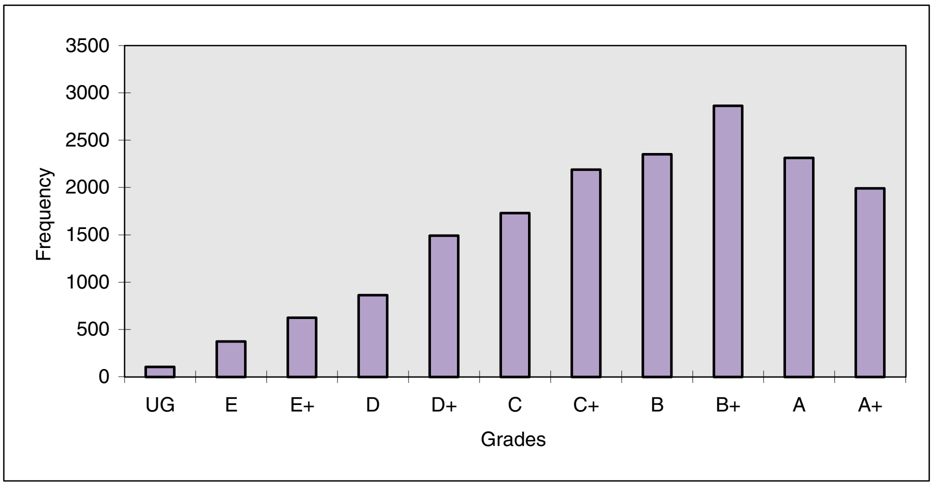
HISTOGRAM OF TOTAL SCORES 2000

Count 16898 Mean 47.09 Standard Deviation 23.85 NA Result 503



HISTOGRAM OF TOTAL GRADES 2000

Count 16898 Mean 6.63 Standard Deviation 2.41 NA Result 503



ENROLMENTS		%
Female	7922	45.5
Male	9479	54.5
Total	17401	

GLOSSARY OF TERMS

Count Number of students undertaking the assessment. This excludes those for whom NA was the result.

Mean This is the ‘average’ score; that is all scores totalled then divided by the ‘Count’.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).