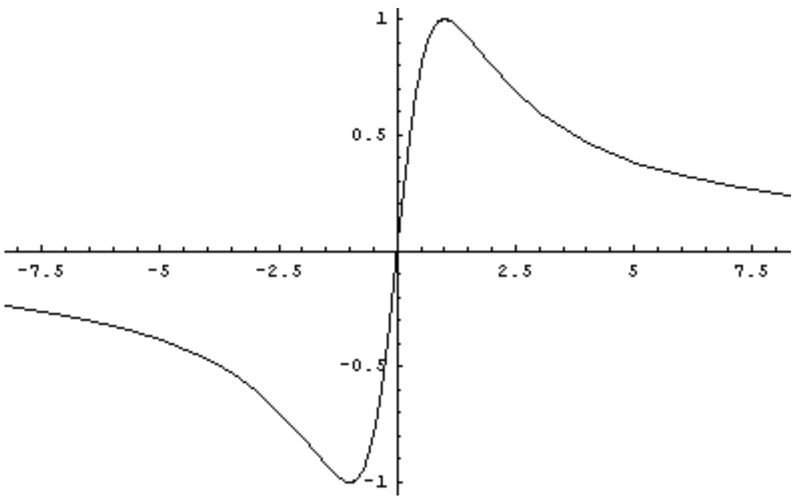
Mathematical Methods (CAS) pilot study: supplementary questions – short answer

Question 1

The following diagram shows part of the graph of a function f:

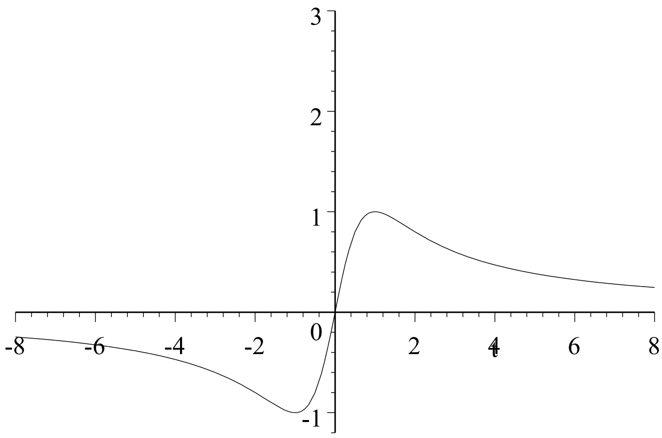


On the same set of axes as the graph of f, draw the corresponding part of the graph of $|f|$.

2 marks

Question 2

The following diagram shows part of the graph of a function f:

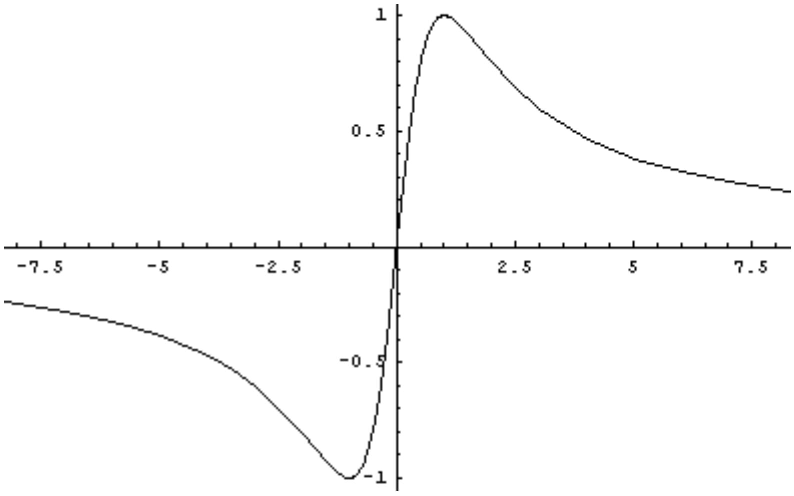


On the same set of axes as the graph of f, draw the corresponding part of the graph of f'.

2 marks

Question 3

The following diagram shows part of the graph of a function f:

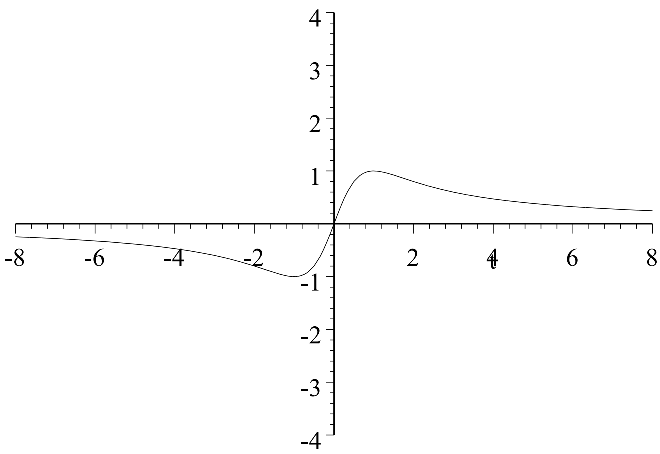


On the same set of axes as the graph of f, draw the corresponding part of the graph of f^2.

2 marks

Question 4

The following diagram shows part of the graph of a function f:



On the same set of axes as the graph of f, draw the corresponding part of the graph of $\dfrac{1}{f}$.

2 marks

Question 5

Consider the function $f: R \to R$, where $f(x) = x^3 + 2x^2 - 5x - 9$. Let P be the point on the graph of f in the first quadrant whose distance from the x-axis is 1. Find the x-coordinate of P.

2 marks

Question 6

a. Let $h(x) = f(x)\sin(x)$. If $f(\frac{\pi}{3}) = a$ and $f'(\frac{\pi}{3}) = b$, express $h'(\frac{\pi}{3})$ in terms of a and b.

b. Find the exact value of $h'(\frac{\pi}{3})$ if $f(x) = \log_e(x)$.

2 + 1 = 3 marks

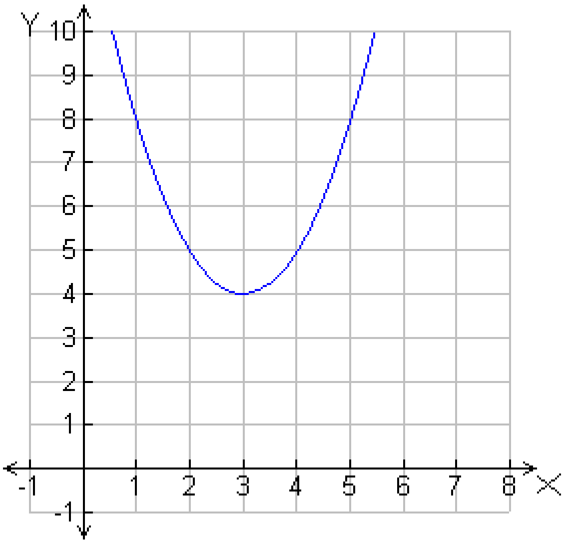
Question 7

Let $g(x) = f(\cos(x))$. If $g'(x) = -3\cos^2(x)\sin(x)$, find $f(x)$.

1 mark

Question 8

Part of the graph of the function *f* with rule $f(x) = (x-3)^2 + 4$ is shown below.



a. Find the distance from the origin to the turning point of the graph of *f*.

b. Let *P* be a point on the graph of *f* such that the distance from the origin to *P* is a minimum. Determine the co-ordinates of the point *P*, correct to two decimal places.

1 + 3 = 4 marks

Question 9

Let the function *f* be defined on the set of real numbers, where $f'(x) = 2 - \cos(x)$

and $f(\pi) = 0$.

a. Find $f(x)$.

b. Explain briefly why *f* has no stationary points.

c. Find the minimum value of the gradient of *f*.

2 + 1 + 1 = 4 marks

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Question 10

a. Find the derivative of $x^3\cos(x)$.

b. Use the product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ with $u = x^3$ to identify v, $\frac{du}{dx}$ and $\frac{dv}{dx}$ in your answer to part **a**.

1 + 1 = 2 marks

Question 11

Economists study the effects on the demand, D, for a product brought about by a change in the price p, in dollars, of the product. They use the price elasticity of demand function, E, defined by:

$$E(p) = \frac{-pD'(p)}{D(p)} \quad \text{where } p > 0.$$

a. If the demand for a product is given by $D(p) = \sqrt{200 - p}$ for prices between 50 and 200 dollars inclusive, find the rule for the price elasticity of demand, E, stating its maximal domain.

b. The demand for a product is said to be elastic when $E(p) > 1$.

Determine the values of p, where $50 < p < 200$, for which the demand for the product given in **a**. is elastic.

1 + 2 = 3 marks

Question 12

a. Sketch the graph of $f(x) = |\cos(x)|$, $0 \le x \le 2\pi$.

b. State the period of f.

c. Find the area of the region enclosed by the graph of f, and the x-axis, between $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

2 + 1 + 2 = 5 marks

Question 13

A quartic polynomial function, f, with the rule $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ is uniquely defined by the following set of conditions:

$$f(-1)=0\,;\; f(0)=4\,;\; f'(1)=-2\,;\; f'(0)=0\,;\; f(2)=-12$$

a. Each of the above conditions can be written as a linear equation in terms of a, b, c, d and e. Write down the corresponding linear equation for each condition:

$f(-1) = 0$ ____

$f(0) = 4$ ____

$f'(1) = -2$ ____

$f'(0) = 0$ ____

$f(2) = -12$ ____

b. Write this system of simultaneous linear equations in matrix form.

c. Solve this system of simultaneous linear equations and state the rule for the corresponding quartic polynomial function.

d. Find the coordinates of any stationary points of f.

2 + 1+ 1 + 2 = 6 marks

Question 14

a. Find the equation of the image of the graph for each of the following functions

under the transformation defined by $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

i. $y = x^2 - 2$;

ii. $y = \dfrac{2}{x-1}$

b. Sketch the graph of the image in **a. ii.** and state the nature of the transformation.

2 + 3 = 5 marks

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Question 15

Consider the following pair of simultaneous equations in terms of k.

$$\begin{aligned} 2x - y &= 1 \\ x + ky &= k \end{aligned}$$

a. Find the solution to the pair of simultaneous equations when $k = 1$ and interpret this solution graphically.

b. Find the solution to the pair of simultaneous equations in terms of k.

c. For the value(s) of k which give no solution, explain what this means graphically.

3 + 2 + 1 = 6 marks

Question 16

Let $q(x) = x^2 + bx + c$. Express $q(x)$ as a product of two linear factors, given that the two solutions of $q(x) = 0$ have a sum of -1 and a product of -6.

2 marks

Question 17

Consider the function, f, with the rule $f(x) = x^3 - 6x^2 + mx + n$.

a. The gradient of f at $x = 1$ is 2. Find the value of m.

b. Given that $(x - 3)$ is a factor of $f(x)$, find the value of n.

2 + 2 = 4 marks

Question 18

An athlete's heart rate, in beats per minute, was monitored during some exercises lasting 10 minutes. Assume that the heart rate, h, in beats per minute, at any time t, in minutes, can be modelled by the function

$$h:[0,10] \to R \text{ with rule } h(t) = at^3 + bt^2 + ct + d$$

The athlete's initial heart rate was 60 beats/minute, and the following additional information was obtained for these exercises.

$$h(2) = 92,\ \ h'(4) = -12, \text{ and } h(10) = 220.$$

a. Identify which piece of information corresponds to each one of the following linear equations:

$100a + 10b + c = 16$ __________________ Equation 1

$48a + 8b + c = -12$ __________________ Equation 2

$4a + 2b + c = 16$ __________________ Equation 3

b. Using these equations, or otherwise, find the rule for h.

c. Write down an appropriate definite integral for the average heart rate in beats per minute during these exercises, and hence find this average heart rate.

2 + 1 + 1 + 1 = 5 marks

Question 19

Consider $f(x) = \dfrac{x^4 - 1}{4}$ and $g(x) = \dfrac{x^3 - 1}{3}$.

a. Factorise $f(x) - g(x)$ over R.

b. Explain why $f(x) \geq g(x)$ for all $x \in R$.

1 + 2 = 3 marks

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Question 20

(*This question highlights a range of items that could be asked. These would not typically be combined in a single short answer question.*)

A continuous random variable, X, has probability density function, f, defined by

$$f(x) = \begin{cases} A(1-x)^2 & \text{if} \quad 0 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$ where A is a positive real number.

Find:

a. the value of A;

b. the probability that X is greater than 2, correct to three decimal places;

c. the probability that X takes a value between 1 and 2, correct to three decimal places;

d. the expected value of X, $E(X)$, correct to three decimal places;

e. the mode of X, correct to three decimal places ;

f. the variance of X, $V(X)$, correct to three decimal places.

2 + 2 + 2 + 2 + 2 +2 = 12 marks

Question 21

a. A jar contains red and black marbles. If n marbles are selected at random from the jar with replacement, write down an expression for the probability of getting exactly three red marbles.

b. Consider the particular case where four marbles are selected at random from the jar with replacement. If the probability, correct to four decimal places, of getting exactly three red marbles is 0.0988, find the probability, correct to four decimal places, of getting exactly one red marble.

2 + 2 = 4 marks

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Question 22

The probability distribution for the discrete random variable X is given below:

x	0	1	2	3
$\Pr(X = x)$	$\frac{1}{2}k^3$	$\frac{1}{2}k^3$	$\frac{3}{4} - 3k^2$	$1 - \frac{1}{4}k$

a. Find k.

b. Find the probability that X takes a value less than 2.

2 + 1 = 3 marks

Question 23

Let $f : R \to R$ where $f(x) = \begin{cases} a(1 - x^2) & \text{for } -1 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$ and a is a real number.

a. Find the value of a for which f could be a probability density function for the continuous random variable X.

b. Find the variance of X, for this probability density function, correct to three decimal places.

c. Find the value of k such that $\Pr(-k \le X \le k) = 0.95$, correct to three decimal places.

2 + 2 + 2 = 6 marks

Mathematical Methods (CAS) pilot study: supplementary questions – short answer

Question 24

Consider the probability density function defined by $p(t) = \begin{cases} te^{-t} & \text{if} \quad t > 0 \\ 0 & \text{elsewhere} \end{cases}$.

a. Find the mode, correct to four decimal places.

b. Find the median, correct to four decimal places.

c. Find the probability that the value of the random variable, T, with probability density function p, is greater than 6, correct to four decimal places.

2 + 2 + 2 = 6 marks

Question 25

If Mary eats out one evening, there is a probability of 0.7 that she will eat at home the following evening. If she eats at home one evening, there is a probability of 0.6 that she will eat at home the following evening. Assume that, whether Mary eats at home or eats out, where she eats each evening depends only on whether she ate at home or ate out on the previous evening. On 1 February this year Mary ate at home.

a. Find the probability that Mary ate at home on the evening of 4 February this year, correct to three decimal places.

b. Find the probability that Mary ate at home each evening from 1 February to 4 February inclusive, this year, correct to three decimal places.

b. Find the percentage of times that Mary would eat at home, in the long term, correct to one decimal place.

2 + 1 + 2 = 5 marks

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Question 26

The amount, in 10 000's of litres, of oil processed by a refinery during a month is a random variable Y which has probability density function given by

$$f(y) = \begin{cases} ky, & 0 \le y < 1 \\ \dfrac{k}{2}(3 - y), & 1 \le y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the exact value of k.

b. What is the probability of processing more than 15 000 litres of oil in a month, correct to three decimal places?

2 + 2 = 4 marks

Question 27

The number, N, in millions, of bacteria in a culture was monitored over an eight hour period. The results are recorded in the following table.

t (hours)	0	1	2	3	4	5	6	7	8
N	9	32	75	144	245	384	567	800	1089

a. Use difference tables to determine whether the most appropriate polynomial model for the data is linear, quadratic, cubic or quartic polynomial function.

b. Determine the equation of the most appropriate polynomial model for the data.

2 + 2 = 4 marks