GENERAL COMMENTS

The number of students presenting for Mathematical Methods Examination 2 in 2001 was 17 468, an increase on the 16 875 who sat in 2000. The entire range of marks from 0 to 55 occurred. Student responses showed that the paper was accessible and provided opportunities for students to demonstrate what they knew. There were excellent papers presented by some students who achieved perfect or near-perfect scores. There were the usual number of students who were unable to do more than a couple of mark's worth of work, despite some marks that could have been gained using knowledge and skills from earlier years. Generally, although the paper was considered to be the same standard as the 2000 paper, students in 2001 performed better, particularly at the upper end of the scale.

The competence demonstrated by students in the use of their graphics calculators was quite varied. Some students tried to use it for almost everything (despite exact answers or the use of calculus being required) and others showed a poor knowledge of how to use it effectively. Most students used the graphics calculator well in appropriate situations. There was evidence of use to check answers (such as in Question 1eii) and of intelligent use where the graphics calculator's potential to help with the question was not obvious. There was also evidence of poor graphics calculator use – a $\boxed{\text{TRACE}}$ button method or similar to find intersections or intercepts is neither accurate nor quick, and many students should be better able to determine an appropriate window when drawing graphs, as noted in the 2000 Report to Teachers and the April 2001 *VCE Bulletin*.

Some parts of questions on the paper (e.g. probability question) can be done quickly using a graphics calculator without any working being shown. If the question is worth more than 1 mark, students risk losing all the available marks if the answer is all that is written down, and it is incorrect. When a question is worth more than 1 mark, it is usual to award a 'method' mark if the student has indicated something of the way they have arrived at the answer. It makes good sense to write down something which may gain this mark even if the answer happens to subsequently be incorrect.

For example, in Question 2e, correct answer (i.e. 0.909) with nothing else, gained 2 marks. An answer of 0.908 with no working scored 0 marks. One mark out of the two was awarded if the answer was incorrect but working, such as one or more of the following, was shown:

- normalcdf(30, ∞, 32, 1.5)
- normal, $X > 30$
- $Z > -1\frac{1}{3}$
- a normal distribution diagram with the appropriate mean and value marked, and the correct area shaded.

Many students lost marks because they:

- did not answer the question asked
- gave decimal answers when an exact answer was required
- gave the wrong number of decimal places
- misread the question in other ways
- did not pay sufficient attention to detail in sketching graphs
- were not sufficiently careful, particularly with algebra.

For example:

- Question 3f: ignoring the 'hence' instruction and finding the area required by integrating the inverse function
- Question 4aiii: not labelling intercepts or asymptotes, despite a clear instruction to do so.

Question 3c: where required to **show** a given result was handled poorly (this skill should be taught and practised). A suitable response was:

$$\frac{dy}{dx} = \frac{-4}{3-4x}$$

Since $x < \frac{3}{4}$, $3 - 4x > 0$.

The denominator of $\frac{dy}{dx}$ is always positive and the numerator always negative, so $\frac{dy}{dx}$ is always negative.

SPECIFIC INFORMATION

Question 1

	Answer	Mean marks	Available marks
a.	29°, 21°	1.61	2
b.	3pm	0.72	1
c.	7am and 11pm	1.28	2
d.	28.46°	1.10	2
ei.	$\frac{dT}{dt} = \frac{\pi}{3}\sin\left(\frac{\pi(t-3)}{12}\right)$	0.40	1
eii.	3.73 to 14.27	0.91	3

This question required a very straightforward application of basic knowledge of circular functions, yet the total mean score was less than half the available marks. While the first four parts were handled well, usually by efficient calculator use, part e often caused problems. Many students were not able to apply the chain rule correctly to differentiate the function and many found only one value of t in part ii, or else failed to express their answer as an interval.

In part a, most students were able to focus on the maximum and minimum values of the cosine function and not launch into differentiation. In part c, a surprisingly large number gave only one answer, usually 7 am, ignoring the other solution which their graphic calculator should have shown them. A significant number of students did not calculate the time for the maximum temperature (the time of the absolute maximum ±2 hours) in part d.

An appropriate response to the 'hence' instruction in part eii was often not clearly presented, with no evidence that the derivative had been used. Teachers should make students aware of the need to express their argument clearly enough to show the method being used in this situation. In this case, the derivative from part i had to be equated to 0.2 to indicate that the correct method was used.

Question 2

	Answer	Mean marks	Available marks
a.	1.6	1.31	2
b.	0.243	0.52	2
c.	0.35	0.62	1
di.	0.271	1.45	3
dii.	22.5	0.59	1
e.	0.909	1.26	2
f.	4	0.32	2
g.	32.82	0.92	3

A number of students who did well on the rest of the paper did not attempt, or poorly attempted, this question. This kind of question shows the need to spend time teaching students how to identify the different probability distributions described in the study design.

Most students could get the correct expected number for part a, although some wanted to round the 1.6 to 2, arguing incorrectly that there must be a whole number of candles. For those who did not get 1.6, a method mark was available if there was an indication that the hypergeometric distribution had been recognised and some numbers correctly substituted in the formula.

Part b could be answered using elementary probability ideas, including a tree diagram, although many students then omitted a 3 from their answer. A method mark was available for demonstrating an attempt that could have led to the correct answer of 0.243. Part c was generally well done and most students worked successfully with a reduced sample space.

Part d was concerned with a binomial probability distribution and most students recognised this. Many failed to use $\Pr(X \geq 24) = \Pr(X = 24) + \Pr(X = 25)$ (probably because they did not correctly interpret '24 or more') and too many made rounding errors. A method mark was available for recognition of the correct distribution and the correct numbers associated with it; for example, Bin(25, 0.9) was sufficient. Good use of a graphics calculator in this question could produce the correct answer very quickly.

Most students were able to identify and correctly use the normal distribution in part e. Some lost marks for replacing $\frac{4}{3}$ with 1.3.

Few students appreciated what was required for part f, despite there being a similar question on the 2000 paper. The equation $1 - (0.5)^n = 0.9375$ had to be derived and solved. Few students were able to do this, some trying things with inverse normal distribution and others using 0.9 instead of 0.5.

In part g, it was necessary to establish that $\frac{30-\mu}{1.5} = -1.882$ and solve to get $\mu = 32.82$. A common error was to use +1.882 and a significant number of students did not manage to find the answer to a sufficient accuracy.

Question 3

	Answer	Mean marks	Available marks
a.	$(0, \log_e(3)), \left(\frac{1}{2}, 0\right)$	1.41	2
b.	$\left(-\infty, \frac{3}{4}\right)$	0.37	1
c.	(see previous comment)	0.65	2
di.	$f^{-1}(x) = \frac{1}{4}(3 - e^x)$	1.41	2
dii.	R	0.51	1
e.	Graph of $f^{-1}(x)$	0.83	2
fi.	$\frac{dy}{dx} = -1 - \log_e(3-4x)$	0.90	2
fii.	$-\frac{1}{2} + \frac{3}{4}\log_e(3)$	0.71	3

It was good to see students tackling later parts of this question when they had not succeeded with early parts; this is an important examination technique which teachers should encourage in their students.

While part a was generally answered well, some students lost marks because they did not find the *coordinates* of the intercepts or because they did not give *exact values*.

In part b, some students relied on the graphics calculator providing the endpoint of the domain instead of using the fact that a logarithm function must have a positive argument. Some of the notation presented for the domain demonstrated poor mathematical communication, since the answer is a set.

Part c was poorly done by many students. Some failed to use the chain rule correctly and many more could not argue meaningfully or well about the sign of the derivative. Only about 10% of students scored full marks for 3c. While most students could gain the first mark in part d for knowing to interchange the variables, poor algebra led to many errors. Part dii was well done; most students knowing that the range of the inverse is the domain of the function.

The graph in part e was generally well answered, although a common error was to label the asymptote as $y = 1$ instead of $y = \frac{3}{4}$, presumably from reading the calculator incorrectly.

In fi, students were required to use the product and chain rules, causing many problems. Of those who were successful, few simplified the expression, making marks in part ii hard to get. Few students knew what to do in part ii although this technique should have been well known and practised. Algebraic manipulation was the cause of many lost marks here.

A sound solution to part f could be:

i. $y = \frac{1}{4}(3-4x)\log_e(3-4x)$

$$\frac{dy}{dx} = \frac{-4(3-4x)}{4(3-4x)} + \frac{-4}{4}\log_e(3-4x) = -1 - \log_e(3-4x)$$

ii. Area = $\int_0^{\frac{1}{2}} \log_e(3-4x)dx$

From i, $\log_e(3-4x) = -1 - \frac{dy}{dx}$ so

$$\int_0^{\frac{1}{2}} \log_e(3-4x)dx = \int_0^{\frac{1}{2}} (-1 - \frac{dy}{dx})dx$$

$$= [-x - y]_0^{\frac{1}{2}}$$

$$= \left[-x - \frac{1}{4}(3-4x)\log_e(3-4x)\right]_0^{\frac{1}{2}}$$

$$= -\frac{1}{2} - \frac{1}{4}\log_e(1) + \frac{3}{4}\log_e(3)$$

$$= -\frac{1}{2} + \frac{3}{4}\log_e(3)$$

A common error was to give the antiderivative of −1 as −1 or 0. Students who (hopefully) found an approximate area using their calculators were not rewarded with any marks. Others found the answer using the right method and others checked their answer with the graphics calculator (a good approach).

Question 4

	Answer	Mean marks	Available marks
ai.	500 000	0.77	1
aii.	1	0.52	1
aiii.	Graph of N(t)	0.77	2
bi.	$\frac{dN}{dt} = \frac{-1}{(t+1)^3} + 6e^{-6t}$	0.90	2
bii.	$t_{\max} = 0.50, N_{\max} = 1.172$	0.62	2
c.	$y = 0.16x^3 - 1.2x^2 + 10$	1.13	5

If a graphics calculator was used for this question, it required careful selection of graphics windows and interpretation of resultant graphs. Part ai could be read from a suitable graph or found by simple substitution and was well done. In part ii, it was difficult to get a value from a graph, and much better to allow $t \to \infty$ in the equation. Part iii required careful use of a graphics calculator and then the drawing and labelling of an asymptote. Through a poor choice of window, some students did not graph important features. It was evident that a number of students had not previously encountered a graph which crosses its asymptote and some strange graphs resulted from them trying to avoid this feature.

In part bi, many students lost marks through incorrect differentiation, particularly of the first term. Part bii required the use of a calculator with sufficient accuracy and students lost marks through rounding. Those who tried to solve $\frac{dy}{dx} = 0$ had no success.

More successful students to enjoyed part c, getting 4 or 5 marks. It required the setting up of a general cubic function with rule $y = ax^3 + bx^2 + cx + d$ and the evaluation of constants from the information given. The technique should not have been unfamiliar to students, and many set about the question in the correct way, while a significant number could not set up the initial function correctly. Some incorrectly wrote $y = x^3 + bx^2 + cx + d$, while many found the necessary algebra too much. An alternative approach is to set up the function in factor form. Another alternative approach started with the derivative:

Let $\frac{dy}{dx} = kx(x-5) = k(x^2 - 5x)$

Then $y = k\left(\frac{x^3}{3} - \frac{5x^2}{2}\right) + c$.

When $x = 0$, $y = 10$, so $c = 10$.

When $x = 5$, $y = 0$, so $k = \frac{12}{25}$.

then $y = \frac{12}{25}\left(\frac{x^3}{3} - \frac{5x^2}{2}\right) + 10$.