Question 1 D

$x-2 \neq 0$

$\therefore$ Vertical asymptote at $x = 2$

$$\begin{array}{r} 1 \\ x-2\overline{)\,x-1} \\ \underline{x-2} \\ 1 \end{array}$$

$f(x) = 1 + \dfrac{1}{x-2}$

$\therefore$ Horizontal asymptote at $y = 1$

Question 2 D

Interchange x and y

$x = \dfrac{1}{2}e^{y+1}$

$2x = e^{y+1}$

$y+1 = \log_e(2x)$

$y = \log_e(2x) - 1$

Question 3 A

$f(x)$ represents a semi circle with centre (0,0) and radius 2. This semi circle is an increasing function for

$-2 \le x \le 0$

$\therefore [-2,0]$

Question 4 B

A maximum value occurs when $\cos\dfrac{\pi(t-1)}{4} = -1$

So $\dfrac{\pi(t-1)}{4} = \pi, 3\pi, 5\pi\ldots.$

$\pi(t-1) = 4\pi, 12\pi, 20\pi\ldots..$

$(t-1) = 4, 12, 20\ldots.$

$t = 5, 13, 21\ldots.$

A maximum value occurs when $t = 5$

Question 5 D

$\sin 2\theta = -\cos 2\theta$

$\tan 2\theta = -1$

$2\theta = \pi - \dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}, 3\pi - \dfrac{\pi}{4}, 4\pi - \dfrac{\pi}{4}$

$0 \le \theta \le 2\pi \qquad 0 \le 2\theta \le 4\pi$

$2\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, \dfrac{11\pi}{4}, \dfrac{15\pi}{4}$

$\theta = \dfrac{3\pi}{8}, \dfrac{7\pi}{8}, \dfrac{11\pi}{8}, \dfrac{15\pi}{8}$

Difference between largest and smallest solution is

$\dfrac{15\pi}{8} - \dfrac{3\pi}{8} = \dfrac{12\pi}{8} = \dfrac{3\pi}{2}$

Question 6 E

This graph has an amplitude of 4 and is not translated up or down.

Question 7 A

$$\begin{aligned} \log_a\left(\frac{b}{c}\right)^2 &= 2\log_a\left(\frac{b}{c}\right) \\ &= 2\left[\log_a b - \log_a c\right] \\ &= 2[1.6 - 2.2] \\ &= 2[-0.6] \\ &= -1.2 \end{aligned}$$

Question 8 E

$P(2) = 8 + 4a + 2b - 24$ (1)

$P(-3) = -27 + 9a - 3b - 24$ (2)

$4a + 2b = 16$ (1a)

$9a - 3b = 51$ (2a)

(1a) × 3 $12a + 6b = 48$

(2a) × 2 $\underline{18a - 6b = 102}$

Add $30a = 150$

$a = 5$

Substituting $a = 5$ in (1a)

$20 + 2b = 16$

$2b = -4 \Rightarrow b = -2$

Question 9 A

$y = b + 2a^{3x}$

$y - b = 2a^{3x}$

$$\frac{y-b}{2} = a^{3x}$$

$$3x = \log_a\left(\frac{y-b}{2}\right)$$

$$x = \frac{1}{3}\log_a\left(\frac{y-b}{2}\right)$$

Question 10 C

Since the vertical asymptote has the equation $x = 1$, then the graph of $\log_e x$ has been translated one unit to the right, so $A = -1$.

When $x = 2$, $y = 3$,

so $\log_e 1 + B = B = y$

$\Rightarrow B = 3$

Question 11 B

Solutions are $x = 0$ (double root) and $x = 3$

$\therefore$ graph has equation $y = Kx^2\,(x - 3)$

When $x < 3$, $y < 0$

When $x > 3$, $y > 0$

$\therefore$ K is positive, so graph could be $y = x^2\,(x - 3)$

$\therefore$ $y = x^3 - 3x^2$

Question 12 A

This graph has amplitude = 3

It is translated 1 unit up.

Period = $\pi = \dfrac{2\pi}{n}$

$\therefore n = 2$

It is a sin graph

When $\theta = 0$, $3\sin 2(\theta + \dfrac{\pi}{4}) + 1 = 4$ so not B, C, D or E

Question 13 B

Point of intersection is the point where

$$x^2 + 1 = \frac{1}{x}$$

Entering the two equations $y = x^2 + 1$ and $y = \dfrac{1}{x}$ in the graphics calculator, it can be seen that 0.682 is closest to the x coordinate for the point of intersection.

Question 14 E

$y = \sin\log_e(x)$

Let $u = \log_e(x)$

$$\frac{du}{dx} = \frac{1}{x}$$

$y = \sin u$

$$\frac{dy}{du} = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{1}{x}$$

$$\frac{dy}{dx} = \cos\log_e x \times \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{\cos\log_e x}{x}$$

Question 15 A

$$f\,'(x) = x^2 \times \frac{d}{dx}\cos 2x + \cos 2x \times \frac{d}{dx}x^2$$

$$= x^2 \times (-2\sin 2x) + \cos 2x \times 2x$$

$$= -2x^2\sin 2x + 2x\cos 2x$$

$$= 2x(\cos 2x - x\sin 2x)$$

Question 16 E

For turning points $f\,'(x) = 0$

$$\frac{(5+e^x)e^x - e^x(e^x)}{(5+e^x)^2} = 0$$

$$\therefore e^x(5+e^x-e^x) = 0$$

$$\therefore 5e^x = 0$$

$$\therefore e^x = 0$$

Which is not possible $x \in R$, so there are no stationary points.

Question 17 E

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + 1}\,dx$$

Let $u = \sin x + 1$

$du = \cos x\,dx$

When $x = 0, u = 1$

When $x = \frac{\pi}{2}, u = 2$

$$\int_1^2 \frac{du}{u} = \log_e u]_1^2$$

$=\log_e 2 - \log_e 1$

$= \log_e 2$

$=0.693$

Question 18 C

$$f(x) = \int -7e^{-2x}dx$$

$$f(x) = \frac{7}{2}e^{-2x} + c$$

When $x = 0$

$$14 = \frac{7}{2} + c$$

$c = 10.5$

Hence, $f(x) = 3.5e^{-2x} + 10.5$

$f(x) = Ae^{-2x} + B$

$A = 3.5, B = 10.5$

Question 19 B

Reading the graph from left to right, $f(x)$ has a negative gradient, then a positive gradient, then a negative gradient. Hence A or B. When $x = 0$, the gradient is positive so not A.

Question 20 B

$$y = \frac{2}{x}$$

$$\int_1^3 y\,dx = 2\int_1^3 \frac{1}{x}\,dx = 2[\log_e x]_1^3$$

$$= 2[\log_e 3 - \log_e 1]$$

$$= 2\log_e 3$$

$$= 2.2$$

which is closest to 2

Question 21 D

$\mu = np$

$6 = 10 \times p$

$0.6 = p$

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\Pr(X = 8) = \binom{10}{8} (0.6)^8 (0.4)^2$$

$$= 0.121$$

Question 22 E

$$\sum \Pr(X = x) = 1$$

$$\frac{6-4a}{8} + \frac{3+2a}{8} + \frac{3-2a}{8} = 1$$

$$\frac{12-4a}{8} = 1$$

$$12 - 4a = 8$$

$$4 = 4a$$

$$1 = a$$

Question 23 D

Binomial with $n = 4$, $p = 0.7$, $x = 3$ or 4

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\Pr(X = 3) + \Pr(X = 4)$$

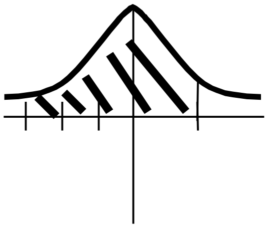
$$= \binom{4}{3}(0.7)^3(0.3)^1 + \binom{4}{4}(0.7)^4(0.3)^0$$

$$= 0.4116 + 0.2401$$

$$= 0.6517$$

which is closest to 0.7

Question 24 C



$\Pr X < Z = 0.7$

$Z = 0.524$

$$Z = \frac{a-\mu}{\sigma}$$

$$0.524 = \frac{a-2}{2.5}$$

$$a - 2 = 1.31$$

$$a = 3.31$$

Question 25 E

Total Outcomes = $\binom{52}{13}$

Favourable Outcomes = $\binom{4}{1} \times \binom{48}{12}$

$$\text{Pr.} = \frac{\binom{4}{1} \times \binom{48}{12}}{\binom{52}{13}}$$

$$= 0.44$$

Question 26 A

$$f^{'}(x) = \frac{(x^2 - 1) \times 1 - x(2x)}{(x^2 - 1)^2}$$

$$f^{'}(x) = \frac{x^2 - 1 - 2x^2}{(x^2 - 1)^2}$$

$$f^{'}(x) = \frac{-(x^2 + 1)}{(x^2 - 1)^2}$$

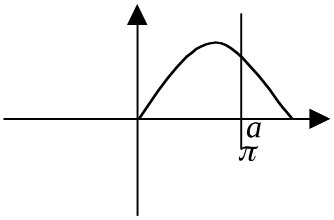
For $x \in R / \{-1, 1\}$ the denominator is always positive and the numerator is always negative

$\therefore f^{'}(x) < 0$

$f^{'}(x) \neq 0$ because $x^2 \neq -1$

$f(x)$ can equal 0 when $x = 0$

Question 27 C



$$\int_0^a 2\sin x dx = \frac{1}{3}\int_a^{\pi} 2\sin x dx$$

$$-2\cos x]_0^a = \frac{1}{3}[-2\cos x]_a^{\pi}$$

$$-6[\cos a - \cos 0] = -2[\cos \pi - \cos a]$$

$$3[\cos a - 1] = [-1 - \cos a]$$

$$3\cos a - 3 = -1 - \cos a$$

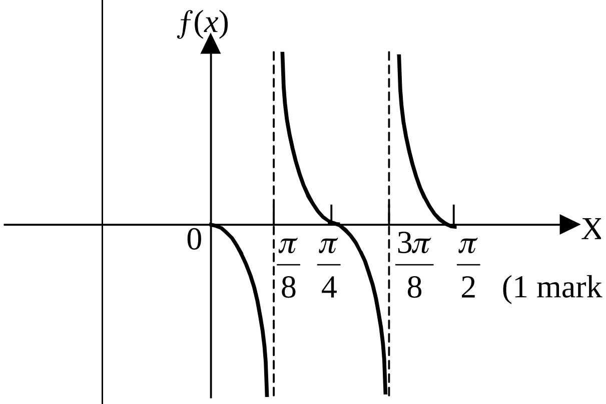
$$4\cos a = 2$$

$$\cos a = \frac{1}{2}$$

$$a = \frac{\pi}{3}$$

Question 1

a.



(1 mark for shape)

(1 mark for asymptote)

b. *R* (1 mark)

c. Reflection in the *X* axis (1 mark)

Question 2

a.

$$\begin{matrix} & & & 1 & & & \\ & & 1 & & 2 & & 1 \\ & 1 & & 3 & & 3 & & 1 \end{matrix}$$ (1 mark)

$(2\sqrt{x})^3 + 3 \times (2\sqrt{x})^2 + 3 \times (2\sqrt{x}) + 1$ (1 mark)

$= 8x^{\frac{3}{2}} + 12x + 6x^{\frac{1}{2}} + 1$

b.

$$\frac{8x^{\frac{3}{2}} + 12x + 6x^{\frac{1}{2}} + 1}{2\sqrt{x}}$$

$= 4x + 6\sqrt{x} + 3 + \frac{1}{2\sqrt{x}}$ (1 mark)

Question 3

a.

$y = 2x^2 - 2x + 3$

$\frac{dy}{dx} = 4x - 2$ (1 mark)

When $x = 1$

$\frac{dy}{dx} = 4 - 2 = 2$ (1 mark)

b.

tan $\theta = 2$

$\theta = \tan^{-1} 2$

$\theta = 63.4^0$

$\theta = 63^0$ to the nearest degree (1 mark)

This is the acute angle

Obtuse angle $= 180 - 63 = 117^0$ (1 mark)

Question 4

a.

$2x - 2 \geq 0$

$2x \geq 2$

$x \geq 1$

Domain: $x \geq 1$

Range: $f(x) \geq 0$ (1 mark)

b.

Interchange *x* and *y*

$x = \sqrt{2y - 2}$

$x^2 = 2y - 2$ (1 mark)

$x^2 + 2 = 2y$

$\frac{1}{2}x^2 + 1 = y$

$f^{-1}(x) = \frac{1}{2}x^2 + 1$ (1 mark)

c.

Domain: $x \geq 0$

Range: $f^{-1}(x) \geq 1$ (1 mark)

Question 5

Probability

$$= \frac{\binom{5}{1}\binom{6}{1}\binom{9}{1}}{\binom{20}{3}} \quad \text{(1 mark)}$$

$$= \frac{5 \times 6 \times 9}{1140}$$

$$= 0.237 \qquad \text{(1 mark)}$$

Question 6

$$g'(x) = \cos\frac{5x}{2} - 2\sin 3x$$

$$g(x) = \int(\cos\frac{5x}{2} - 2\sin 3x)dx \quad \text{(1 mark)}$$

$$\text{g}(x) = \frac{2}{5}\sin\frac{5x}{2} + \frac{2}{3}\cos 3x + c$$

$$g(0) = \frac{2}{5} \times 0 + \frac{2}{3} \times 1 + c = 1$$

$$g(0) = 0 + \frac{2}{3} + c = 1$$

$$\therefore c = \frac{1}{3} \quad \text{(1 mark)}$$

$$\text{g}(x) = \frac{2}{5}\sin\frac{5x}{2} + \frac{2}{3}\cos 3x + \frac{1}{3} \quad \text{(1 mark)}$$

Question 7

a.

$$3e^{-\frac{x}{2}}\cos(2x) = 0$$

$$3e^{-\frac{x}{2}} \neq 0$$

$$\therefore \cos(2x) = 0 \quad 0 \le 2x \le 4\pi \quad \text{(1 mark)}$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \qquad \text{(1 mark)}$$

b.

$$y = e^{\frac{1}{x}}$$

Let $u = \frac{1}{x}$

$$y = e^u$$

$$\frac{dy}{du} = e^u$$

$$\frac{du}{dx} = -\frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$\frac{dy}{dx} = e^u \times (-\frac{1}{x^2}) \quad \text{(1 mark)}$$

$$\frac{dy}{dx} = -\frac{1}{x^2}e^{\frac{1}{x}}$$

But $e^{\frac{1}{x}}$ is always positive, and $\frac{1}{x^2}$ is always positive

So $\frac{dy}{dx}$ is always negative and therefore decreasing.

However $x \neq 0$ since x is in the denominator. (1 mark)

END OF SUGGESTED SOLUTIONS
2002 Mathematical Methods Trial Examination 1

KILBAHA PTY LTD (Publishers in Education)
ABN 47 065 111 373
PO BOX 2227 KEW VIC 3101 AUSTRALIA
TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com