MAV Mathematical Methods Examination 2 Solutions

Question 1

$V = \pi r^2 l + \frac{4}{3}\pi r^3 \text{ cm}^3$

a. S. A. $= 2\pi rl + 4\pi r^2 \text{ cm}^2$ **[A]**

b. If S. A. $= 400 \text{ cm}^2$

$400 = 2\pi rl + 4\pi r^2$ **[M]**

$\frac{400 - 4\pi r^2}{2\pi r} = l$

$\Rightarrow l = \frac{200 - 2\pi r^2}{\pi r}$ cm **[A]**

c. $V = \pi r^2\left(\frac{200 - 2\pi r^2}{\pi r}\right) + \frac{4}{3}\pi r^3$ **[M]**

$= r\left(200 - 2\pi r^2\right) + \frac{4}{3}\pi r^3$

$= 200r - \frac{2}{3}\pi r^3 \text{ cm}^3$ **[A]**

d. $V > 0 \Rightarrow 200r - \frac{2}{3}\pi r^3 > 0$

$r > 0 \Rightarrow 200 - \frac{2}{3}\pi r^2 > 0$

$\Rightarrow \frac{2}{3}\pi r^2 < 200$

$\Rightarrow r^2 < \frac{300}{\pi}$

$\Rightarrow r < \sqrt{\frac{300}{\pi}} \quad (r > 0)$

Domain of V: $0 < r < \sqrt{\frac{300}{\pi}}$ cm **[A][A]**

e. $350 = 200r - \frac{2}{3}\pi r^3$

Solve for r : $\frac{2}{3}\pi r^3 - 200r + 350 = 0$
From TABLE, $r = 8.7$ or 1.8.
Ans : $r = 8.7$ cm **[A]**

f. $\frac{dV}{dr} = 200 - 2\pi r^2$; max where $\frac{dV}{dr} = 0$ **[M]**

$\Rightarrow 100 - \pi r^2 = 0 \Rightarrow r = \frac{10}{\sqrt{\pi}}$ **[A]**

$l = 0$ **[A]**

i.e. a sphere **[A]**

g. Max volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times \frac{1000}{\pi\sqrt{\pi}}$ **[M]**

$= \frac{4000}{3\sqrt{\pi}} \text{ cm}^3$ **[A]**

Question 2

a. mark (5.1, 1.49), (12.3, 1.01) **[A][A]**

b. amplitude $= \frac{1}{2}(1.49 - 1.01)$; $a = 0.24$ **[A]**

$c \rightarrow$ vertical translation $= 1.01 + 0.24$

$c = 1.25$ **[A]**

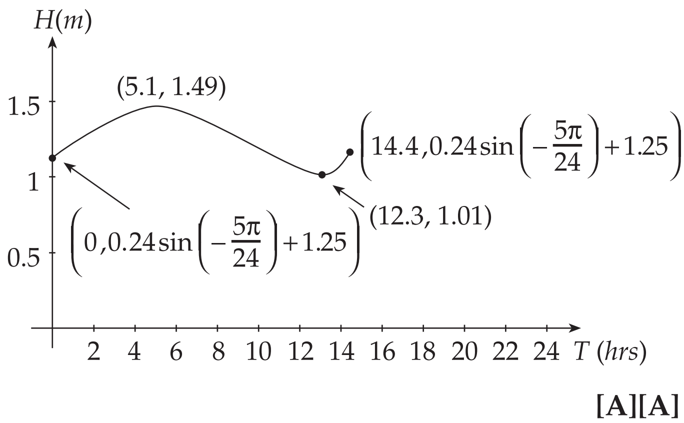
c. $n = \frac{2\pi}{\text{period}}$;

high tide to low tide $= \frac{1}{2}$ period **[M]**

$\therefore n = \frac{2\pi}{2(12.3 - 5.1)}$ **[M]**

$= \frac{\pi}{7.2} = \frac{5\pi}{36}$

d.



[A][A]

e. Use calculator to find $H = 1.2$, or Solve

$$1.2 = 0.24\sin\left[\frac{5\pi}{36}(T-1.5)\right]+1.25$$

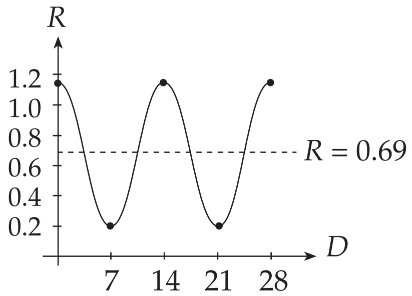
$T = 1{:}01$ am **[A]**

$$\frac{dH}{dT} = 0.1024 \text{ m/hr}$$ **[A]**

f. $R_{max} = 1.18$ metres **[A]**

g. $R_{min} = 0.2$ metres **[A]**

h. $R = 0.49\cos\left(\frac{\pi}{7}D\right)+0.69$



[A][A]

Question 3

a. $f(x) = 3e^{2-x}-1$; $g(x) = x$

Point E; $f(x) = 4 \Rightarrow 3e^{2-x}-1 = 4$

$$\Rightarrow e^{2-x} = \frac{5}{3}$$

$$\Rightarrow 2-x = \log_e\frac{5}{3}$$

$$x = 2-\log_e\frac{5}{3}$$

$\left(2-\log_e\frac{5}{3}, 4\right)$ **[A]**

Point G; $f(x) = 0 \Rightarrow 3e^{2-x}-1 = 0$

$$\Rightarrow e^{2-x} = \frac{1}{3}$$

$$\Rightarrow 2-x = -\log_e 3$$

$$\Rightarrow x = 2+\log_e 3$$

$(2+\log_e 3, 0)$ **[A]**

Point F; $3e^{2-x}-1 = x$
from calculator: $x = 2$
$(2, 2)$ **[A]**

b. Area AFG $= \text{area of } \triangle + \int_{x_1}^{x_2}(3e^{2-x}-1)dx$ **[M]**

$$= 2+\int_{2}^{2+\log_e 3}(3e^{2-x}-1)dx$$ **[M]**

$$= 2+\left[-3e^{2-x}-x\right]_2^{2+\log_e 3}$$ **[M]**

Area $= (4-\log_e 3)$ sq. metres **[A]**

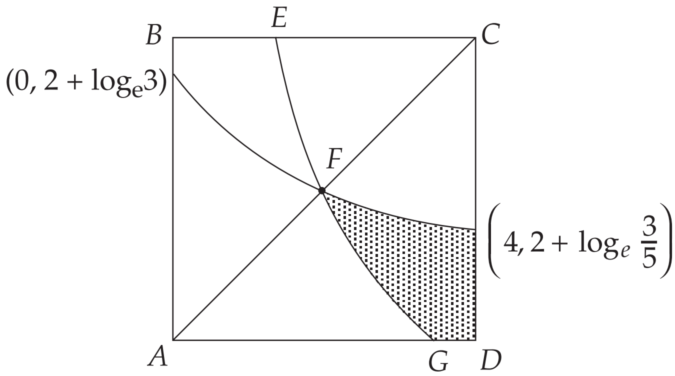
c. Inverse: Let $x = 3e^{2-y}-1$ **[M]**

$$\frac{x+1}{3} = e^{2-y}$$

$$2-y = \log_e\left(\frac{x+1}{3}\right)$$

$$y = 2-\log_e\left(\frac{x+1}{3}\right)$$

$$f^{-1}(x) = 2-\log_e\left(\frac{x+1}{3}\right)$$ **[A]**



sketch **[A]**
end points **[A]**

d. New panel area

$$= \int_2^{2+\log_e 3}\left(\left(f^{-1}(x)\right)-f(x)\right)dx + \int_{2+\log_e 3}^{4} f^{-1}(x)$$

OR

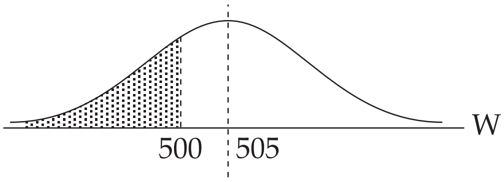
$$= \int_2^4 f^{-1}(x) - \int_2^{2+\log_e 3} f(x)dx$$ **[M]**

$= 3.4459 - 0.9014$ **[M]**

$= 2.5$ square metres **[A]**

Question 4

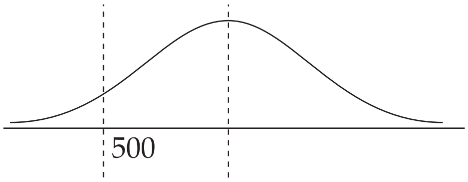
a.



Pr (W< 500) = normal cdf(–1 E 99, 500, 505, 10) **[M]**

≈ 0.3085 **[A]**

b.



Pr $(W \geq 500) = 0.95$

OR

PR $(W < 500) = .05$

inv normal (.05, 0, 1) $\rightarrow Z = -1.64485$ **[M]**

$$z = \frac{x - \mu}{\sigma}$$

If $\sigma = 10$ then $\frac{500 - \mu}{10} = -1.64485$

$\Rightarrow 500 - \mu = -16.4485$

$\Rightarrow \mu = 516.4485$ gm **[A]**

c. If $\mu = 505$ then $\frac{500 - 505}{\sigma} = -1.64485$ **[M]**

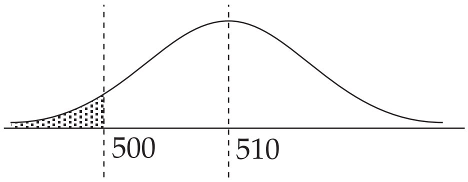
$\Rightarrow \frac{5}{\sigma} = 1.64485$

$\Rightarrow \sigma = 3.0398$ **[A]**

d. In case (**b**) the mean is higher and would cost the company too much.

In case (**c**) the standard deviation of 3 gms may be difficult to achieve, but it would be preferable from a cost point of view. **[A][A]**

e.



$\mu = 510; \quad \sigma = \sqrt{40}$

Pr(bag < 500 gm)

$= \text{normalcdf}(-1\text{E}99,\ 500,\ 510,\ \sqrt{40})$

$= 0.056923$ **[M]**

Let X be the number of bags which weigh less than 500 gm.

$\Pr(X \geq 3) = 1 - \Pr(X \leq 2)$

$= 1 - \text{bimomcdf}(5, 0.056923, 2)$ **[M]**

≈ 0.0017 **[A]**

f. Hypergeometric

N = 25 D = 5

$n = 5$ $x \geq 3$

Pr(box rejected) =

$$\frac{\left({}^{20}C_2\right)\left({}^{5}C_3\right) + \left({}^{20}C_1\right)\left({}^{5}C_4\right) + \left({}^{20}C_0\right)\left({}^{5}C_5\right)}{\left({}^{25}C_5\right)}$$ **[M]**

≈ 0.0377 **[A]**