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Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 1: Facts, Skills and Applications Task

Part I Multiple-choice Question Booklet

Reading time 15 minutes
Writing time 1 hour 30 minutes

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).

Part I consists of this question booklet and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer booklet.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

A detachable formula sheet for use in both parts is in the centrefold of this booklet.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer booklet (Part II).

You may retain this question booklet.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2002 VCE Mathematical Methods Units 3 & 4 Examination 1.

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Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 1: Facts, Skills and Applications Task

Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\qquad$ $\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$ $\qquad$ $\int e^{ax}dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$ $\qquad$ $\int \frac{1}{x}dx = \log_e(x) + c$, for $x > 0$

$\frac{d}{dx}(\sin ax) = a\cos ax$ $\qquad$ $\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$

$\frac{d}{dx}(\cos ax) = -a\sin ax$ $\qquad$ $\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

$\frac{d}{dx}(\tan ax) = \frac{a}{\cos^2 ax} = a\sec^2 ax$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ $\qquad$ quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and probability

$\Pr(A) = 1 - \Pr(A')$ $\qquad$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$ $\qquad$ variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$ $= \Sigma x^2 p(x) - \mu^2$
binomial	${}^nC_x p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\,{}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $N(0, 1)$.		

Table 1: Normal Distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	-5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

PART I

Structure of Booklet

Number of questions	Number of questions to be answered	Marks
27	27	27

Materials

Question booklet of 14 pages with a detachable formula sheet in the centrefold.

Answer sheet for multiple-choice questions.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the centre of this booklet during reading time.

Ensure that you write your **name** and your **teacher's name** in the spaces provided on the cover of the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer booklet (Part II).

SECTION A

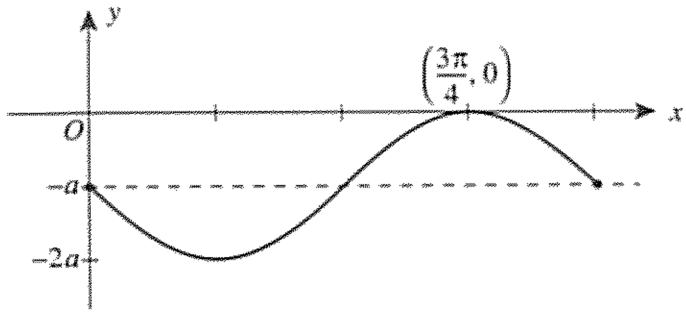
Specific Instructions for Part I

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

Question 1



The graph shown above is best represented by the equation

A. $y = -2a\sin 2(x-a)$

B. $y = a\cos 2x - a$

C. $y = -2a\cos 2x - a$

D. $y = -a\sin \frac{1}{2}x - a$

E. $y = -a\sin 2x - a$

Question 2

The coefficient of the x^2 term in the expansion of $(3x-2)^6$ is

A. 4320
B. −4320
C. 2160
D. −2160
E. 1080

Question 3

The range of the function $f(x) = \log_2 x$ for $x \in (0, 2]$ is

A. R

B. $R^+ \cup \{0\}$

C. $[0, 1]$

D. $(-\infty, 1]$

E. $(-\infty, 2]$

Question 4

$\log_e x - 3\log_e 2x + 2\log_e 3x$ is equal to

A. $\log_e\left(\frac{8}{9}\right)$

B. $\log_e 9 - \log_e 8$

C. $\log_e\left(\frac{9x^3}{8}\right)$

D. $\log_e\left(\frac{9x^6}{8}\right)$

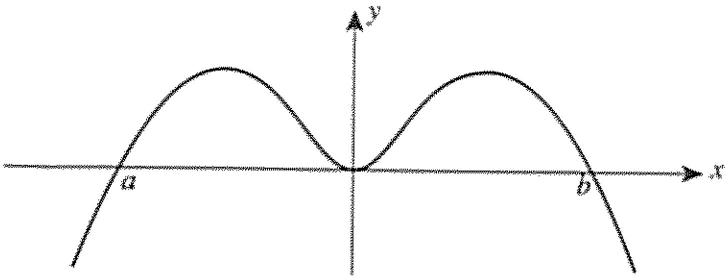
E. $\frac{9}{8}$

Question 5

Which of the following gives the solution or solutions to the equation $\log_2 x(x-1) = 1$?

A. 1 only
B. −2 and 1
C. −1 only
D. 2 only
E. −1 and 2

Question 6



The most likely equation for the polynomial graph shown is

A. $y = x(x-a)(x-b)$

B. $y = x^2(x-a)(x-b)$

C. $y = -x^2(x+a)(x-b)$

D. $y = x^2(a-x)(x-b)$

E. $y = x^2(a-x)(b-x)$

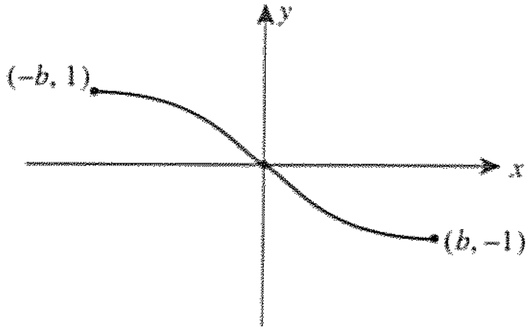
Question 7

The number of solutions to the equation $2\sin 3x = \log_{10} 30x$ is

A. 1
B. 2
C. 3
D. 4
E. 5

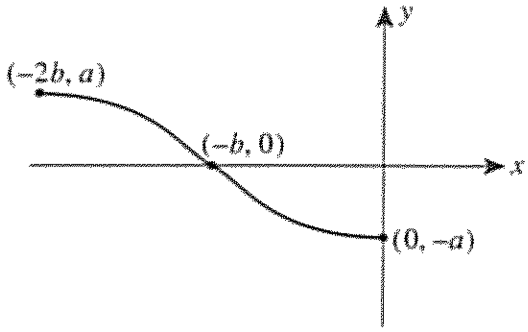
Question 8

The graph of $y = f(x)$ is shown below.

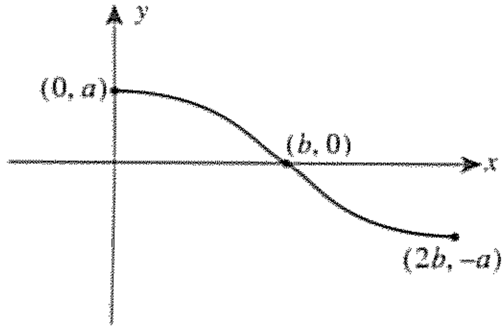


Which of the following graphs best represents $y = af(x - b)$ if $a, b > 0$?

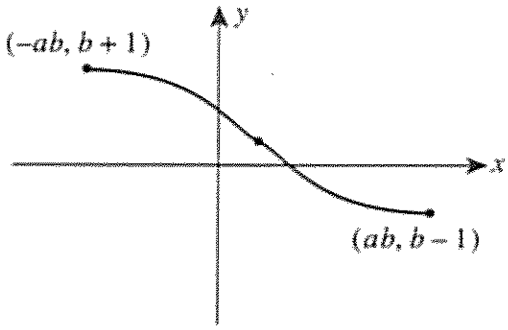
A.



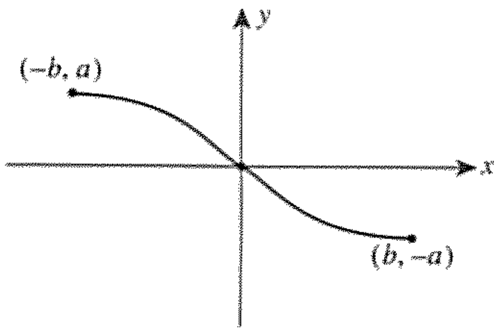
B.



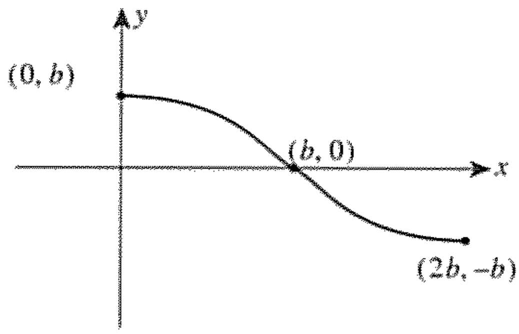
C.



D.

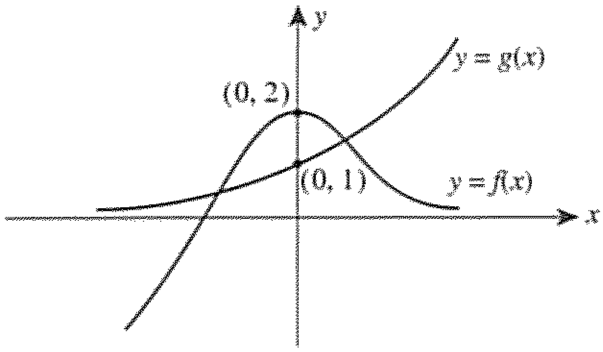


E.



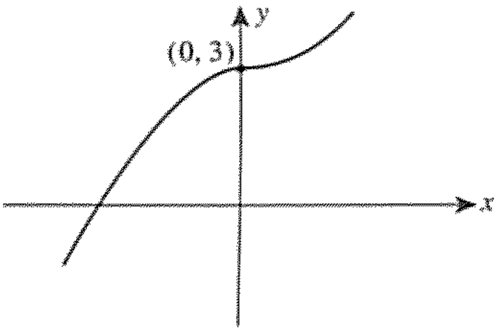
Question 9

The graphs of $y = f(x)$ and $y = g(x)$ are shown on the set of axes below.

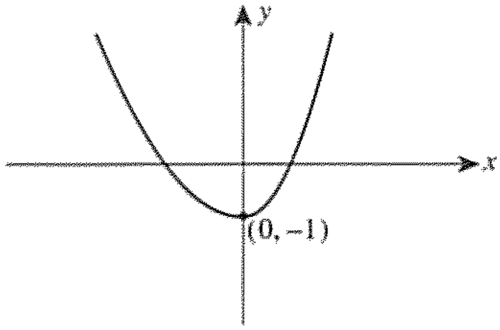


Which of the following is most likely to be the graph of $y = f(x) - g(x)$?

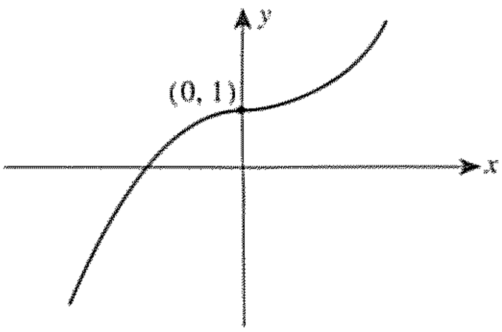
A.



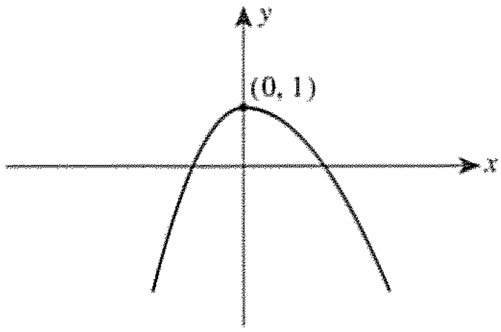
B.



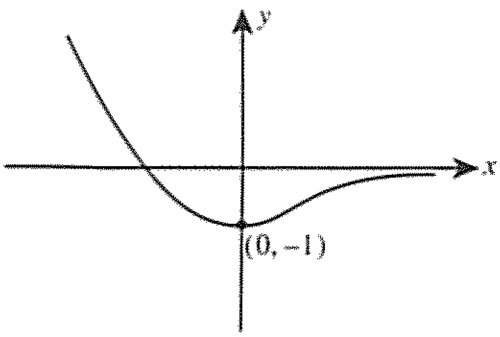
C.



D.

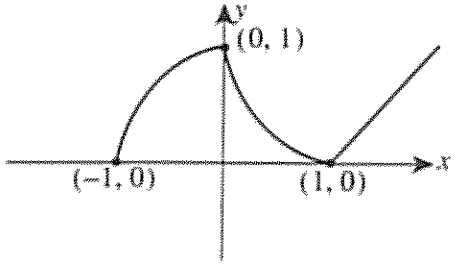


E.



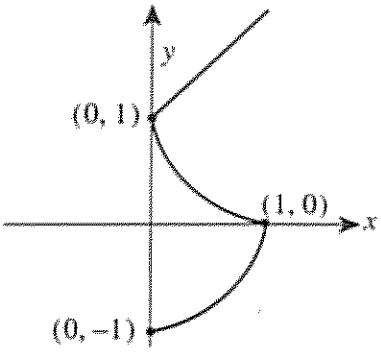
Question 10

The graph $y = f^{-1}(x)$ is shown on the set of axes below.

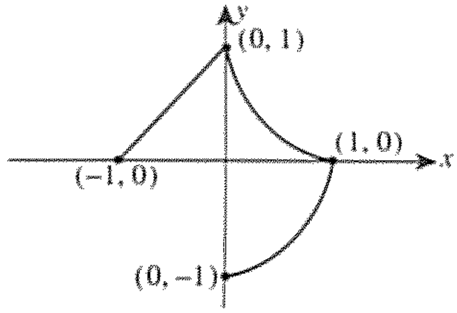


Which of the following is most likely to be the graph of $y = f(x)$?

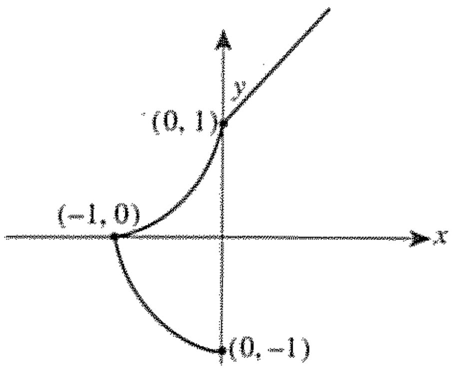
A.



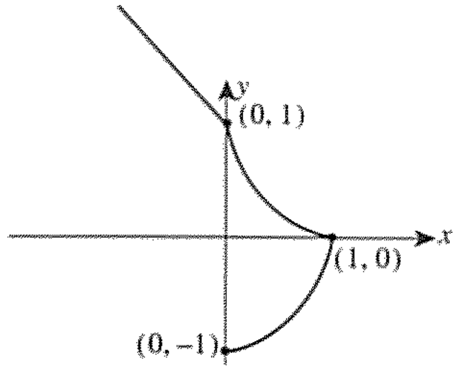
B.



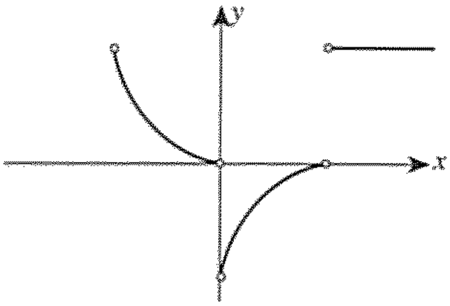
C.



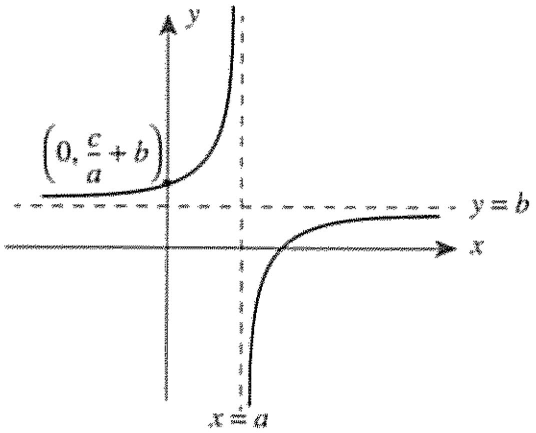
D.



E.



Question 11



The most appropriate equation for the graph shown above, if a, b and $c \in R^+$, is

A. $y = \dfrac{c - bx}{a - x}$

B. $y = \dfrac{c}{b - x} + a$

C. $y = \dfrac{c}{x - b} + a$

D. $y = \dfrac{c}{a - x} + b$

E. $y = \dfrac{c}{x - a} + b$

Question 12

$f(x)$ is e^{x^2+1}. Which of the following is $f'(x)$?

A. $2xe^{x^2+1}$

B. $2xe^{2x}$

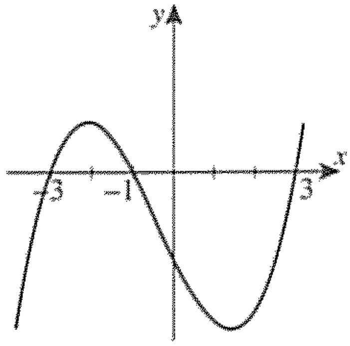
C. $2x$

D. $(x^2 + 1)e^{x^2}$

E. $(x^2 + 1)e^{2x}$

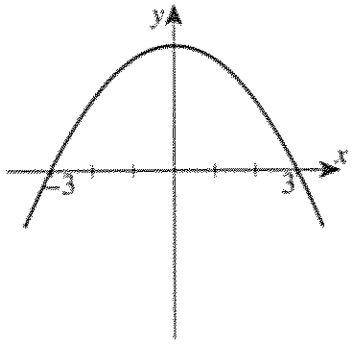
Question 13

The graph of $y = f(x)$ is shown below.

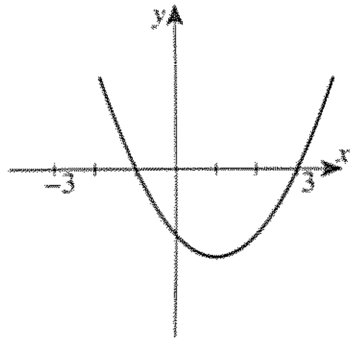


Which one of the following is most likely to be the graph of the derivative function with the equation $y = f'(x)$?

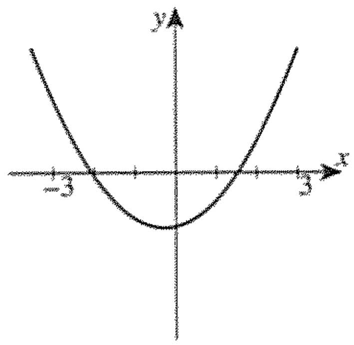
A.



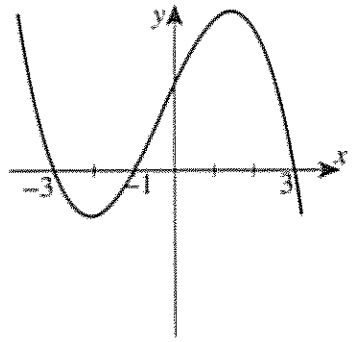
B.



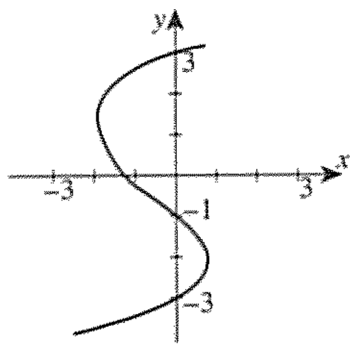
C.



D.



E.



Question 14

The derivative of $\log_e\left(\frac{1}{\sin x}\right)$ with respect to x is

A. $-\frac{1}{\tan x}$

B. $\frac{1}{\tan x}$

C. $-\tan x$

D. $\frac{1}{\cos x}$

E. $\tan x$

Question 15

If $y = \frac{\sin \pi x}{e^x}$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

A. 0

B. -1

C. π

D. $-\pi$

E. $\frac{1}{\pi}$

Question 16

The derivative of $e^x \log_e 2x$ is

A. $e^x\left(\frac{1}{x} + \log_e 2x\right)$

B. $e^x(1 + \log_e 2x)$

C. $-e^x(1 + \log_e 2x)$

D. $e^x x + \log_e 2x$

E. $e^x\left(\frac{1}{2x} + \log_e 2x\right)$

Question 17

The equation of the normal to the curve of the function with equation $y = e^{-\frac{1}{2}x} - 1$ at the point where $x = 0$ is

A. $y = x$

B. $y = -2x$

C. $y = 2x$

D. $y = -\frac{1}{2}x$

E. $y = \frac{1}{2}x$

Question 18

Using the left rectangle approximation with rectangles of width 1, the area of the region bounded by the curve $y = x^2 + 1$, the x axis, and the lines $x = 1$ and $x = 5$ is approximated by

A. 30 sq. units

B. 34 sq. units

C. 46 sq. units

D. $45\frac{1}{3}$ sq. units

E. 58 sq. units

Question 19

An antiderivative of $\frac{1}{2(5x+2)^2}$ is

A. $\frac{1}{10(5x+2)}$

B. $\frac{1}{10(5x+2)^3}$

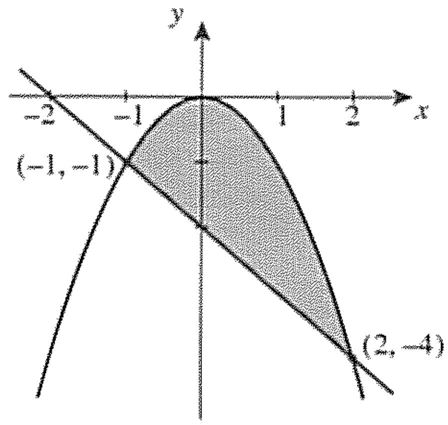
C. $\frac{-1}{10(5x+2)^3}$

D. $\frac{-10}{10(5x+2)^3}$

E. $\frac{-1}{10(5x+2)}$

Question 20

The graphs of function $y = -x^2$ and $y = -(x + 2)$ are shown in the diagram below.



The area shaded (in square units) is equal to

A. $1\frac{1}{2}$

B. $4\frac{1}{2}$

C. $-4\frac{1}{2}$

D. $5\frac{1}{6}$

E. $\frac{5}{6}$

Question 21

If $\int_a^{\frac{3\pi}{4}} \sin 2x\,dx = 0$ then a is most likely to be

A. $\frac{\pi}{4}$

B. 0

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

E. π

Question 22

A is a binomial random variable with $\text{E}(A) = 2$ and $\text{Var}(A) = \frac{3}{2}$. As a result $\Pr(A = 2)$ is equal to

A. $\left(\frac{1}{4}\right)^2$

B. ${}^4C_2\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^2$

C. ${}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

D. ${}^8C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6$

E. ${}^8C_2\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^2$

Question 23

On a school committee composed of eight members, five favour the proposition that students should wear protective hats throughout the entire year, while three consider it only necessary during term 1 and term 4.

What is the probability that a randomly chosen subcommittee of four will contain **exactly** three who favour all year protection?

A. $\frac{1}{7}$

B. $\frac{2}{7}$

C. $\frac{3}{7}$

D. $\frac{4}{7}$

E. $\frac{5}{7}$

Question 24

If Z is a normal random variable with $\mu = 0$ and $\sigma = 1$, then $\Pr(-0.3 < Z < 0.5)$ is equal to

A. $\Pr(Z < 0.5) - \Pr(Z > -0.3)$

B. $\Pr(Z < 0.5) - 1 + \Pr(Z > -0.3)$

C. $\Pr(Z < 0.5) + \Pr(Z < -0.3)$

D. $\Pr(Z < -0.3) - \Pr(Z < 0.5)$

E. $\Pr(Z < -0.3) + \Pr(Z > 0.5)$

Question 25

Jasmeet recently purchased a bug zapper to hang out under her patio. Over a period of 20 days, she has been daily counting the number of insects zapped in the collection tray at the bottom of her device. The results are given in the table below.

Number of insects (x)	0	1	2	3	4	5	6
Number of days zapper contained x	1	2	1	4	7	4	1

The mean number of insects, per day, that Jasmeet found was equal to

A. 1.95
B. 3
C. 3.5
D. 3.9
E. 6.1

Question 26

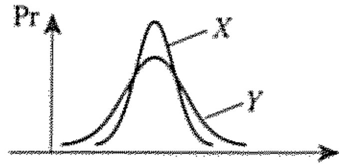
Joel, a keen golfer, is selecting some tees from a gift box he has recently received. The box contains only 35 yellow and 25 red tees. He randomly selects 10 tees.

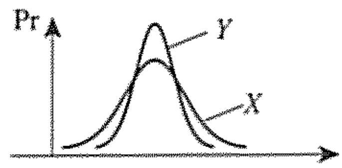
The expected number of yellow tees and the variance for the number of yellow tees that Joel selects is closest to

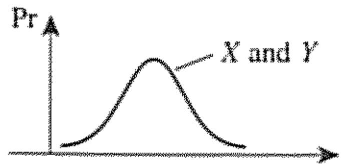
A. 5, 1.4
B. 5, 2.4
C. 6, 1.4
D. 6, 2.1
E. 6, 2.4

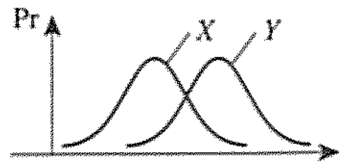
Question 27

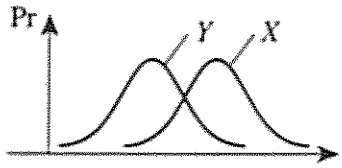
A sample of 5 is selected from a display containing dark and milk chocolates. X is the number of dark chocolates in the sample when the sample is consumed and Y is the number of dark chocolates in the sample when it is made with replacement. Which of the following graphs best represents the distributions described?

A.


B.


C.


D.


E.


END OF MULTIPLE-CHOICE QUESTION BOOKLET

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Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 1: Facts, Skills and Applications Task

Part II Question and Answer Booklet

Reading time 15 minutes
Writing time 1 hour 30 minutes

Student's Name: ______________________________

Teacher's Name: ______________________________

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).

Part I consists of a separate question booklet and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of this question and answer booklet.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

A detachable formula sheet for use in both parts is in the centrefold of the Part I question booklet.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this booklet (Part II).

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2002 VCE Mathematical Methods Units 3 & 4 Examination 1.

PART II

Structure of Booklet

Number of questions	Number of questions to be answered	Marks
6	6	23

Materials

Question and answer booklet of 6 pages.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the centre of the Part I booklet during reading time.

Ensure that you write your **name** and your **teacher's name** in the spaces provided on the cover of this booklet.

A decimal approximation will not be accepted if an exact answer is required to a question.

Where an exact answer is required to a question, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses should be in English.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer booklet (Part II).

Specific Instructions for Part II

Answer **all** questions in this part in the spaces provided.

Question 1

Consider the function $f(x) = \log_e(2x - 1)$.

a. State the maximal domain of $f(x)$.

b. Determine the equation of $f^{-1}(x)$.

c. State the range of $f^{-1}(x)$.

1 + 2 + 1 = 4 marks

Question 2

Calculate the **exact** solutions to the equation $\sqrt{3} + 2\sin\frac{x}{2} = 0$ for $-2\pi \le x \le 0$.

3 marks

Question 3

Consider the equation $y = x^2 \log_e 3x$.

Find

a. $\frac{dy}{dx}$

b. the exact value of the gradient of the tangent when $x = \frac{1}{3}$.

c. the equation of the tangent when $x = \frac{1}{3}$.

2 + 1 + 2 = 5 marks

Question 4

Given $y = e^x$, find in terms of h an expression without exponential or logarithmic notation, for the approximate increase in y as x is increased from $\log_e 2$ to $h + \log_e 2$.

2 marks

Question 5

A recent telephone survey concluded that 54% of Australians favour the Government's decision to support stem-cell research.

a. If 1769 calls were made, what would be the expected number of non-supporters of the Government decision?

b. If 5 people were telephoned at random, what is the probability that at least two will favour the Government decision? Express your answer to the nearest percentage.

$1 + 2 = 3$ marks

Question 6

For a particular class, the marks in a Maths Methods test are normally distributed with a mean of 67. The probability that a randomly selected student from the class will obtain a mark of less than 50 is 0.1.

a. What is the standard deviation of the marks in the class? Express your answer to two decimal places.

b. Chrissy, a student in the class, decides to attempt some self-paced learning in order to improve her score. She purchases a computer program sold as a box of 6 disks. As a promotional gimmick, one or more of these disks can be 'tagged' for a prize. It is known that these disks have a chance of 0.13 of being tagged for a prize. Find, to 4 decimal places, the probability that Chrissy purchases a box containing fewer than three prize disks, if she knows it contains at least one prize disk.

3 + 3 = 6 marks

END OF QUESTION AND ANSWER BOOKLET

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Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 1: Facts, Skills and Applications Task

Suggested Solutions

PART I

Question 1

This graph has the basic shape of a $y = -\sin x$ graph. $\frac{3}{4}$ of its period is $\frac{3\pi}{4}$, so its period is π.

Period $= \frac{2\pi}{n} \Rightarrow n = 2$. The amplitude is a and it has been translated down by a units. The equation is therefore $y = -a\sin 2x - a$.

Answer E

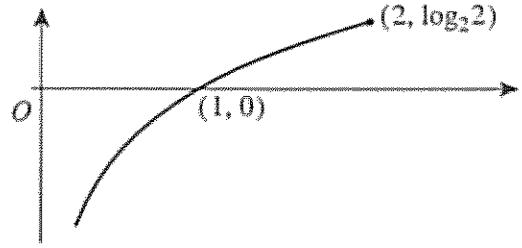
Question 2

The term containing x^2 is ${}^6C_2(3x)^2(-2)^4 = 15(3)^2(-2)^4x^2 = 2160x^2$.

Answer C

Question 3

The graph of $f(x)$ is shown below.



The range of f is $(-\infty, 1]$, as $\log_2 2 = 1$.

Answer D

Question 4

$$\begin{aligned}\log_e x - 3\log_e 2x + 2\log_e 3x &= \log_e x - \log_e(2x)^3 + \log_e(3x)^2\\ &= \log_e \frac{x \times (3x)^2}{(2x)^3}\\ &= \log_e \frac{9}{8}\\ &= \log_e 9 - \log_e 8.\end{aligned}$$

Answer B

Question 5

$$\begin{aligned}\log_2 x(x-1) &= 1\\ \therefore x^2 - x &= 2\\ x^2 - x - 2 &= 0\\ (x-2)(x+1) &= 0\\ x &= 2 \text{ or } -1.\end{aligned}$$

Answer E

Question 6

The graph shown is a negative quartic with x-intercepts a and b. Therefore linear factors of $x-a$ and $x-b$ exist. We notice a turning point on the x-axis when $x=0$; hence x^2 is a factor in the equation.

We look for $y=-x^2(x-a)(x-b)$, which can also be expressed as $y=x^2(a-x)(x-b)$.

Answer D

Question 7

Use of the graphic calculator is required. Entering `Y1=2sin(3x)` and `Y2=log(3x)`, we observe 3 intersections. Hence there are 3 solutions.

Answer C

Question 8

The graph required has $y=f(x)$ translated b units to the right and dilated by a factor a away from the x-axis in the y direction.

Answer B

Question 9

The easiest way to generate the resultant graph is to add $-g(x)$ to $f(x)$. This results in a y-intercept of $(0, 1)$, and $y \to -\infty$ as $x \to -\infty$ and as $x \to \infty$.

Answer D

Question 10

We look for a reflection in the line $y=x$ and all inverse coordinates will have x and y values interchanged.

Answer A

Question 11

The vertical asymptote $x=a$ results from equating the denominator to zero. This is a negative rectangular hyperbola, hence the denominator is $a-x$. The horizontal asymptote is obtained when b is added to $\frac{c}{a-x}$.

The y-intercept (when $x=0$) confirms that D is correct.

Answer D

Question 12

$f(x)=e^{x^2+1}$. By the chain rule, $f'(x)=e^{x^2+1}(2x)=2xe^{x^2+1}$.

Answer A

Question 13

$f(x)$ is a positive cubic, hence $f'(x)$ is a positive quadratic. The stationary points of $f(x)$ are the x-intercepts of $f'(x)$. The turning points of $f'(x)$ correspond to the maximum negative gradient of $f(x)$.

Answer C

Question 14

$f(x) = \log_e\left(\frac{1}{\sin x}\right) = \log_e((\sin x)^{-1}) = -\log_e(\sin x)$.

Using the chain rule, $f'(x) = -\frac{\cos x}{\sin x} = -\tan x$.

Answer A

Question 15

Using the quotient rule, $\frac{dy}{dx} = \frac{e^x \pi\cos \pi x - (\sin \pi x)e^x}{e^{2x}} = \frac{e^x(\pi\cos \pi x - \sin \pi x)}{e^{2x}}$.

At $x = 0$, $\frac{dy}{dx} = \frac{e^0(\pi\cos 0 - \sin 0)}{e^0} = \pi$.

Alternatively, the product rule could be used with $y = e^{-x}(\sin \pi x)$:

$\frac{dy}{dx} = e^{-x}(\pi\cos \pi x) + (\sin \pi x)(-e^{-x}) = e^{-x}(\pi\cos \pi x - \sin \pi x)$.

At $x = 0$, $\frac{dy}{dx} = e^0(\pi\cos 0 - \sin 0) = \pi$.

Answer C

Question 16

Using the product rule, $\frac{dy}{dx} = e^x\left(\frac{1}{x}\right) + (\log_e 2x)e^x = e^x\left(\frac{1}{x} + \log_e 2x\right)$.

Answer A

Question 17

At $x = 0$, $y = e^{-\frac{1}{2}(0)} - 1 = 0$.

The gradient of the tangent, m_T is given by $\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{x}{2}}$. At $x = 0$, $m_T = -\frac{1}{2}$.

$m_T m_N = -1$, so at $x = 0$, $m_N = \frac{-1}{-\frac{1}{2}} = 2$.

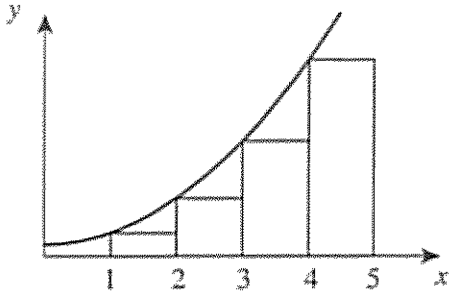
Hence the equation of the normal at $(0, 0)$ is $y - 0 = 2(x - 0)$

$$y = 2x.$$

Answer C

Question 18

Using the left rectangle approximation for $y = x^2 + 1$



When $x = 1$, $y = 2$.

When $x = 2$, $y = 5$.

When $x = 3$, $y = 10$.

When $x = 4$, $y = 17$.

$$\begin{aligned}\text{Approximate area} &= 2 + 5 + 10 + 17 \\ &= 34 \text{ square units.}\end{aligned}$$

Answer B

Question 19

$$\begin{aligned}\int \frac{1}{2(5x+2)^2}dx &= \frac{1}{2}\int (5x+2)^{-2}dx \\ &= \frac{1}{2}\left[\frac{(5x+2)^{-1}}{5(-1)}\right] + C \\ &= -\frac{1}{10}\left(\frac{1}{5x+2}\right) + C.\end{aligned}$$

An antiderivative is $-\frac{1}{10}\left(\frac{1}{5x+2}\right)$.

Answer E

Question 20

$$\begin{aligned}\text{Area} &= \int_{-1}^{2} -x^2 - (-x-2)dx \\ &= \int_{-1}^{2} -x^2 + x + 2\ dx \\ &= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right]_{-1}^{2} \\ &= \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) \\ &= \frac{9}{2}\text{square units.}\end{aligned}$$

Answer B

Question 21

$$\int_a^{\frac{3\pi}{4}} \sin 2x = 0$$

$$\left[-\frac{1}{2}\cos 2x\right]_a^{\frac{3\pi}{4}} = 0$$

$$-\frac{1}{2}\cos\frac{3\pi}{2} - \left(-\frac{1}{2}\cos 2a\right) = 0$$

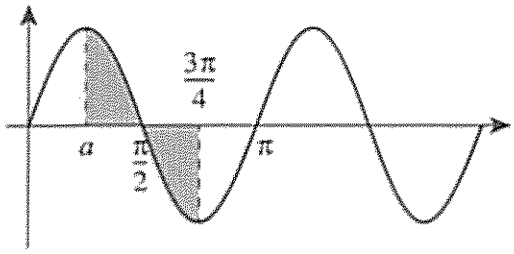
$$0 + \frac{1}{2}\cos 2a = 0$$

$$\cos 2a = 0$$

$$2a = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

$$a = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$$

OR



Balancing areas above and below the x-axis, $a = \frac{\pi}{4}$.

Answer A

Question 22

$E(A) = np = 2$ and $Var(A) = \frac{3}{2} = np(1-p)$.

$$\therefore \frac{3}{2} = 2(1-p)$$

$$\therefore (1-p) = \frac{3}{4}$$

$$\therefore p = \frac{1}{4}$$

As $E(A) = np = 2$, $n = \frac{2}{\frac{1}{4}} = 8$.

For a binomial random variable, $Pr(A=2) = {}^nC_2 p^2(1-p)^{n-2} = {}^8C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6$.

Answer D

Question 23

This is a hypergeometric experiment. Let X be the number who favour all-year protection.

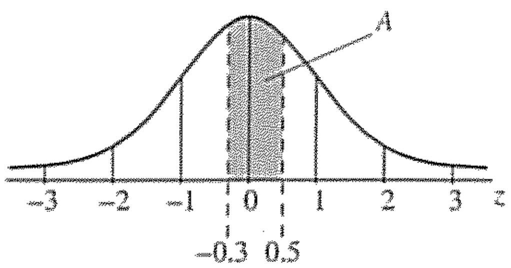
Then $Pr(X=3) = \dfrac{\binom{5}{3}\binom{3}{1}}{\binom{8}{4}} = \dfrac{10 \times 3}{70} = \dfrac{3}{7}$.

Answer C

Question 24

Z has a standard normal distribution.

We require area A.



$$\begin{aligned} A &= \Pr(Z < 0.5) - \Pr(Z < -0.3) \\ &= \Pr(Z < 0.5) - \Pr(Z > 0.3) \\ &= \Pr(Z < 0.5) - (1 - \Pr(Z < 0.3)) \\ &= \Pr(Z < 0.5) - 1 + \Pr(Z < 0.3) \\ &= \Pr(Z < 0.5) - 1 + \Pr(Z > -0.3). \end{aligned}$$

Answer B

Question 25

Let X be the number of insects zapped in the tray.

x	0	1	2	3	4	5	6
$\Pr(X = x)$	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{1}{20}$	$\frac{4}{20}$	$\frac{7}{20}$	$\frac{4}{20}$	$\frac{1}{20}$

The mean number of insects zapped is given by

$$E(X) = 0 + \frac{2}{20} + \frac{2}{20} + \frac{12}{20} + \frac{28}{20} + \frac{20}{20} + \frac{6}{20} = \frac{7}{2}.$$

Answer C

Question 26

This is a hypergeometric experiment with $n = 10$, $N = 60$ and $D = 35$.

Let X be the number of yellow tees selected. Then $E(X) = n\frac{D}{N} = \frac{35}{60} \times 10 = 5.83$

and $\text{Var}(X) = \dfrac{nD(N-D)(N-n)}{N^2(N-1)} = \dfrac{(10 \times 35)(25)(50)}{3600(59)} = 2.06$.

Answer D

Question 27

X is hypergeometric, Y is binomial. By experiment it can be found that $E(X) = E(Y)$ and $SD(X) < SD(Y)$ when n is small. The graph in A is the only representation of this situation.

Answer A

PART II

Question 1

a. The smallest x value is obtained by letting $2x - 1 = 0$, i.e. when $x = \frac{1}{2}$.

There is no upper limit on the value of x. Hence the maximal domain is $\left(\frac{1}{2}, \infty\right)$. [A]

(Note that $x = \frac{1}{2}$ is not included in the domain as $\log_e 0$ is indeterminate.)

b. For the inverse function, swap x and y,

i.e. $x = \log_e(2y - 1)$ [M]

$$e^x = 2y - 1$$

$$y = \frac{e^x + 1}{2}$$

$$\therefore f^{-1}(x) = \frac{e^x + 1}{2}$$ [A]

c. The domain of $f^{-1}(x)$ is the range of $f(x)$ and the range of $f^{-1}(x)$ is the domain of $f(x)$.

So, the range of $f^{-1}(x) = \left(\frac{1}{2}, \infty\right)$. [A]

Question 2

$$\sqrt{3} + 2\sin\frac{x}{2} = 0$$

$$2\sin\frac{x}{2} = -\sqrt{3}$$

$$\sin\frac{x}{2} = -\frac{\sqrt{3}}{2}.$$ [A]

$$\therefore \frac{x}{2} = -\frac{\pi}{3}, -\pi + \frac{\pi}{3}$$

$$\therefore x = -\frac{2\pi}{3}, -\frac{4\pi}{3}.$$ [A][A]

Question 3

a. $y = x^2 \log_e 3x$.

Using the product rule, [M]

$$\frac{dy}{dx} = x^2\left(\frac{1}{x}\right) + (\log_e 3x)2x$$

$$= x + 2x\log_e 3x.$$ [A]

b. The gradient of the tangent is $\frac{dy}{dx}$. When $x = \frac{1}{3}$, $\frac{dy}{dx} = \frac{1}{3} + 2 \times \frac{1}{3} \times \log_e 1$

$$= \frac{1}{3}, \text{ as } \log_e 1 = 0.$$ [A]

c. The partial equation of the tangent is given by $y = \frac{1}{3}x + c$.

Substitute in $\left(\frac{1}{3}, 0\right)$: $0 = \frac{1}{9} + c$ [M]

$\therefore c = -\frac{1}{9}$.

Hence the equation of the tangent is $y = \frac{1}{3}x - \frac{1}{9}$. [A]

Question 4

An approximation for δy is $\frac{dy}{dx} \times \delta x = e^x \times h$. [M]

When $x = \log_e 2$, $\delta y \approx e^{\log_e 2} \times h$

$= 2h$. [A]

Question 5

a. Let X be the number of Australians in favour of the Government's decision.

$\therefore \text{E}(X) = np$

$= 1769 \times 0.54$

$= 955.26$

$\therefore \text{E}(X') = 1769 - 955.26$

$= 813.74$.

So we expect 813 (or 814) people to be non-supporters of the Government decision. [A]

(Alternatively, $\text{E}(X') = 1769 \times (1 - 0.54) = 813.74$.)

b. $X \sim \text{Bi}(n = 5, p = 0.54)$ and $\Pr(X \geq 2) = 1 - [\Pr(X = 0) + \Pr(X = 1)]$. [M]

$\Pr(X = 0) = {}^5C_0(0.54)^0(0.46)^5$

$= (0.46)^5$

$= 0.0206$.

$\Pr(X = 1) = {}^5C_1(0.54)^1(0.46)^4$

$= 0.1209$.

Hence $\Pr(X \geq 2) = 1 - [0.0206 + 0.1209]$

$= 0.8585$

$= 86\%$, to the nearest percentage point. [A]

(or $1 - \text{binomcdf}(5, 0.54, 1) = 1 - 0.1415 = 0.8585$)

Question 6

a. $X \sim N(67, \sigma^2)$.

$$\Pr(X < 50) = 0.1$$

$$\therefore \Pr\left(Z < \frac{50-67}{\sigma}\right) = 0.1$$ [M]

$$\therefore \Pr\left(Z < -\frac{17}{\sigma}\right) = 0.1$$

$$\therefore 1 - \Pr\left(Z < \frac{17}{\sigma}\right) = 0.1$$

$$\therefore \Pr\left(Z < \frac{17}{\sigma}\right) = 0.9$$

$$\frac{17}{\sigma} = 1.28155$$ [A]

Hence $\sigma = 13.27$. [A]

b. Let X be the number of prize disks in a box.

$$\Pr(X = 0) = \binom{6}{0}(0.13)^0(0.87)^6$$

$$= 0.43363$$

$$\Pr(X = 1) = \binom{6}{1}(0.13)^1(0.87)^5$$

$$= 0.38877$$

$$\Pr(X = 2) = \binom{6}{2}(0.13)^2(0.87)^4$$

$$= 0.14523$$ [A]

$$\therefore \Pr(X < 3 \mid X \geq 1) = \frac{\Pr(1 \leq X \leq 2)}{\Pr(X \geq 1)}$$ [M]

$$= \frac{0.38877 + 0.14523}{1 - 0.43363}$$ [A]

$$= 0.9428$$