Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 2: Analysis Task

Reading time 15 minutes
Writing time 1 hour 30 minutes

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
4	4	55

Materials

Question and answer booklet of 10 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set square and aids for curve sketching.

The task

Detach the formula sheet from the centre of this booklet during reading time.

Ensure that you write your **name** and your **teacher's name** in the space provided on the front of this booklet.

All written responses should be in English.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2002 VCE Mathematical Methods Units 3 & 4 Examination 2.

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Formula Sheet

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\frac{d}{dx}\log_e x = \frac{1}{x}$

$\frac{d}{dx}(\sin ax) = a\cos ax$

$\frac{d}{dx}(\cos ax) = -a\sin ax$

$\frac{d}{dx}(\tan ax) = \frac{a}{\cos^2 ax} = a\sec^2 ax$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x}dx = \log_e x + c$, for $x > 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$

$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X=x)$	mean	variance
general	$p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$ $= \Sigma x^2 p(x) - \mu^2$
binomial	${}^nC_x p^x(1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\,{}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $N(0, 1)$.		

Table 1: Normal Distribution – cdf

X	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Instructions

Answer **all** questions.
A decimal approximation will not be accepted if an exact answer is required to a question.
Where an exact answer is required to a question, appropriate working must be shown.
The instruction **use calculus** requires students to show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

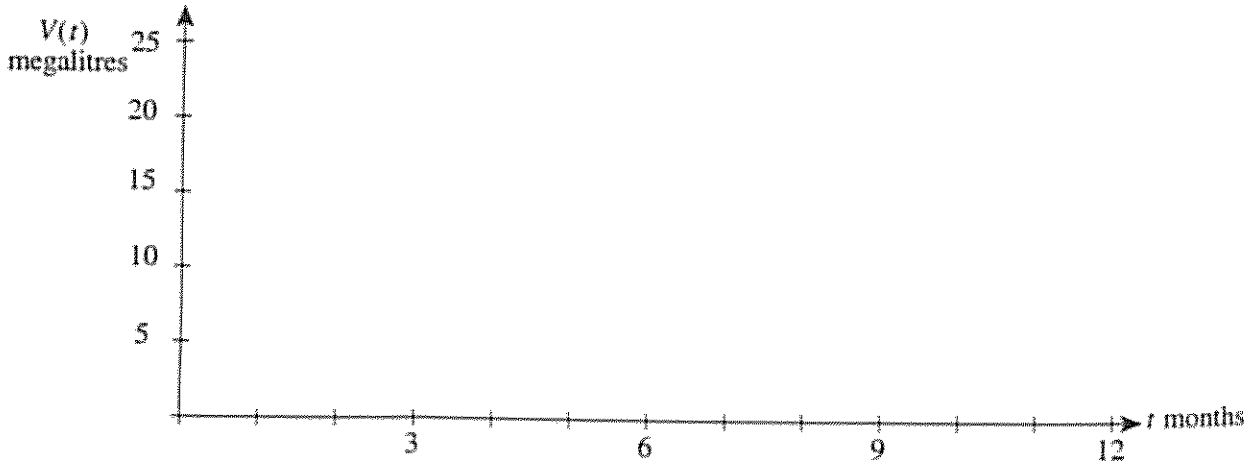
Question 1

The volume of water, $V(t)$ megalitres, in a small reservoir is known to be modelled by the equation

$$V(t) = 10\sin\left(\frac{t}{2} + 3\right) + 15 \text{ for } 0 \le t \le 12$$

where t is the number of months in a certain year.

a. On the axes below, sketch a graph of $V(t)$ versus t for the entire year. Clearly show important features including endpoints and turning points to one decimal place.



3 marks

b. Find the function for the **rate of change** of volume with respect to time.

____

1 mark

c. Calculate the **exact** values of t when the volume is at its minimum and maximum values during the year.

____

____

____

____

____

4 marks

d. Correct to the nearest 0.1 month, for how long during the year is the volume of water in the reservoir less than 7.5 megalitres?

2 marks
Total 10 marks

Question 2

The design of a new stadium roof is being considered by an architect. A diagram of the proposed stadium (Figure 1) and a cross-sectional view of the stadium on Cartesian axes (Figure 2) are shown.

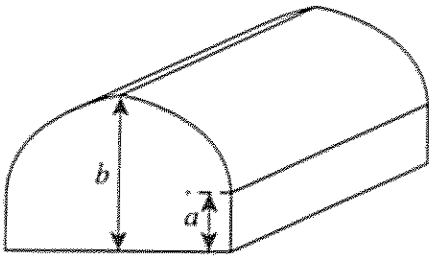


Figure 1

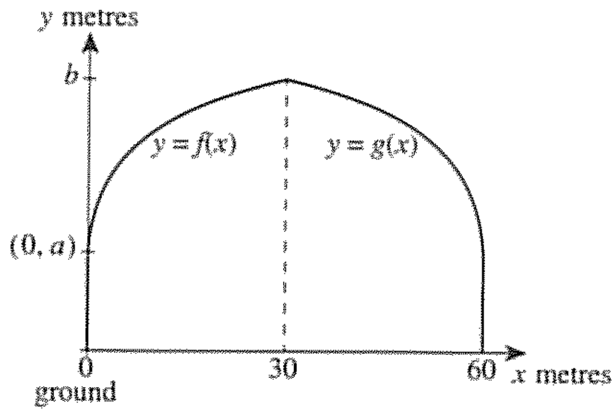


Figure 2

The equation of the left side of the curved roof $(y = f(x))$ is given by $f : [0, 30] \rightarrow R$, where $f(x) = 5\log_e(x + 1) + 10$.

a. Calculate the value of a (the height of the side wall).

____

1 mark

b. Calculate (to the nearest centimetre) the maximum height of the roof above the ground (b).

____

1 marks

The right side of the roof is to be symmetrical to the left side about the line $x = 30$. Its equation, $y = g(x)$, can be deduced by reflecting the graph of $y = f(x)$ in the y axis and translating it 60 units to the right.

c. Deduce the equation for $g(x)$.

____

2 marks

d. State the domain and range of $g(x)$.

____

____

2 marks

The architect needs to know the slope of the roof at various points so that roofing materials can be designed.

e. Calculate an expression in terms of x for the slope of the roof at any x value

i. when $0 < x < 30$.

1 mark

ii. when $30 < x < 60$.

1 mark

f. Calculate the exact slope of the roof when

i. $x = 15$.

1 mark

ii. $x = 45$.

1 mark

The architect considered bricking in one entire end wall enclosed under the roof span.

g. Write an expression in terms of $f(x)$ for the exact area (A m^2) of one end of the stadium enclosed under the entire cross sectional span of the roof.

2 marks

h. Use calculus to calculate this exact area given that

$$\int \log_e(x+1)\,dx = (x+1)\log_e(x+1) - x.$$

3 marks

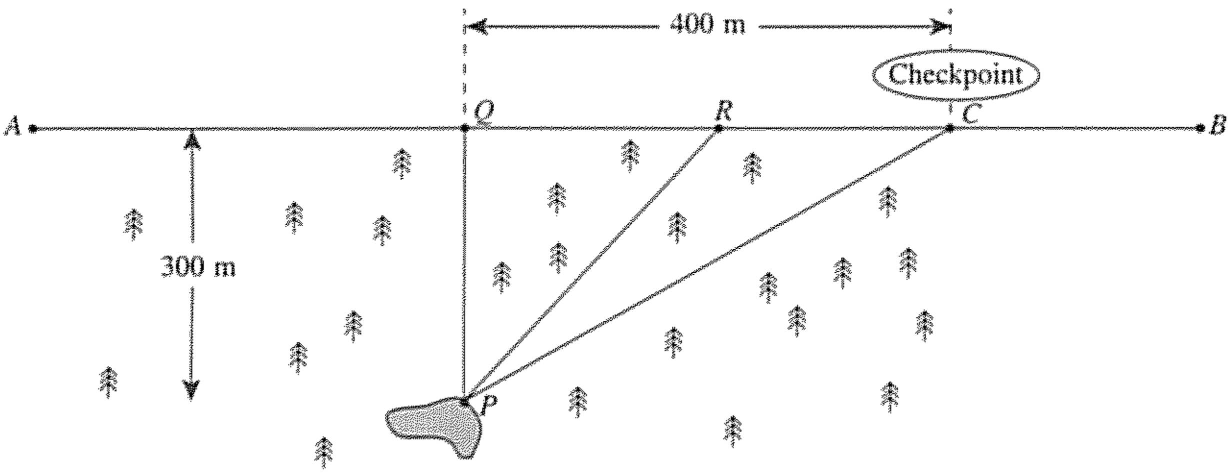
i. Use your calculator to find this area in square metres, correct to 1 decimal place.

1 mark

Total 16 marks

Question 3

Paolo, an experienced orienteer, is standing by a pond P in a heavily wooded forest. He has 5 minutes to report to checkpoint C, along a straight track AB, otherwise he will be disqualified – and he must walk, not run.



Q is a point on the road directly opposite his position. The distances from P to Q and from Q to the checkpoint are as shown in the diagram. Paolo averages 12 km/h when walking along the track and 6 km/h through the forest.

a. Find the exact time, in minutes, that Paolo will take to walk in a straight line from the pond to the checkpoint.

2 marks

b. How long, to the nearest minute, will Paolo take to walk from the pond to the checkpoint via Q?

1 mark

Paolo wishes to get to the checkpoint in the quickest time possible. He believes he can do this by walking through the forest to a point R and then along the track to the checkpoint.

c. If R is x metres from Q, show that the time taken along this route is

$$\frac{2\sqrt{90000 + x^2} + 400 - x}{200} \text{ minutes.}$$

2 marks

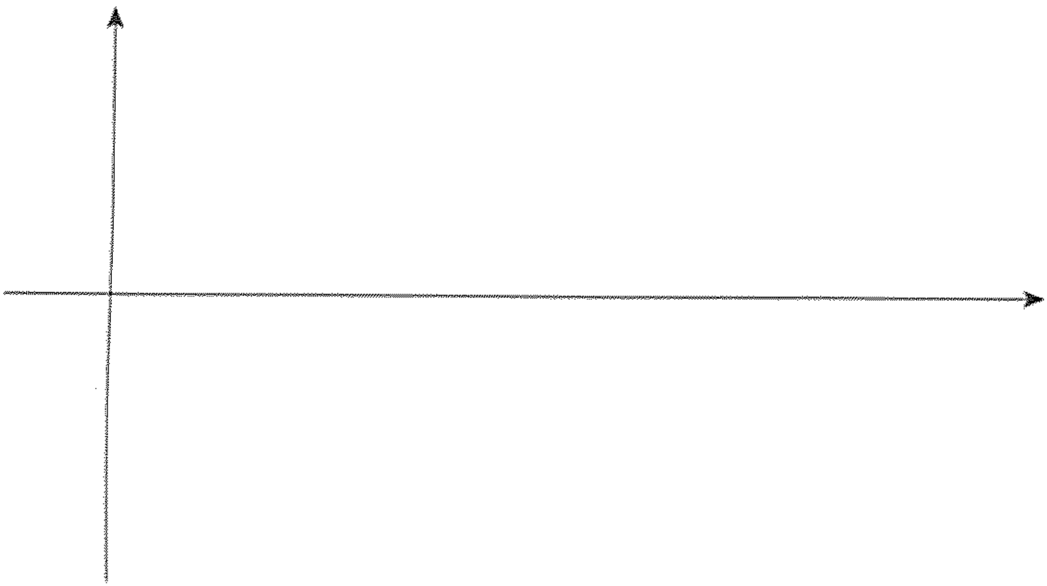
d. Using calculus, find the time in minutes for Paolo to get to the checkpoint as quickly as possible. Express your answer to 1 decimal place.

4 marks

e. **i.** Find $T'(170)$ and $T'(180)$ and hence justify that the time found in **d.** is the quickest possible.

3 marks

ii. Sketch the graph of the time function, clearly showing the endpoints, and hence verify this time to 4 decimal places.



2 marks

f. Will Paolo be disqualified? Give reasons for your answer.

1 mark

Total 15 marks

Question 4

Isaias wants to enter his rally car into the gruelling Arnhem desert off-road race. As this race requires precision driving and fierce concentration, the organisers put the applicants through a preliminary driving test. The probability that an applicant passes this test on their first attempt is 0.6.

For your answers in **a.**, express all probabilities to 4 decimal places.

a. **i.** What is the probability that 6 out of 10 applicants pass the test on their first attempt?

1 mark

ii. What is the probability that at least 6 out of 10 applicants pass the test on their first attempt?

1 mark

iii. What is the probability that at most 6 out of 10 applicants pass the test on their first attempt?

1 mark

b. A group of 250 candidates attempt the preliminary driving test. Calculate the mean and standard deviation of the number of candidates who pass on their first attempt.

2 marks

Isaias passes the preliminary driving test and is invited to participate in the rally. The probability of developing engine trouble for every day's full driving is 1% and is independent for each day.

c. What is the exact probability that a rally car will not have engine trouble on any particular day?

1 mark

d. What is the probability that a car will not have developed engine trouble during the first 2 days? Express your answer correct to 4 decimal places.

1 mark

e. What is the probability that a car will have developed engine trouble during the first D days?

1 mark

It is discovered that engine trouble in the particular make of car that Isaias is driving is caused by the overheating of an electronic sensor. The manufacturer of the sensor has found that the temperature at which a randomly chosen sensor fails is normally distributed with a mean of 94.5°C and a standard deviation of 5.7°C.

f. What proportion of sensors will operate at 100°C? Express your answer correct to the nearest percentage.

3 marks

Due to the regular failure of the sensors in this make of car, the manufacturer decides to quote a safe operating temperature at which 99% of the sensors will work.

g. What temperature, to the nearest degree, should the supplier quote?

3 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOKLET

Trial Examination 2002

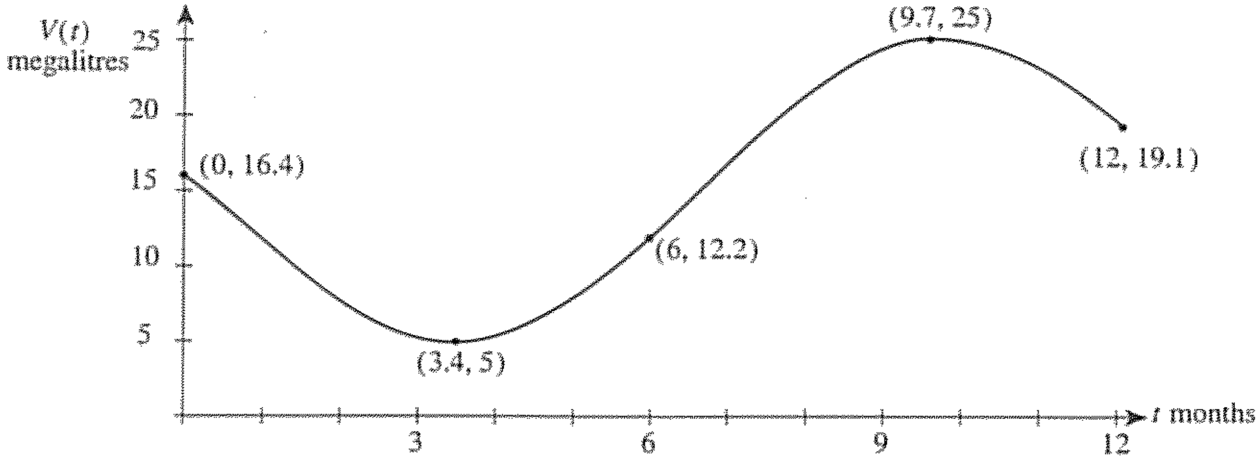
VCE Mathematical Methods Units 3 & 4

Examination 2: Analysis Task

Suggested Solutions

Question 1

a.



Basic shape and location [A]
Endpoints, coordinates and labelling [A][A]

b. Rate of change $= V'(t)$

$$= 5\cos\left(\frac{t}{2}+3\right),\ 0 \le t \le 12$$ [A]

c. Minimum and maximum occur when $5\cos\left(\frac{t}{2}+3\right) = 0$.

$$\therefore \cos\left(\frac{t}{2}+3\right) = 0$$ [M]

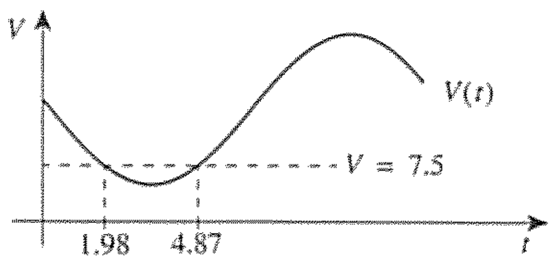
$$\therefore \frac{t}{2}+3 = \cos^{-1}(0)$$

$$= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$ [M]

$$\therefore \frac{t}{2} = \frac{\pi}{2}-3, \frac{3\pi}{2}-3, \frac{5\pi}{2}-3$$

(Note that $\frac{\pi}{2} - 3 < 0$.) $\therefore t = 3\pi - 6$ (min), $5\pi - 6$ (max). [A][A]

d.



When $V = 7.5$, $t = 1.98$ and 4.87.

The volume is less than 7.5 during the year [M]
for $4.87 - 1.98 = 2.9$ months. [A]

Question 2

a. When $x = 0$, $f(0) = 5\log_e 1 + 10$.

As $\log_e 1 = 0$, $f(0) = 10$.

$\therefore a = 10$ and so the height of the wall is 10 m. [A]

b. $b = f(30)$

$= 5\log_e(30 + 1) + 10$

$= 5\log_e 31 + 10$

≈ 27.17 m. [A]

The maximum height above the ground is 27.17 m.

c. Reflection in y axis: $g(x) = f(-x)$

Translation 60 units to the right: $g(x) = f(x - 60)$

Combining these gives:

Reflected curve in y axis: $g(x) = 5\log_e(-x + 1) + 10$ [M]

Translated curve (after reflection): $g(x) = 5\log_e(-(x - 60) + 1) + 10$

$\therefore g(x) = 5\log_e(-x + 61) + 10$ [A]

d. The domain of $g(x)$ is $[30, 60]$ or $30 \le x \le 60$. [A]

The range of $g(x)$ is $[10, 27.17]$ or $[10, 5\ln 31 + 10]$. [A]

e. **i** $f'(x) = \dfrac{5}{x+1}$, $0 < x < 30$. [A]

ii. $g'(x) = 5\left(\dfrac{-1}{-x+61}\right)$ or $\dfrac{-5}{-x+61}$, $30 < x < 60$. [A]

f. **i** $f'(15) = \dfrac{5}{16}$ [A]

ii. $g'(45) = \dfrac{-5}{-45+61}$

$= -\dfrac{5}{16}$ [A]

g. $A = 2\displaystyle\int_0^{30} f(x)dx$

Terminals [A]

$2 \times$ integral with dx [A]

h. $A = 2\displaystyle\int_0^{30} (5\log_e(x+1) + 10)dx$

$= 10\displaystyle\int_0^{30} (\log_e(x+1) + 2)dx$ [M]

$= 10\Big[(x+1)\log_e(x+1) - x + 2x\Big]_0^{30}$ [A]

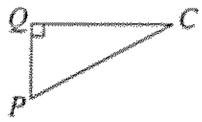
$= 10(31\log_e 31 + 30)$

$= 310\log_e 31 + 300$ [A]

i. $1364.5\ \text{m}^2$ [A]

Question 3

a. PQC is a 3-4-5 right angled triangle



$$\therefore PC = \sqrt{(300)^2 + (400)^2}$$
$$= 500 \text{ m.}$$

$$\therefore T_{PC} = \frac{0.5}{6}$$
$$= 0.08\dot{3} \text{ hr}$$
$$= 5 \text{ min.}$$ [A]

b. $$T_{\text{total}} = T_{\text{forest}} + T_{\text{track}}$$
$$= \frac{0.3}{6} + \frac{0.4}{12}$$ [M]
$$= 0.05 + 0.0\dot{3}\dot{3}$$
$$= 0.083$$
$$= 5 \text{ min}.$$ [A]

c. $$T_{PRC} = T_{PR} + T_{RC}$$
$$d_{PR} = \sqrt{300^2 + x^2}$$
$$d_{RC} = 400 - x$$

Hence $$T_{PRC} = \frac{\sqrt{300^2 + x^2}}{6000} + \frac{400 - x}{12000}$$ [A]
$$= \frac{2\sqrt{300^2 + x^2} + 400 - x}{12000} \text{ hr}$$

and so $$T_{PRC} = \frac{2\sqrt{300^2 + x^2} + 400 - x}{12000} \times 60 \text{ min}$$
$$= \frac{2\sqrt{90000 + x^2} + 400 - x}{200} \text{ min}$$ [A]

d. Minimum time is where $\frac{dT}{dx} = 0$. [M]

$$T = \frac{2(90000 + x^2)^{1/2}}{200} + \frac{400 - x}{200}$$
$$= \frac{1}{100}(90000 + x^2)^{1/2} + 2 - \frac{1}{200}x.$$

$$\frac{dT}{dx} = \frac{1}{200}(90000 + x^2)^{-1/2} \times 2x - \frac{1}{200}$$
$$= \frac{1}{200}\left(\frac{2x}{\sqrt{90000 + x^2}} - 1\right).$$ [A]

$$\frac{dT}{dx} = 0 \text{ when } \frac{2x}{\sqrt{90000 + x^2}} - 1 = 0.$$

$$\therefore \frac{2x}{\sqrt{90000 + x^2}} = 1$$

$$\begin{aligned}
\therefore 2x &= \sqrt{90000 + x^2} \\
\therefore 4x^2 &= 90000 + x^2 \\
\therefore 3x^2 &= 90000 \\
\therefore x^2 &= 30000 \\
\therefore x &= 173.2051 \text{ m.}
\end{aligned}$$ [A]

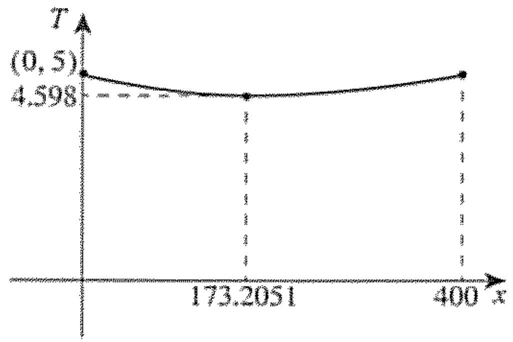
$$\begin{aligned}
T_{\min} &= \frac{2\sqrt{90000 + 30000} + 400 - 173.2051}{200} \\
&= 4.598 \\
&= 4.6 \text{ min (to 1 d.p.).}
\end{aligned}$$ [A]

e. i $$T'(170) = \frac{1}{200}\left[\frac{2(170)}{\sqrt{90000 + 170^2}} - 1\right] = -0.699 \times 10^{-5}$$ [A]

$$T'(180) = \frac{1}{200}\left[\frac{2(180)}{\sqrt{90000 + 180^2}} - 1\right] = 0.0000145$$ [A]

Since the gradient changes from negative to positive, the time found in **d.** is a minimum. [A]

ii. A graphic calculator is useful here.



General shape and minimum [A]

Endpoints [A]

f. All possible routes will give a time within $[4.598, 5]$ and so Paolo will not be disqualified. [A]

Question 4

a. **i** Let the random variable X be the number of applicants who pass on their first attempt.
X has a binomial distribution with $n = 10$ and $p = 0.6$.

$$\Pr(X = 6) = \binom{10}{6}(0.6)^6(0.4)^4$$
$$= 0.2508. \quad [A]$$

Alternatively, using the graphic calculator, binompdf(10, 0.6, 6) = 0.2508.

ii.
$$\Pr(X \geq 6) = \Pr(X = 6) + \Pr(X = 7) + \Pr(X = 8) + \Pr(X = 9) + \Pr(X = 10)$$
$$= 0.2508 + 0.21499 + 0.12093 + 0.04031 + 0.0060466$$
$$= 0.6331. \quad [A]$$

Alternatively, using the graphic calculator, 1 – binomcdf(10, 0.6, 5) = 0.6331.

iii.
$$\Pr(X \leq 6) = 1 - \Pr(X > 6)$$
$$= 1 - (0.21499 + 0.12093 + 0.04031 + 0.0060466)$$
$$= 1 - 0.382277$$
$$= 0.6177. \quad [A]$$

Alternatively, using the graphic calculator, binomcdf(10, 0.6, 6) = 0.6177.

b.
$$E(X) = np$$
$$= 250 \times 0.6$$
$$= 150 \quad [A]$$

$$SD(X) = \sqrt{np(1-p)}$$
$$= \sqrt{150(0.4)}$$
$$= 7.746 \quad [A]$$

c. $1 - 0.01 = 0.99$ [A]

d. $(0.99)^1(0.99)^1 = (0.99)^2 = 0.9801$. [A]

e. $1 - (0.99)^d$. [A]

f. Let T = overheating temperature.
This is a normal distribution with $\mu = 94.5$ and $\sigma = 5.7$.

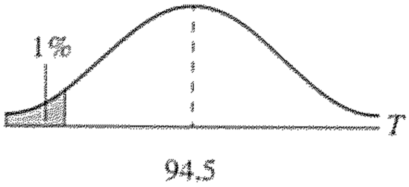
$$\text{So, } \Pr(T \leq 100) = \Pr\left(Z \leq \frac{100 - 94.5}{5.7}\right) \quad [A]$$
$$= \Pr(Z \leq 0.965).$$

Using graphic calculator or tables, $\Pr(T \leq 100) = 0.8328$. [A]

The proportion of sensors that will operate (not overheat) at 100°C is

$$1 - 0.8328 = 0.1672$$
$$= 17\%. \quad [A]$$

g. We require the temperature at which 1% fail.



So, $\Pr(T < t) = 0.01$. [M]

Using the graphic calculator or reverse tables to look up 0.99, $z = -2.326$ [A]

and so $\dfrac{t - 94.5}{5.7} = -2.326$

$$\therefore t = (-2.326 \times 5.7) + 94.5$$
$$= 81.24°\text{C}.$$

To the nearest °C, $t = 81°\text{C}$. [A]