2003 Mathematical Methods (CAS) Pilot Study GA 2: Written examination 1

GENERAL COMMENTS

The number of students who sat for this examination was 269. Marks ranged from 4 to 49 out of a maximum of 50. Student responses showed that the paper was accessible and provided opportunities for students to demonstrate what they knew. Of the whole cohort, 35 students (13.0 per cent) scored 90 per cent or more of the available marks, and 74 per cent scored 50 per cent or more of the available marks. The mean mark for the paper was 32.3, comprised of a mean of 19.1 marks on the multiple-choice part (27 marks in total) and a mean of 13.2 marks on the short-answer part (23 marks in total). The median score for the paper was 30 marks. This is a slight increase compared with 2002.

Overall, the symbolic facility of CAS was used well. There was no discernable advantage seen by the assessors of one CAS compared to another, although techniques for dealing with the mathematics varied according to the CAS in some questions. For Part 2 of the paper, there were many very successful responses and several students were able to work through most questions completely to obtain full marks. There was little evidence to suggest that not making a reasonable attempt at Part 2 was due to lack of available time.

Students should be familiar with instructions such as:
- a decimal approximation will not be accepted if an exact answer is required to a question
- in questions where more than 1 mark is available, appropriate working must be shown.

It was pleasing to see that where working was shown, correct mathematical notation was generally used.

Student facility with graphical representation is an area for further improvement. Both graphs required in Part 2 involved the absolute value function in some way, as part of a composite function (Question 3) and in its own right (Question 5). Some students were not able to recognise and deal with it appropriately. Care needs to be taken in specifying asymptotes and showing asymptotic behaviour.

SPECIFIC INFORMATION

Part 1 – Multiple choice

Content that needs further attention includes the definition of the definite integral as the limiting value of a sum, and its corresponding representation in integral form – this is fundamental to the understanding of the definite integral. In Question 17, only 17 per cent of students selected the correct answer, while a further 9 per cent selected the integral representation with the limits of integration reversed and 33 per cent selected the integral representation involving an antiderivative of the function. Other content that would benefit from further attention is the relation between the graph of an antiderivative function and the graph of the original function (Question 12), properties of antiderivatives and definite integrals (Question 14), transformations of functions (Question 5), the relationship of $f(x \pm y)$, $f(xy)$ and $f(\frac{x}{y})$ to values of $f(x)$ and $f(y)$ for different functions f (Question 18), informal treatment of the fundamental theorem of calculus (Question 15) and the hypergeometric distribution (Question 27).

This table indicates the approximate percentage of students choosing each distractor. The correct answer is the shaded alternative.

Question	A	B	C (%)	D	E
1	3	5	5	80	7
2	97	1	2	0	0
3	13	67	3	16	1
4	1	0	0	98	1
5	12	20	11	3	54
6	78	2	8	3	9
7	5	4	87	0	4
8	12	6	2	78	2
9	3	84	2	8	3
10	6	4	84	2	4

11	13	10	69	3	5
12	48	22	17	6	7
13	92	0	3	3	2
14	2	8	30	9	51
15	8	8	17	9	58
16	8	10	75	6	1
17	9	33	21	20	17
18	29	57	6	6	2
19	4	81	4	10	1
20	3	6	61	13	17
21	71	4	22	2	1
22	2	5	10	3	80
23	6	6	11	75	2
24	10	1	5	80	4
25	4	18	6	67	5
26	21	65	10	3	1
27	1	59	7	9	24

Part 2 – Short answer

Question 1

1a

Marks	0	1	2	Average
%	23	22	55	**1.32**

Correct response: $f(1) = 6$ so $a + b + c = 6$
and $f\,'(1) = 4$ so $2a + b = 4$.
Hence $a = c - 2$ and $b = 8 - 2c$

Most students were able to determine these two equations in *a*, *b* and *c*, for which a method mark was awarded. Some students incorrectly determined a unique solution for *a*, *b* and *c* from these two equations – they should have known that this is not possible. Some students tried to write the system in matrix form, but appeared to want to have a 3 × 3 matrix of coefficients when there should only have been a 2 × 3 matrix. Some students took the two equations, obtained another equation from part b and solved these simultaneously for *a*, *b* and *c* and returned these answers for part a. However, this is incorrect, as the question asked to find the values of *a* and *b* **in terms of** *c*, and only the first two of the three equations were available to be used at this stage.

1b

Marks	0	1	2	Average
%	38	8	54	**1.16**

Correct response: Solve $\int_0^1 ((c-2)x^2 + (8-2c)x + c)dx = 6$ for *c*, and therefore $c = 8$.

This answer could also be obtained by doing the integration and obtaining the equation $\frac{a}{3} + \frac{b}{2} + c = 6$; this combined with the two equations from part a. could then be solved to get the value for *c*. A method mark was awarded for either $\int_0^1 ((c-2)x^2 + (8-2c)x + c)dx = 6$ (or an expression of similar complexity if the students had solved the equations from part a. incorrectly) or for $\frac{a}{3} + \frac{b}{2} + c = 6$. A student who had made a minor error in part a was able to obtain full marks for this question if their working using incorrect results from part a. was correct and of the same complexity.

Question 2

Marks	0	1	2	Average
%	30	15	55	**1.24**

Correct response: $\tan(2\pi x) = -\sqrt{3}$

so $2\pi x = \ldots, \frac{-\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, \ldots$

hence $x = \frac{1}{3}, \frac{5}{6}$

This question was not as well done as it could have been. A method mark was awarded if students were able to obtain an intermediate result of $2\pi x = \frac{-\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{5\pi}{3}$, or one of the two answers. Some students gave approximate answers, rather than exact values as required, others gave one of the answers, and some gave more than the two which were correct. Some students gave unhelpful forms such as $-4\tan(\frac{\pi x}{2})^3 + 4\tan(\frac{\pi x}{2})$... This is the result when the given equation is entered directly into some CAS. Where this occurs, students should be prepared to rearrange the original equation, in this case to $\tan(2\pi x) = -\sqrt{3}$, in order to have their CAS provide exact solutions. Other students wrote expressions such as $x = 0.5(@n1 - 0.3333)$, which is a general form of solution (where $@n1$ indicates a parameter), and not an exact value. Students who obtain these general forms should be able to interpret them in context and write their solutions using conventional mathematical notation rather than CAS specific syntax.

Question 3

3a

Marks	0	1	Average
%	57	43	**0.43**

Correct response: $\boldsymbol{R}\backslash\{-3\}$ or $(-\infty, -3) \cup (-3, \infty)$ or $\{x \in \boldsymbol{R}: x \neq -3\}$

This was reasonably well done. There are still some students who write $\boldsymbol{R}/\{-3\}$ instead of $\boldsymbol{R}\backslash\{-3\}$, and some who use $\cap$ instead of $\cup$ (incorrectly) in the second form of the domain. Some also wrote $\boldsymbol{R}-3$, or just -3, which are insufficient in correctly specifying the domain. Some students wrote $(-3, \infty)$ or $x > -3$, indicating that they were either not aware of the absolute value function inside the natural logarithm, or they did not know how to deal with it.

3b

Marks	0	1	2	3	Average
%	18	5	15	62	**2.21**

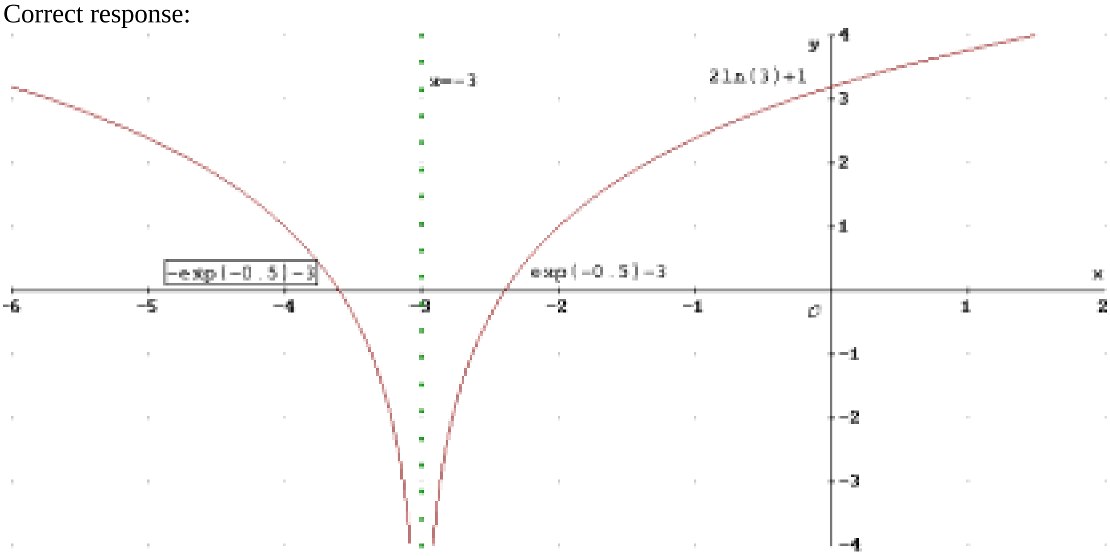
Correct response: Graph cuts x-axis when $f(x) = 0$, that is when $x = e^{-0.5} - 3$ or $-e^{-0.5} - 3$.
Graph cuts y-axis when $x = 0$, that is when $y = 2\log_e(3) + 1$.
Hence, the points where the graph of $y = f(x)$ cuts the x- and y-axes are $(e^{-0.5} - 3, 0)$, $(-e^{-0.5} - 3, 0)$ and $(0, 2\log_e(3) + 1)$ respectively.

Some students did not give exact values, and some missed the second x-intercept, because they were not aware of the absolute value function inside the natural logarithm. Instead of writing $\log_e(3)$, some students appeared to write the '3' as a superscript.

3c

Marks	0	1	2	Average
%	34	28	38	**1.03**

Correct response:



Some students only drew the right branch of the function. Some did not show the asymptotic behaviour correctly, particularly by having the branches suddenly stop at a circle somewhere on the asymptote. Also students should take care to clearly show asymptotic behaviour – the function should not curve away from the asymptote, nor touch it.

Some students who were aware of the absolute value function inside the natural logarithm function incorrectly decided that the corresponding graph could not show negative values, and had the image of the graph from $x = -e^{-0.5} - 3$ to $x = e^{-0.5} - 3$ reflected in the x-axis. Some students failed to label the asymptote, and some labelled it incorrectly, often as $x = 3$.

Question 4

4a

Marks	**0**	**1**	**Average**
%	13	87	**0.87**

Correct response: 0.567

Generally well done, although a few students had rounding errors.

4b

Marks	**0**	**1**	**Average**
%	24	76	**0.76**

Correct response: Area = $\int_0^k (-x+1-(1-e^{-x}))dx = \int_0^k (e^{-x} - x)dx$

Generally well done. Some students had the negative of the integrand, while others were careless and omitted the brackets surrounding $(1 - e^{-x})$ or simplified it incorrectly. Because of potential difficulty in identifying the shaded area on the diagram of some examination scripts, a mark was awarded for giving the definite integral corresponding to the other clearly identifiable closed region on the diagram. The definite integral in this case consisted of the two definite integrals $\int_0^k (1-e^{-x})dx + \int_k^1 (-x+1)dx$, and evaluates to 0.23 correct to 2 decimal places.

4c

Marks	**0**	**1**	**Average**
%	22	78	**0.78**

Correct response: 0.27

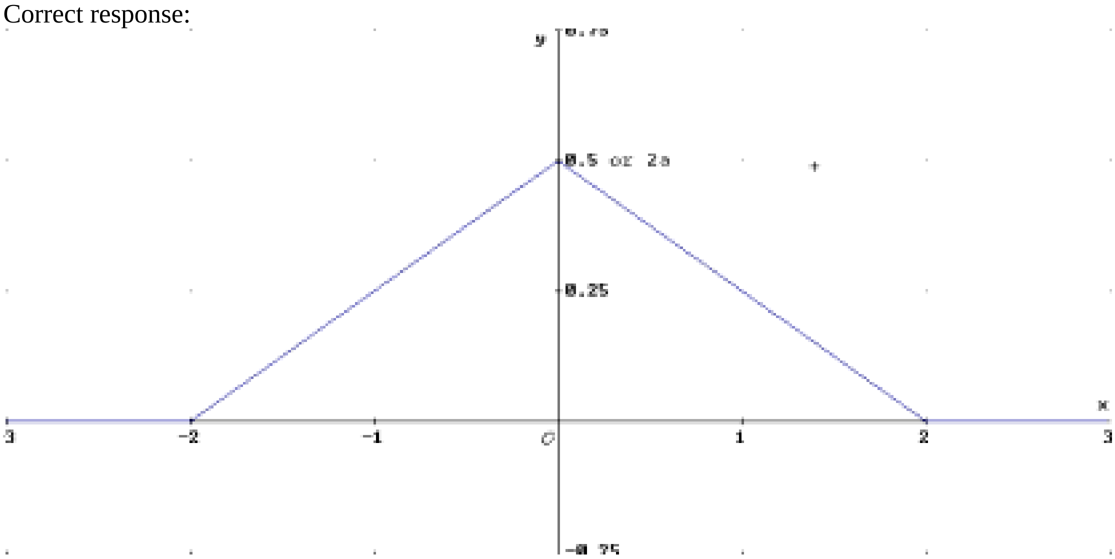
Generally well done. Students who had the definite integral for the other region at part b were awarded the mark if they had an answer of 0.23 in this part.

Question 5

5a

Marks	**0**	**1**	**2**	**Average**
%	34	37	29	**0.95**

Correct response:



In general this question was not well done as students were unable to draw a graph that was the correct shape and had appropriate scales on the horizontal and vertical axes. Many students tried to draw something resembling a normal distribution curve. Many students forgot to attach values to the markers on the horizontal axis. A label of either 2a or ½ was acceptable for the point of intersection of the curve with the vertical axis. Some students also extended their graphs below the horizontal axis, which, by definition, is not the case for the graph of a probability density function.

5b

Marks	0	1	2	Average
%	45	7	48	**1.02**

Correct response: Area = 0.5 × 4 × 2*a* = 1 so *a* = 0.25.

Alternatively, students could solve $\int_{-2}^{2} a(2-|x|)dx = 1$ for *a*. Many students tried to do this, but did not suitably deal with the absolute value function, and obtained an incorrect value of 0.125 for *a*. A method mark was awarded for indicating an area calculation or for $\int_{-2}^{2} a(2-|x|)dx = 1$.

Question 6

6a

Marks	0	1	Average
%	68	32	**0.32**

Correct response: $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

This was a straightforward probability calculation and was not as well done as might have been expected. Several students gave an answer of $\frac{7}{9}$, which they had obtained by using the transition matrix to work out the probabilities of working out in the gym on the Tuesday night and on the Wednesday night, and then multiplying them together. However, the probability for Wednesday calculated this way also includes the possibility of going for a swim on Tuesday evening.

6b

Marks	0	1	2	3	Average
%	52	18	9	21	**0.98**

Correct response: Transition matrix $T = \begin{bmatrix} 0 & \frac{2}{3} \\ 1 & \frac{1}{3} \end{bmatrix}$

$$T^4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{26}{81} \\ \frac{55}{81} \end{bmatrix}$$

Probability of having a swim on the Friday of that week is $\frac{26}{81}$.

Some students made errors by having the wrong transition matrix, the wrong power for the transition matrix, typically 3 or 5, or the wrong initial state, using $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ or $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$ instead of $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. A few students used the transpose of the transition matrix, which is also correct, provided multiplications by matrices of order 1 × 2 are on the left. One method mark was awarded for writing down the correct transition matrix and another for writing T^4× some initial state matrix. Other methods involved use of a tree diagram or simply listing all possibilities. Students who did the latter frequently made mistakes in assigning the probabilities, not taking into account their conditional nature.

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