VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS

Written examination 2
(Analysis task)

Monday 10 November 2003

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	55

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 15 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- Where an **exact** answer is required to a question, appropriate working must be shown.
- In questions where more than one mark is available, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A manufacturer makes metal rods whose lengths are normally distributed with mean 140.0 cm and standard deviation 1.2 cm.

a. Find the probability, correct to three decimal places, that a randomly selected metal rod is longer than 141.5 cm.

2 marks

b. A rod has a **size fault** if its length is not within d cm either side of the mean. The probability of a rod having a size fault is 0.15. Find the value of d, correct to one decimal place.

2 marks

c. A random sample of 12 rods is taken from a crate containing a very large number of rods. Find the probability, correct to three decimal places, that the sample contains exactly 2 rods with a **size fault**.

2 marks

d. A particular box of 25 rods has 4 rods in it which have **size faults**. A sample of 12 rods is withdrawn without replacement. Find the probability, correct to three decimal places, that the sample contains at least 2 rods with a size fault.

2 marks

Question 1 – continued

e. The sales manager is considering at what price, x dollars, to sell each rod. The materials cost \$5. The rods are sorted into three bins. 15 % of all the rods manufactured have a size fault, and another 17 % of all the rods manufactured have other faults. The profit, Y dollars, is a random variable whose probability distribution is shown in the table below.

Bin	Description	Profit(\$$y$)	Pr($Y = y$)
A	Good rods – these are sold for \$$x$ each	$x - 5$	k
B	Rods with a size fault – these are not sold but are recycled	0	0.15
C	Rods with other faults – these are sold at a discount of \$3 each	$x - 8$	0.17

i. Find the value of k.

ii. Find the mean of Y in terms of x.

iii. Hence or otherwise find, correct to the nearest cent, the selling price of good rods so that the mean profit is zero.

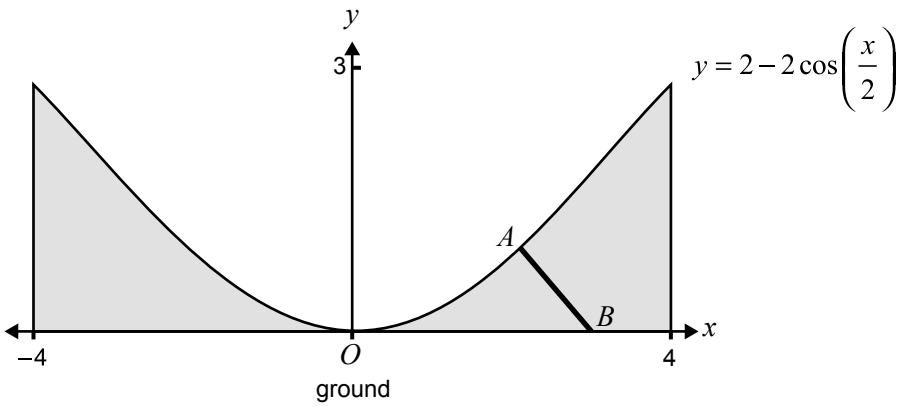
iv. The rods are stored in the bins until there is a large number ready to be sold.
What proportion of the rods ready to be sold are good rods?

1 + 1 + 1 + 1 = 4 marks

Total 12 marks

Question 2

Andrew is making a skateboard ramp. He draws a cross-section diagram with coordinate axes as shown below.



The curve has the equation $y = 2 - 2\cos\left(\frac{x}{2}\right)$, $-4 \leq x \leq 4$. All measurements are in metres; the horizontal length of the structure is 8 metres.

a. How many metres above the ground is the highest point of the ramp? Give your answer correct to two decimal places.

1 mark

b. Use calculus to show that the gradient of the ramp is always less than or equal to 1.

2 marks

c. Use calculus to find the area of the shaded region, correct to two decimal places.

3 marks

There is a supporting beam AB on the structure as shown. A is a point on the curve one metre vertically above the x-axis. B is a point on the x-axis such that AB is normal to the curve at A.

d. **i.** Find the exact value of the x-coordinate of A.

ii. Use calculus to find the exact value of the gradient of the normal to the curve at A.

iii. Find the exact value of the length of AB.

2 + 2 + 3 = 7 marks

Total 13 marks

Working space

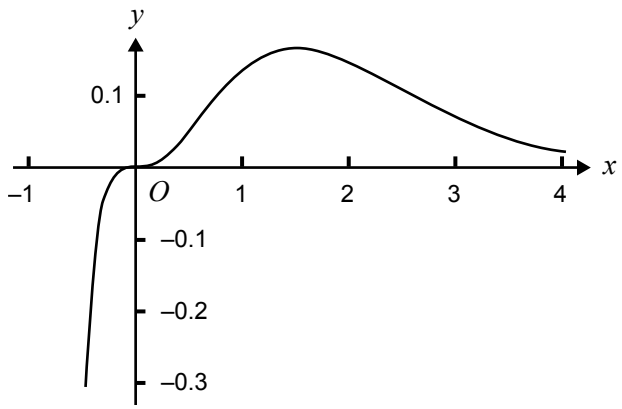
Question 3

Consider the function $f: R \rightarrow R,\ f(x) = x^3e^{-2x}$.

a. $f'(x)$ may be written as $f'(x) = e^{-2x}(ax^3 + bx^2)$ where a and b are real constants. Find the exact values of a and b.

1 mark

b. The graph of $y = f(x)$ is as shown.



Find the exact coordinates of the two stationary points and state their nature.

2 marks

c. **i.** Show that, at the point on the graph where $x = 1$, an equation of the tangent is $y = e^{-2}x$.

ii. Write down an equation of the tangent to the curve at the point (0, 0).

iii. Show that the tangents of parts **i.** and **ii.** are the only two tangents to the curve which pass through the origin.

3 + 1 + 3 = 7 marks

d. **i.** Use calculus to find the exact values of the real constants p, q and k such that $\frac{d}{dx}\left[\left(4x^3 + px^2 + qx + 3\right)e^{-2x}\right]$ is equal to $k\,x^3e^{-2x}$.

ii. Hence use calculus to find the exact value of the area of the region bounded by the curve with equation $y = x^3e^{-2x}$, where $x \geq 0$, and by the tangent to the curve where $x = 1$.

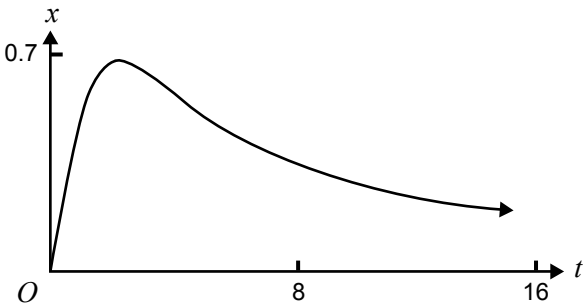
3 + 3 = 6 marks

Total 16 marks

Working space

Question 4

A tranquilliser is injected into a muscle from which it enters the bloodstream. The concentration, x mg/L, of the tranquilliser in the bloodstream, may be modelled by the equation $x = \frac{3t}{5+t^2}$, $t \geq 0$, where t is the number of hours after the injection is given. The graph of this equation is shown.



a. The tranquilliser is effective when the concentration in the bloodstream is at least 0.4 mg/L.
Find, correct to two decimal places, the length of time in hours for which the tranquilliser is effective, according to this model.

3 marks

b. Use calculus to find the exact number of hours, after the injection, when the tranquilliser concentration is greatest. Also find the exact value of this maximum concentration.

4 marks

c. According to this model, the derivative of x with respect to t gives a measure of the rate of absorption of tranquilliser into the bloodstream.

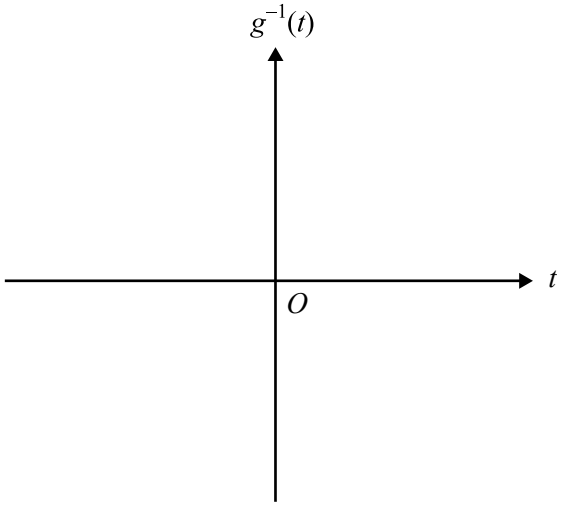
According to this model, how many hours after the injection is the rate of absorption into the bloodstream 0.25 mg/L/h?

Give your answer correct to two decimal places.

1 mark

d. **i.** What is the least value of a such that the function g: $[a, \infty) \rightarrow R$, $g(t) = \frac{3t}{5+t^2}$ has an inverse function?

ii. For this value of a, sketch the graph of g^{-1} on the axes below. Label any end-point with its coordinates. Label any asymptote with its equation.



iii. Find the rule for g^{-1}.

1 + 2 + 3 = 6 marks

Total 14 marks

Working space

END OF QUESTION AND ANSWER BOOK