

VCE
Mathematical Methods
Trial Examination 1

Suggested Solutions

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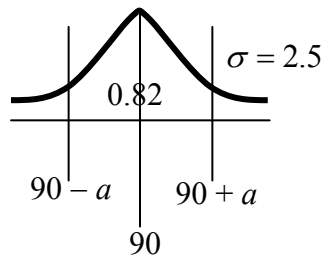
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| <p>Question 1 C</p> $ \begin{array}{r} \overline{)2x+2} \\ \underline{2x-10} \\ +12 \\ \overline{)12} \\ \underline{10} \\ 2 \\ \underline{2} \\ 0 \end{array} $ $= 2 + \frac{12}{x-5}$ | <p>Question 2 B</p> $P(-1) = 1 - 1 + a - b + 30 = 0$ $\Rightarrow a - b = -30 \quad (1)$ $P(2) = 16 + 8 + 4a + 2b + 30 = 0$ $\Rightarrow 4a + 2b = -54 \quad (2)$ $(1) \times 4 \rightarrow 4a - 4b = -120 \quad (1a)$ $(2) - (1a) \rightarrow 6b = 66$ $b = 11$ <p>Substituting $b = 11$ in (1)</p> $a - 11 = -30$ $a = -19$ |
| <p>Question 3 B</p> <p>To have an inverse that is a function $f(x)$ must be a one-to-one function. In the domains given, the only function that is one-to-one is $f(x) = x^3 - x$</p> | <p>Question 4 D</p> <p>The basic graph of $\log_e(x)$ which passes through the point $(1,0)$ and has an asymptote $x = 0$ is translated a units to the right and b units up, to pass through the point $(a+1, b)$. The asymptote is translated a units to the right to become the vertical line $x = a$</p> |
| <p>Question 5 E</p> $\frac{\sqrt{3} \sin\left(\frac{3\pi}{2}x\right)}{\cos\left(\frac{3\pi}{2}x\right)} = -1 \quad 0 \leq x \leq 2$ $\sqrt{3} \tan\left(\frac{3\pi}{2}x\right) = -1 \quad 0 \leq \frac{3\pi}{2}x \leq 3\pi$ $\tan\left(\frac{3\pi}{2}x\right) = -\frac{1}{\sqrt{3}}$ $\frac{3\pi}{2}x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}$ $3\pi x = \frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}$ $x = \frac{5}{9}, \frac{11}{9}, \frac{17}{9}$ $\text{Sum} = \frac{5}{9} + \frac{11}{9} + \frac{17}{9} = \frac{33}{9} = \frac{11}{3}$ | <p>Question 6 A</p> <p>Domain of $f(x)$ is $R^+ \cup \{0\}$</p> <p>Range of $f(x)$ is $[1, \infty)$</p> <p>Domain of $f^{-1}(x) = \text{range of } f(x)$</p> <p>$\therefore$ Domain of $f^{-1}(x)$ is $x \geq 1$</p> |

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| <p>Question 7 E This graph is translated 2 units to the right and 3 units up, so maximum at $(-1 + 2, 16 + 3)$ i.e. $(1, 19)$ minimum at $(3 + 2, -16 + 3)$ i.e. $(5, -13)$</p> | <p>Question 8 C X intercept when $y = 0$ $3\log_3(x + 2) - 1 = 0$ $3\log_3(x + 2) = 1$ $\log_3(x + 2) = \frac{1}{3}$ $x + 2 = 3^{\frac{1}{3}} = \sqrt[3]{3}$ $x = \sqrt[3]{3} - 2$</p> |
| <p>Question 9 C $\frac{dy}{dx} = -6\cos(2x)$ This cos graph has a maximum of 6 and a minimum of -6</p> | <p>Question 10 A Product rule $f'(x) = x^4(-3e^{-3x}) + e^{-3x} \times 4x^3$ $= x^3e^{-3x}(4 - 3x)$</p> |
| <p>Question 11 E $\frac{dy}{dx} = \frac{(x^3 + 2)4x - 2x^2(3x^2)}{(x^3 + 2)^2} = 0$ (Quotient rule) $\frac{4x^4 + 8x - 6x^4}{(x^3 + 2)^2} = 0$ $\frac{8x - 2x^4}{(x^3 + 2)^2} = 0$ $8x - 2x^4 = 0$ $2x(4 - x^3) = 0$ $x(4 - x^3) = 0$</p> | <p>Question 12 C Chain rule $\frac{dV}{dx} = \frac{dV}{dy} \frac{dy}{dx}$ $\frac{dV}{dy} = 5(2y - 3)^4 \times 2 = 10(2y - 3)^4$ $\frac{dy}{dx} = 2(2x - 3) \times 2 = 4(2x - 3)$ $\frac{dV}{dx} = 10(2y - 3)^4 \times 4(2x - 3)$ $= 40(2y - 3)^4(2x - 3)$</p> |
| <p>Question 13 E $f(x) = \int (3 - x)(5 - 2x)dx$ $= \int (15 - 6x - 5x + 2x^2)dx$ $= \int (15 - 11x + 2x^2)dx$ $= 15x - \frac{11x^2}{2} + \frac{2x^3}{3} + c$ $66 = 90 - 198 + 144 + c$ $66 - 36 = c$ $c = 30$ $f(x) = \frac{2x^3}{3} - \frac{11x^2}{2} + 15x + 30$</p> | <p>Question 14 A Area $= \int_0^4 (4ax - ax^2)dx$ $= [2ax^2 - \frac{ax^3}{3}]_0^4$ $= (32a - \frac{64a}{3}) - 0$ $= \frac{96a}{3} - \frac{64a}{3}$ $= \frac{32a}{3} = 10\frac{2}{3}a$</p> |

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| <p>Question 15 E</p> $A = \int_{-1}^2 [f(x) - g(x)] dx$ | <p>Question 16 A</p> $A = \int \frac{2x^2}{x} dx - \int \frac{1}{x} dx + \int \frac{1}{x-2} dx$ $= x^2 - \log_e(x) + \log_e(x-2)$ $= x^2 + \log_e\left(\frac{x-2}{x}\right)$ |
| <p>Question 17 B</p> $A = \frac{1}{2}[(f(0) + f(1)) + (f(1) + f(2)) + (f(2) + f(3)) + (f(3) + f(4))]$ $= \frac{1}{2}f(0) + f(1) + f(2) + f(3) + \frac{1}{2}f(4)$ $= \frac{1}{2} + 3^1 + 3^2 + 3^3 + \frac{1}{2} \times 3^4$ $= \frac{1}{2} + 3 + 9 + 27 + \frac{81}{2}$ $= 80$ | <p>Question 18 E</p> $2y = x - 6$ $\Rightarrow y = \frac{1}{2}x - 3$ $m = \frac{1}{2}$ <p>gradient of perpendicular line $\times m = -1$ gradient of perpendicular line = -2</p> $y = -ax - b$ $\therefore a = 2$ $y = -2x - b$ <p>When $x = 4, y = -1$</p> $-1 = -8 - b$ $-b = 7$ $b = -7$ |
| <p>Question 19 E</p> $f(x+h) \approx f(x) + hf'(x)$ $f(x+0.2) \approx f(x) + 0.2 \times f'(x)$ $f(x) = SA = 6x^2$ $f'(x) = 12x$ $\Delta SA = f(x+0.2) - f(x) \approx 0.2 \times f'(x)$ $= 0.2 \times 12x$ <p>When $x = 10$</p> $\Delta SA = 0.2 \times 12 \times 10 = 24 \text{ cm}^2$ | <p>Question 20 A</p> $x^2 - 2x + 3 = Ax^2 + 2A + Bx^2 + Bx + Cx + C$ $x^2 - 2x + 3 = (A+B)x^2 + (B+C)x + (2A+C)$ <p>Equating coefficients</p> $A+B=1 \quad (1)$ $B+C=-2 \quad (2)$ $2A+C=3 \quad (3)$ $(2)-(1) \rightarrow C-A=-3$ $\therefore -2A+2C=-6 \quad (4)$ $(4)+(3) \rightarrow 3C=-3$ $\Rightarrow C=-1$ |

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| <p>Question 21 C The domain is R and the range is $[2, \infty)$. This is the graph of $y = x^3$ translated 3 units to the right and 2 units up, so local minimum is $(3, 2)$. The dilation is by a factor of 4 parallel to the y axis.</p> | <p>Question 22 A Vertical asymptote when $2x - 4 = 0$ $\Rightarrow x = 2$ Graph of $y = \frac{5}{2x - 4}$ is moved 3 units up by $+3$ so the horizontal asymptote is $y = 3$</p> |
| <p>Question 23 E $\text{Pr} = \frac{17}{17 + 23} = \frac{17}{40}$</p> | <p>Question 24 A For A, when $x = 0$, $\text{Pr} = \frac{1}{4}$ When $x = 1$, $\text{Pr} = \frac{3}{4}$ $\sum \text{Pr} = \frac{1}{4} + \frac{3}{4} = 1$ \therefore this is a probability function.</p> |
| <p>Question 25 B Hypergeometric $\sigma = \sqrt{n \frac{D}{N} \left(1 - \frac{D}{N}\right) \left(\frac{N-n}{N-1}\right)}$ $D = 8, N = 18, n = 5$ $\sigma = \sqrt{5 \times \frac{8}{18} \left(1 - \frac{8}{18}\right) \left(\frac{18-5}{18-1}\right)} = 0.97$</p> | <p>Question 26 A Binomial with $n = 10, p = \frac{1}{6}, x = 9$ or 10 $\text{Pr}(X = 9) + \text{Pr}(x = 10)$ $= \binom{10}{9} \left(\frac{1}{6}\right)^9 \left(\frac{5}{6}\right)^1 + \binom{10}{10} \left(\frac{1}{6}\right)^{10} \left(\frac{5}{6}\right)^0$ $= \binom{10}{9} \left(\frac{1}{6}\right)^9 \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^{10}$</p> |

Question 27 E



$$\Pr(90 - a < X < 90 + a) = 0.82$$

$$\Pr(X < (90 + a)) - \Pr(X < (90 - a)) = 0.82$$

$$\Pr(X < (90 + a)) - \Pr(X > (90 + a)) = 0.82$$

$$\Pr(X < (90 + a)) - (1 - \Pr(X < (90 + a))) = 0.82$$

$$2 \Pr(X < (90 + a)) - 1 = 0.82$$

$$2 \Pr(X < (90 + a)) = 1.82$$

$$\Pr(X < (90 + a)) = 0.91$$

$$Z = \frac{x - \mu}{\sigma} = \frac{90 + a - 90}{2.5} = \frac{a}{2.5}$$

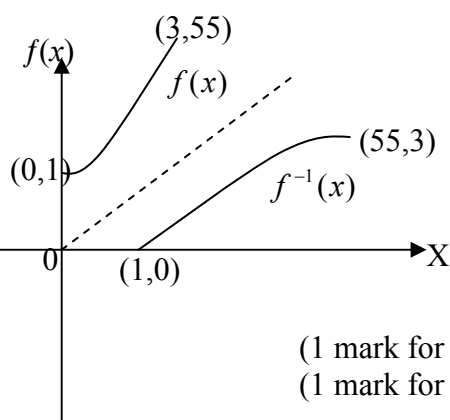
$$\Pr(Z < \frac{a}{2.5}) = 0.91$$

$$\frac{a}{2.5} = 1.341$$

$$a = 2.5 \times 1.341 = 3.3525$$

Question 1

a. & c.



(1 mark for graph a.)
(1 mark for graph c.)

b. Range of $f(x)$ is $[1,55]$ (1 mark)

Question 2

a.

$$g(x) = 2\left[x^2 - 6x + \frac{31}{2}\right]$$

$$= 2\left[x^2 - 6x + 9 - 9 + \frac{31}{2}\right] \quad (1 \text{ mark})$$

$$= 2\left[(x-3)^2 - \frac{18}{2} + \frac{31}{2}\right]$$

$$= 2\left[(x-3)^2 + \frac{13}{2}\right]$$

$$= 2(x-3)^2 + 13 \quad (1 \text{ mark})$$

b.

$f(x)$ is translated 3 units to the right parallel to the X axis (1 mark)
 $f(x)$ is translated 13 units up parallel to the Y axis (1 mark)
 $f(x)$ is dilated by a factor of 2 parallel to the Y axis (1 mark)

Question 3

a.

When $x = 0, y = 0$ (0,0) (1 mark)

When $x = -2, y = 4e^{-2}$ $(-2, 4e^{-2})$ (1 mark)

b.

$\frac{dy}{dx} = x^2e^x + e^x \times 2x = 0$ for tangents that are horizontal. (1 mark)

$$\therefore xe^x(x+2) = 0$$

$\Rightarrow x = 0$ or $x = -2, e^x > 0$ for all x (1 mark)

Question 4

a.

$$\sin(2\pi x)(2\cos(2\pi x) + \sqrt{3}) = 0 \quad (1 \text{ mark})$$

$$\sin(2\pi x) = 0 \text{ or } 2\cos(2\pi x) = -\sqrt{3} \quad 0 \leq x \leq 2$$

$$\sin(2\pi x) = 0 \text{ or } \cos(2\pi x) = -\frac{\sqrt{3}}{2} \quad 0 \leq 2\pi x \leq 4\pi$$

$$2\pi x = 0, \pi, 2\pi, 3\pi, 4\pi \text{ or } 2\pi x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6},$$

$$3\pi - \frac{\pi}{6}, 3\pi + \frac{\pi}{6} \quad (1 \text{ mark})$$

$$2x = 0, 1, 2, 3, 4 \text{ or } 2x = \frac{5}{6}, \frac{7}{6}, \frac{17}{6}, \frac{19}{6}$$

$$x = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{12}, \frac{7}{12}, \frac{17}{12}, \frac{19}{12} \quad (1 \text{ mark})$$

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| <p>Question 5</p> $\log_a(x^2) + \log_a(5) = \log_a(a) + \log_a(8x - 3a)$ $\log_a(5x^2) = \log_a(a(8x - 3a)) \quad (1 \text{ mark})$ $5x^2 = 8ax - 3a^2$ $5x^2 - 8ax + 3a^2 = 0 \quad (1 \text{ mark})$ $x = \frac{8a \pm \sqrt{64a^2 - 60a^2}}{10}$ $x = \frac{8a \pm \sqrt{4a^2}}{10}$ $x = \frac{10a}{10} \text{ or } \frac{6a}{10}$ $x = a \text{ or } 0.6a \quad (1 \text{ mark})$ | <p>Question 6</p> <p>a. Hypergeometric $N = 20, D = 4, n = 6 \quad x \geq 1$ $\Pr(X \geq 1) = 1 - \Pr(X = 0) \quad (1 \text{ mark})$</p> $= 1 - \frac{\binom{4}{0} \binom{16}{6}}{\binom{20}{6}} = 0.7934 \quad (1 \text{ mark})$ <p>b. Pr not defective and defective and not defective and defective = $\frac{16}{20} \times \frac{4}{20} \times \frac{16}{20} \times \frac{4}{20} = 0.0256$ (1 mark)</p> <p>c. Pr rejects = $\Pr(X \geq 1) = 1 - \Pr(X = 0)$ $= 1 - \binom{4}{0} \left(\frac{4}{20}\right)^0 \left(\frac{16}{20}\right)^4 = 1 - \left(\frac{16}{20}\right)^4$ Pr accepts batch = $1 - \Pr \text{ rejects}$ $= 1 - \left[1 - \left(\frac{16}{20}\right)^4\right]$ $= \left(\frac{16}{20}\right)^4 = 0.4096$</p> |
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END OF SUGGESTED SOLUTIONS
2004 Mathematical Methods Trial Examination 1

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