VCE
Mathematical Methods
Trial Examination 1

Suggested Solutions

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Question 1 C

$$x-5\overline{)2x+2}$$ quotient 2

$$2x-10$$

$$+12$$

$$=2+\frac{12}{x-5}$$

Question 2 B

$P(-1)=1-1+a-b+30=0$

$\Rightarrow a-b=-30$ (1)

$P(2)=16+8+4a+2b+30=0$

$\Rightarrow 4a+2b=-54$ (2)

$(1)\times 4 \rightarrow 4a-4b=-120$ (1a)

$(2)-(1a) \rightarrow 6b=66$

$b=11$

Substituting $b=11$ in (1)

$a-11=-30$

$a=-19$

Question 3 B

To have an inverse that is a function $f(x)$ must be a
one-to-one function. In the domains given, the only
function that is one-to-one is $f(x)=x^3-x$

Question 4 D

The basic graph of $\log_e(x)$ which passes through the point (1,0) and has an asymptote $x=0$ is translated a units to the right and b units up, to pass through the point $(a+1,b)$. The asymptote is translated a units to the right to become the vertical line $x=a$

Question 5 E

$$\frac{\sqrt{3}\sin\left(\frac{3\pi}{2}x\right)}{\cos\left(\frac{3\pi}{2}x\right)}=-1 \qquad 0\le x\le 2$$

$$\sqrt{3}\tan\left(\frac{3\pi}{2}x\right)=-1 \qquad 0\le \frac{3\pi}{2}x\le 3\pi$$

$$\tan\left(\frac{3\pi}{2}x\right)=-\frac{1}{\sqrt{3}}$$

$$\frac{3\pi}{2}x=\frac{5\pi}{6},\frac{11\pi}{6},\frac{17\pi}{6}$$

$$3\pi x=\frac{5\pi}{3},\frac{11\pi}{3},\frac{17\pi}{3}$$

$$x=\frac{5}{9},\frac{11}{9},\frac{17}{9}$$

$$\text{Sum } = \frac{5}{9}+\frac{11}{9}+\frac{17}{9}=\frac{33}{9}=\frac{11}{3}$$

Question 6 A

Domain of $f(x)$ is $R^+\cup\{0\}$

Range of $f(x)$ is $[1,\infty)$

Domain of $f^{-1}(x)=$ range of $f(x)$

$\therefore$ Domain of $f^{-1}(x)$ is $x\ge 1$

Question 7 E
This graph is translated 2 units to the right and 3 units up,
so maximum at (−1 + 2, 16 + 3) i.e.(1,19)
minimum at (3 + 2, −16 + 3) i.e. (5, −13)

Question 8 C
X intercept when $y = 0$

$3\log_3(x+2) - 1 = 0$

$3\log_3(x+2) = 1$

$\log_3(x+2) = \dfrac{1}{3}$

$x + 2 = 3^{\frac{1}{3}} = \sqrt[3]{3}$

$x = \sqrt[3]{3} - 2$

Question 9 C

$\dfrac{dy}{dx} = -6\cos(2x)$

This cos graph has a maximum of 6 and a minimum of -6

Question 10 A
Product rule

$f'(x) = x^4(-3e^{-3x}) + e^{-3x} \times 4x^3$

$= x^3e^{-3x}(4 - 3x)$

Question 11 E

$\dfrac{dy}{dx} = \dfrac{(x^3+2)4x - 2x^2(3x^2)}{(x^3+2)^2} = 0$ (Quotient rule)

$\dfrac{4x^4 + 8x - 6x^4}{(x^3+2)^2} = 0$

$\dfrac{8x - 2x^4}{(x^3+2)^2} = 0$

$8x - 2x^4 = 0$

$2x(4 - x^3) = 0$

$x(4 - x^3) = 0$

Question 12 C
Chain rule

$\dfrac{dV}{dx} = \dfrac{dV}{dy}\dfrac{dy}{dx}$

$\dfrac{dV}{dy} = 5(2y-3)^4 \times 2 = 10(2y-3)^4$

$\dfrac{dy}{dx} = 2(2x-3) \times 2 = 4(2x-3)$

$\dfrac{dV}{dx} = 10(2y-3)^4 \times 4(2x-3)$

$= 40(2y-3)^4(2x-3)$

Question 13 E

$f(x) = \int (3-x)(5-2x)dx$

$= \int (15 - 6x - 5x + 2x^2)dx$

$= \int (15 - 11x + 2x^2)dx$

$= 15x - \dfrac{11x^2}{2} + \dfrac{2x^3}{3} + c$

$66 = 90 - 198 + 144 + c$

$66 - 36 = c$

$c = 30$

$f(x) = \dfrac{2x^3}{3} - \dfrac{11x^2}{2} + 15x + 30$

Question 14 A

$\text{Area} = \int_0^4 (4ax - ax^2)dx$

$= [2ax^2 - \dfrac{ax^3}{3}]_0^4$

$= (32a - \dfrac{64a}{3}) - 0$

$= \dfrac{96a}{3} - \dfrac{64a}{3}$

$= \dfrac{32a}{3} = 10\dfrac{2}{3}a$

Question 15 E $A = \int_{-1}^{2} [f(x) - g(x)]dx$	**Question 16 A** $A = \int \frac{2x^2}{x} dx - \int \frac{1}{x} dx + \int \frac{1}{x-2} dx$ $= x^2 - \log_e(x) + \log_e(x-2)$ $= x^2 + \log_e\left(\frac{x-2}{x}\right)$
Question 17 B $A = \frac{1}{2}[(f(0) + f(1)) + (f(1) + f(2)) +$ $(f(2) + f(3)) + (f(3) + f(4))]$ $= \frac{1}{2} f(0) + f(1) + f(2) + f(3) + \frac{1}{2} f(4)$ $= \frac{1}{2} + 3^1 + 3^2 + 3^3 + \frac{1}{2} \times 3^4$ $= \frac{1}{2} + 3 + 9 + 27 + \frac{81}{2}$ $= 80$	**Question 18 E** $2y = x - 6$ $\Rightarrow y = \frac{1}{2}x - 3$ $m = \frac{1}{2}$ gradient of perpendicular line $\times\, m = -1$ gradient of perpendicular line $= -2$ $y = -ax - b$ $\therefore a = 2$ $y = -2x - b$ When $x = 4, y = -1$ $-1 = -8 - b$ $-b = 7$ $b = -7$
Question 19 E $f(x+h) \approx f(x) + hf'(x)$ $f(x+0.2) \approx f(x) + 0.2 \times f'(x)$ $f(x) = SA = 6x^2$ $f'(x) = 12x$ $\Delta SA = f(x+0.2) - f(x) \approx 0.2 \times f'(x)$ $= 0.2 \times 12x$ When $x = 10$ $\Delta SA = 0.2 \times 12 \times 10 = 24 \text{ cm}^2$	**Question 20 A** $x^2 - 2x + 3 = Ax^2 + 2A + Bx^2 + Bx + Cx + C$ $x^2 - 2x + 3 = (A+B)x^2 + (B+C)x + (2A+C)$ Equating coefficients $A + B = 1$ (1) $B + C = -2$ (2) $2A + C = 3$ (3) $(2) - (1) \rightarrow C - A = -3$ $\therefore -2A + 2C = -6$ (4) $(4) + (3) \rightarrow 3C = -3$ $\Rightarrow C = -1$

Question 21 C
The domain is R and the range is $[2,\infty)$.

This is the graph of $y = x^3$ translated 3 units to the right and 2 units up, so local minimum is (3,2). The dilation is by a factor of 4 parallel to the y axis.

Question 22 A
Vertical asymptote when $2x - 4 = 0$

$\Rightarrow x = 2$

Graph of $y = \dfrac{5}{2x-4}$ is moved 3 units up by $+3$

so the horizontal asymptote is $y = 3$

Question 23 E

$$\Pr = \frac{17}{17+23} = \frac{17}{40}$$

Question 24 A

For **A**, when $x = 0, \Pr = \dfrac{1}{4}$

When $x = 1, \Pr = \dfrac{3}{4}$

$$\sum \Pr = \frac{1}{4} + \frac{3}{4} = 1$$

$\therefore$ this is a probability function.

Question 25 B
Hypergeometric

$$\sigma = \sqrt{n\frac{D}{N}\left(1-\frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)}$$

$D = 8, N = 18, n = 5$

$$\sigma = \sqrt{5 \times \frac{8}{18}\left(1-\frac{8}{18}\right)\left(\frac{18-5}{18-1}\right)} = 0.97$$

Question 26 A

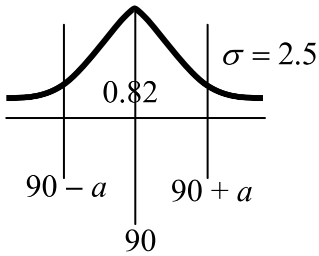
Binomial with $n = 10, p = \dfrac{1}{6}, x = 9$ or 10

$\Pr(X = 9) + \Pr(x = 10)$

$$= \binom{10}{9}\left(\frac{1}{6}\right)^9\left(\frac{5}{6}\right)^1 + \binom{10}{10}\left(\frac{1}{6}\right)^{10}\left(\frac{5}{6}\right)^0$$

$$= \binom{10}{9}\left(\frac{1}{6}\right)^9\left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^{10}$$

Question 27 E



$\Pr(90 - a < X < 90 + a) = 0.82$

$\Pr(X < (90 + a)) - \Pr(X < (90 - a)) = 0.82$

$\Pr(X < (90 + a)) - \Pr(X > (90 + a)) = 0.82$

$\Pr(X < (90 + a)) - (1 - \Pr(X < (90 + a)) = 0.82$

$2\Pr(X < (90 + a)) - 1 = 0.82$

$2\Pr(X < (90 + a)) = 1.82$

$\Pr(X < (90 + a)) = 0.91$

$Z = \dfrac{x - \mu}{\sigma} = \dfrac{90 + a - 90}{2.5} = \dfrac{a}{2.5}$

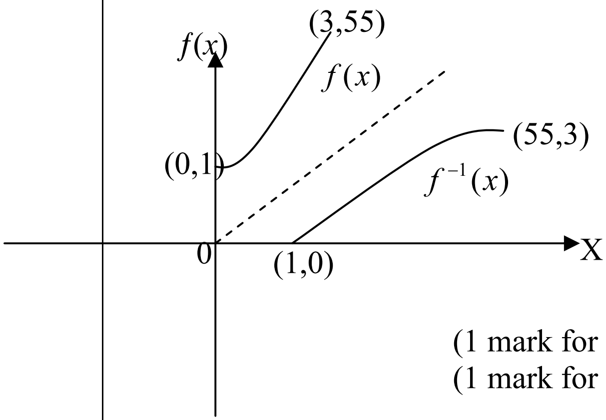
$\Pr(Z < \dfrac{a}{2.5}) = 0.91$

$\dfrac{a}{2.5} = 1.341$

$a = 2.5 \times 1.341 = 3.3525$

Question 1

a. & c.



(1 mark for graph a.)
(1 mark for graph c.)

b. Range of $f(x)$ is [1,55] (1 mark)

Question 2

a.

$g(x) = 2[x^2 - 6x + \frac{31}{2}]$

$= 2[x^2 - 6x + 9 - 9 + \frac{31}{2}]$ (1 mark)

$= 2[(x-3)^2 - \frac{18}{2} + \frac{31}{2}]$

$= 2[(x-3)^2 + \frac{13}{2}]$

$= 2(x-3)^2 + 13$ (1 mark)

b.

$f(x)$ is translated 3 units to the right parallel to the X axis (1 mark)

$f(x)$ is translated 13 units up parallel to the Y axis (1 mark)

$f(x)$ is dilated by a factor of 2 parallel to the Y axis (1 mark)

Question 3

a.

When $x = 0, y = 0$ (0,0) (1 mark)

When $x = -2, y = 4e^{-2}$ $(-2, 4e^{-2})$ (1 mark)

b.

$\frac{dy}{dx} = x^2e^x + e^x \times 2x = 0$ for tangents that are horizontal. (1 mark)

$\therefore xe^x(x+2) = 0$

$\Rightarrow x = 0$ or x = -2, $e^x > 0$ for all x (1 mark)

Question 4

a.

$\sin(2\pi x)(2\cos(2\pi x) + \sqrt{3}) = 0$ (1 mark)

$\sin(2\pi x) = 0$ or $2\cos(2\pi x) = -\sqrt{3}$ $\quad 0 \le x \le 2$

$\sin(2\pi x) = 0$ or $\cos(2\pi x) = -\frac{\sqrt{3}}{2}$ $\quad 0 \le 2\pi x \le 4\pi$

$2\pi x = 0, \pi, 2\pi, 3\pi, 4\pi$ or $2\pi x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}, 3\pi + \frac{\pi}{6}$ (1 mark)

$2x = 0, 1, 2, 3, 4$ or $2x = \frac{5}{6}, \frac{7}{6}, \frac{17}{6}, \frac{19}{6}$

$x = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{12}, \frac{7}{12}, \frac{17}{12}, \frac{19}{12}$ (1 mark)

Question 5

$\log_a(x^2) + \log_a(5) = \log_a(a) + \log_a(8x - 3a)$

$\log_a(5x^2) = \log_a(a(8x - 3a))$ (1 mark)

$5x^2 = 8ax - 3a^2$

$5x^2 - 8ax + 3a^2 = 0$ (1 mark)

$x = \dfrac{8a \pm \sqrt{64a^2 - 60a^2}}{10}$

$x = \dfrac{8a \pm \sqrt{4a^2}}{10}$

$x = \dfrac{10a}{10}$ or $\dfrac{6a}{10}$

$x = a$ or $0.6a$ (1 mark)

Question 6

a.
Hypergeometric

$N = 20, D = 4, n = 6 \quad x \geq 1$

$\Pr(X \geq 1) = 1 - \Pr(X = 0)$ (1 mark)

$= 1 - \dfrac{\binom{4}{0}\binom{16}{6}}{\binom{20}{6}} = 0.7934$ (1 mark)

b.
Pr not defective and defective and not defective and defective $= \dfrac{16}{20} \times \dfrac{4}{20} \times \dfrac{16}{20} \times \dfrac{4}{20} = 0.0256$

(1 mark)

c.
Pr rejects $= \Pr(X \geq 1) = 1 - \Pr(X = 0)$

$= 1 - \binom{4}{0}\left(\dfrac{4}{20}\right)^0\left(\dfrac{16}{20}\right)^4 = 1 - \left(\dfrac{16}{20}\right)^4$

Pr accepts batch $= 1 - $ Pr rejects

$= 1 - \left[1 - \left(\dfrac{16}{20}\right)^4\right]$

$= \left(\dfrac{16}{20}\right)^4 = 0.4096$

END OF SUGGESTED SOLUTIONS
2004 Mathematical Methods Trial Examination 1

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