Year 2004

VCE
Mathematical Methods
Trial Examination 2

Suggested Solutions

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Question 1

a.

$t = 0$

$T = 50e^0 + 20 = 50 + 20 = 70^0 C$ (1 mark)

b.

$45 = 50e^{-3k} + 20$ (1 mark)

$25 = 50e^{-3k}$

$0.5 = e^{-3k}$

$-3k = \log_e(0.5)$

$k = 0.231$ (1 mark)

c.

$35 = 50e^{-0.231t} + 20$

$15 = 50e^{-0.231t}$

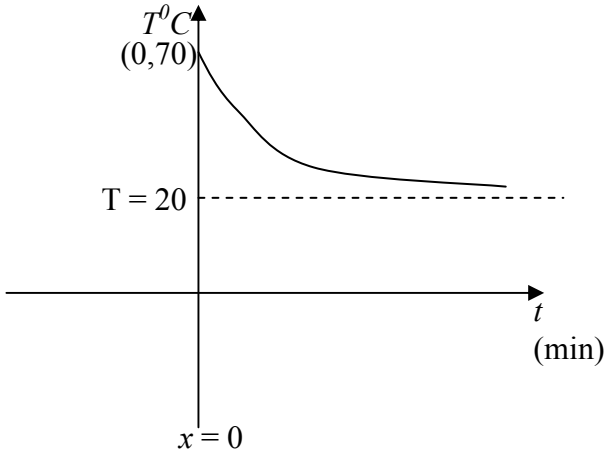
$0.3 = e^{-0.231t}$

$\log_e(0.3) = -0.231t$

$t = 5.2$ min. (1 mark)

d.

- 1 mark for shape
- 1 mark for y intercept
- 1 mark for equation of asymptote.



e.

From (c) when $T = 35, t = 5.2$

$\frac{dT}{dt} = -50 \times 0.231e^{-0.231t}$ (1 mark)

When $t = 5.2$

$\frac{dT}{dt} = -11.55e^{-0.231 \times 5.2} = -3.47$

Temperature is decreasing

at a rate of $3.47^0 C / \min$ (1 mark)

f.(i)

There is a turning point at (3,0)

$\frac{dT}{dt} = 2a(3t-b)\times 3 = 6a(3t-b) = 0$ for T.P.

When $t = 3$, $6a(9-b) = 0$

$\Rightarrow 9-b = 0$ since $a > 0$

$\Rightarrow b = 9$ (1 mark)

f.(ii)

When $t = 4$, $T = 5$

$5 = a(12-9)^2$

$9a = 5$

$a = \frac{5}{9}$ (1 mark)

f.(iii)

$\frac{5}{9}(3t-9)^2 = 50e^{-0.231t} + 20$

In graphics calculator enter

$y_1 = \frac{5}{9}(3t-9)^2 - (50e^{-0.231t} + 20)$

Go to table and look for the answer closest to zero. This is 5.6 to one decimal place. (1 mark)

Question 2

a.

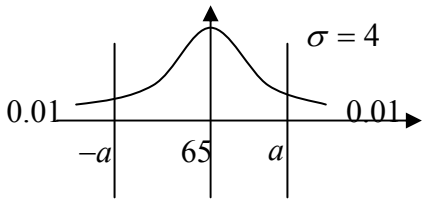
$\Pr(X > 75) = \Pr(Z > 2.5)$ $Z = \frac{75-65}{4}$

$Z = 2.5$ (1 mark)

$\Pr(Z > 2.5) = 1 - \Pr(Z < 2.5)$

$= 1 - 0.9938 = 0.0062$ (1 mark)

b.



$\Pr(Z < -a) = \Pr(Z > a) = 1 - \Pr(Z < a) = 0.01$

$\Rightarrow \Pr(Z < a) = 0.99$

$a = 2.326$

$-a = -2.326$ (1 mark)

$-2.326 = \frac{x-65}{4}$

$x = 65 - 9.304 = 55.7$ (1 mark)

c.
Total probability = 1
Probability acceptable = 1 – [0.01 + 0.0062] = 0.9838

(1 mark)

d.(i)	Acceptable	Not Acceptable
Y	4	–6
Pr(Y=y)	0.9838	0.0162

- (1 mark for each value)

d.(ii)

$$E(X) = \sum x \Pr(X = x)$$

$$= (4 \times 0.9838) - (6 \times 0.0162) = 3.838 \text{ per trip}$$ (1 mark)

Expected value for 80 trips = 80×3.838

= \$307 (1 mark)

e.

$$\Pr(55.7 < X < 75) / X > 65)0.5$$

$$= \frac{\Pr(55.7 < X < 75 / X > 65) \cap X > 65}{\Pr(X > 65)}$$ (1 mark)

$$= \frac{\Pr(65 < X < 75)}{0.5}$$

$$= \frac{\Pr(X < 75) - \Pr(X < 65)}{0.5}$$

$$= \frac{0.9938 - 0.5}{0.5}$$

$$= \frac{0.4938}{0.5} = 0.9876$$ (1 mark)

f.
Binomial: Number of independent trials and only two outcomes.

$$\Pr(X \geq 1) = 1 - \Pr(X = 0)$$ (1 mark)

$$= 1 - \binom{20}{0}(0.0062)^0(0.9938)^{20}$$

$$= 1 - (0.9938)^{20} = 0.117$$ (1 mark)

g.

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) = 0.285$$

$$\Pr(X = 0) = 0.715$$ (1 mark)

$$(0.9938)^n = 0.715$$

$$n \log_e(0.9938) = \log_e(0.715)$$

$$n = \frac{\log_e(0.715)}{\log_e(0.9938)} = 54$$ (1 mark)

Question 3

a.

$y = \frac{1}{2}e^{-x}(\sin(x) - \cos(x))$

$\frac{dy}{dx} = \frac{1}{2}e^{-x}(\cos(x) + \sin(x)) +$

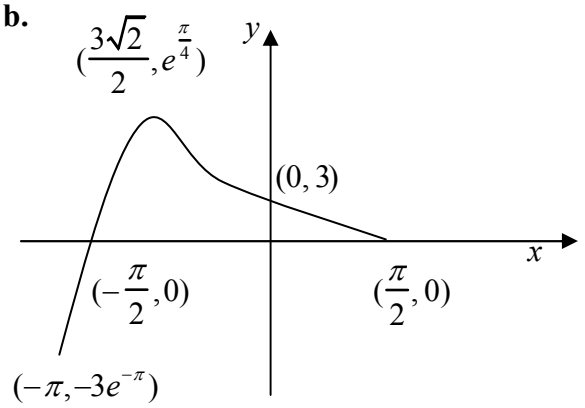
$(\sin(x) - \cos(x))(-\frac{1}{2}e^{-x})$ (1 mark)

$= \frac{1}{2}e^{-x}[\cos(x) + \sin(x) - \sin(x) + \cos(x)]$

$= \frac{1}{2}e^{-x} \times 2\cos(x)$

$= e^{-x}\cos(x)$ (1 mark)

b.



X intercept when $y = 0$

$3e^{-x}\cos(x) = 0$

But $3e^{-x} \neq 0$

$\therefore \cos(x) = 0 \Rightarrow x = -\frac{\pi}{2}, \frac{\pi}{2}$ (1 mark)

Y intercept when $x = 0$

$y = 3e^{0}\cos(0) = 3$ (1 mark)

Turning point when $f'(x) = 0$

$3e^{-x}(-\sin(x)) + \cos(x)(-3e^{-x}) = 0$ (1 mark)

$-3e^{-x}[\sin(x) + \cos(x)] = 0$

$\Rightarrow \sin(x) = -\cos(x)$

$\Rightarrow \tan(x) = -1$ (1 mark)

$\Rightarrow x = -\frac{\pi}{4}$ (1 mark)

When $x = -\frac{\pi}{4}, y = 3e^{\frac{\pi}{4}}\cos(-\frac{\pi}{4})$

$y = \frac{3}{\sqrt{2}}e^{\frac{\pi}{4}} = \frac{3\sqrt{2}}{2}e^{\frac{\pi}{4}}$ (1 mark)

End point when $x = -\pi, y = 3e^{-\pi}\cos(-\pi) = -3e^{-\pi}$ (1 mark)

(1 mark for shape of graph)

Question 3 (continued)

c.

$$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} 3e^{-x}\cos(x)dx$$

$$= 3\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-x}\cos(x)dx$$

$$= 3[\frac{1}{2}(\sin(x)-\cos(x))e^{-x}]_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \qquad \text{(1 mark)}$$

$$= \frac{3}{2}[(\sin\left(\frac{\pi}{2}\right)-\cos\left(\frac{\pi}{2}\right))\times e^{-\frac{\pi}{2}}$$

$$-(\sin\left(-\frac{\pi}{4}\right)-\cos\left(-\frac{\pi}{4}\right))\times e^{\frac{\pi}{4}}] \quad \text{(1 mark)}$$

$$= \frac{3}{2}[(1-0)e^{-\frac{\pi}{2}}-(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}})e^{\frac{\pi}{4}}]$$

$$= \frac{3}{2}[e^{-\frac{\pi}{2}}+\frac{2}{\sqrt{2}}e^{\frac{\pi}{4}}]$$

$$= \frac{3}{2}[e^{-\frac{\pi}{2}}+\sqrt{2}e^{\frac{\pi}{4}}] \qquad \text{(1 mark)}$$

Question 4

a.

$20,000 = a + 5b$ (1)

$10,000 = a + 10b$ (2)

(1)-(2) $\rightarrow 10,000 = -5b$

$\Rightarrow b = -2,000$ (1 mark)

Substituting in (1)

$20,000 = a - 10,000$

$a = 30,000$ (1 mark)

b.

$C = 1,500 + 0.2(a + bs)$ (1 mark)

$C = 1,500 + 0.2(30,000 - 2000s)$

$C = 1,500 + 6,000 - 400s$ (1 mark)

$C = 7,500 - 400s$

c.

$P =$ Selling Price $-$ Cost Price

$P = s(30,000 - 2,000s) - (7,500 - 400s)$ (1 mark)

$P = 30,000s - 2,000s^2 - 7,500 + 400s$

$P = 30,400s - 2,000s^2 - 7,500$ (1 mark)

d.

$\frac{dP}{ds} = 30,400 - 4,000s = 0$ for TP

$\Rightarrow 4,000s = 30,400$

$s = \frac{30,400}{4,000}$

$s = \$7.60$ (1 mark)

Since curve of $P = 30,400s - 2,000s^2 - 7,500$ is concave down, then maximum profit when $s = \$7.60$ (1 mark)

Question 4 (continued)

e.

$P_{\max} = 30,400 \times 7.6 - 2,000(7.6)^2 - 7,500$

$= 231,040 - 115,520 - 7,500$

$= \$108,020$ (1 mark)

f.

To make this profit, $s = \$7.60$

$\therefore n = 30,000 - 2,000 \times 7.6$

$\therefore n = 14,800$ (1 mark)

g.

$P = 0$

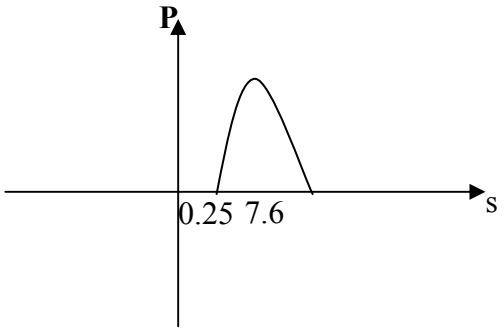
$\therefore 30,400s - 2,000s^2 - 7,500 = 0$

$\Rightarrow -20s^2 + 304s - 75 = 0$

$$s = \frac{-304 \pm \sqrt{304^2 - 6000}}{-40}$$

$s = 25$ cents (1 mark)

h.



$n = a + bs$

$12,000 = 30,000 - 2,000s$

$2,000s = 18,000$

$s = 9$ (1 mark)

When $s = 9$

$P = 30,400 \times 9 - 2,000 \times 81 - 7,500$

$P = \$104,100$ (1 mark)

END OF SUGGESTED SOLUTIONS
2004 Mathematical Methods Trial Examination 2

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