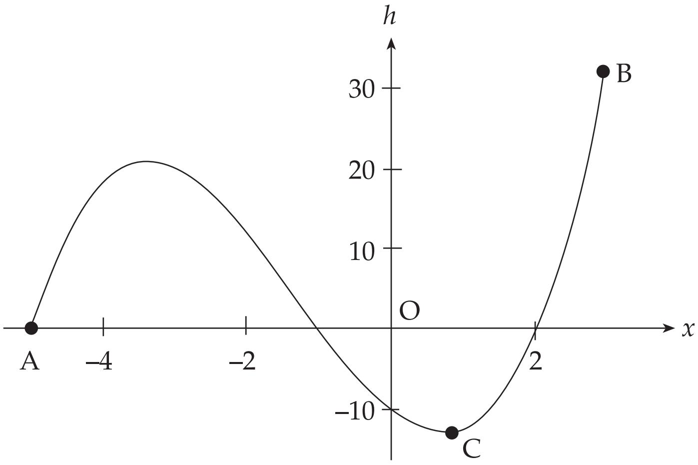
Examination 2 (58 Marks)

Question 1

The side view of a section of a roller coaster ramp at Mars Park is shown on the graph below. The graph is $y = h_1(x)$, where

$$h_1: [-5, 3] \to R, \text{ where } h_1(x) = (x + 5)(x + 1)(x - 2)$$

where x is the horizontal distance in metres from O and h_1 is the vertical distance from the ground in metres.



a Find the coordinates of the points A and B, the endpoints on the graph. 2 marks

An underground tunnel is to be built between points A and C, where $x = \frac{1}{2}$ at C.

b Find the equation of the line which passes through these two points.

2 marks

c Use calculus to show that the line in **part b** is a tangent to $y = h_1(x)$ at C.

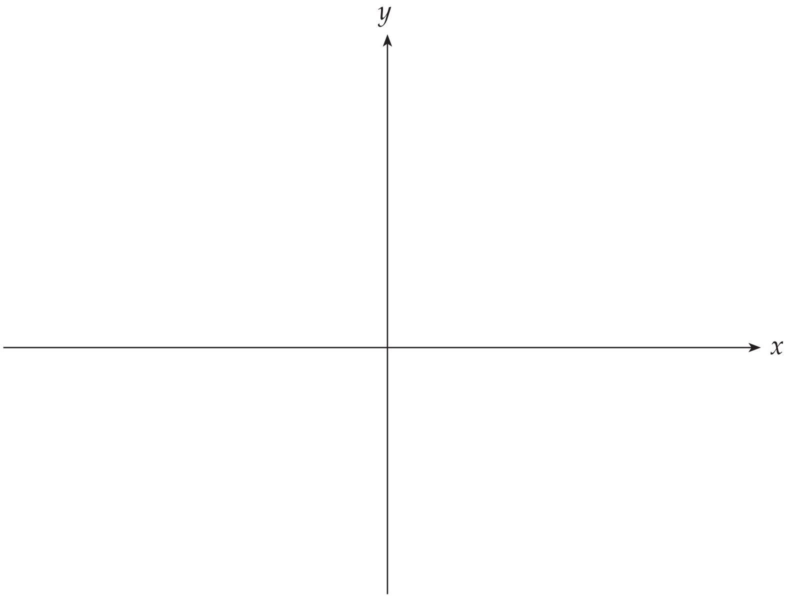
3 marks

The side view of the return section of the roller coaster in the same location is given by the rule:

$$h_2: [-5, 3] \rightarrow R, \text{ where } h_2(x) = \frac{1}{2}e^{\frac{1}{2}x}(x+5)(x+1)(x-2)$$

x is the horizontal distance in metres from O and h_2 is the vertical distance from the ground in metres.

d Sketch the graph of $y = h_2(x)$ on the axes below. Give stationary points, end-points and cuts on axes correct to two decimal places.



4 marks

e How much higher is $h_2(x)$ than $h_1(x)$ at $x = 3$? Give your answer correct to two decimal places.

1 mark

f For what values of x is the graph of $y = h_2(x)$ steeper than the graph of $y = h_1(x)$? Give your answers correct to two decimal places.

2 marks

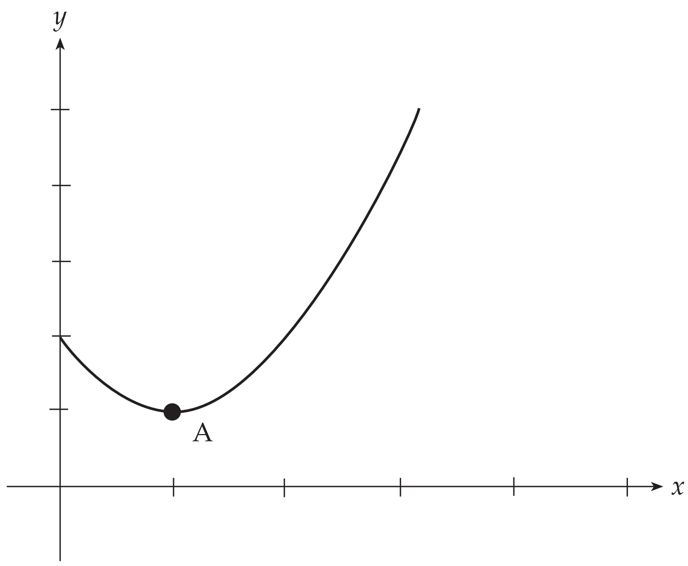
g $h_2'(x) = \frac{1}{4}e^{\frac{1}{2}x}(ax^3 + bx^2 + cx + d)$. Find values for a, b, c and d.

3 marks

Total 17 marks

Question 2

$f: [0, \infty) \to R$, where $f(x) = ax^2 + bx + 2$, where a and b are real constants. The graph of $y = f(x)$ is shown below.



a Give the x-coordinate of the stationary point A in terms of a and b.

1 mark

b Explain why b has to be negative.

1 mark

c The stationary point A is on the line $y = x$. Use this information in sketching the graph of the inverse relation of $y = f(x)$. Draw your sketch on the axes above.

2 marks

d Show that the rule for the inverse relation is $y = \dfrac{-b \pm \sqrt{4a(x-2) + b^2}}{2a}$.

3 marks

e Since $x = ax^2 + bx + 2$ at A, show that $a = \dfrac{b^2 - 2b}{8}$, where $a > 0$ and $b < 0$.

2 marks

f Hence, find b, if $a = 1$.

1 mark

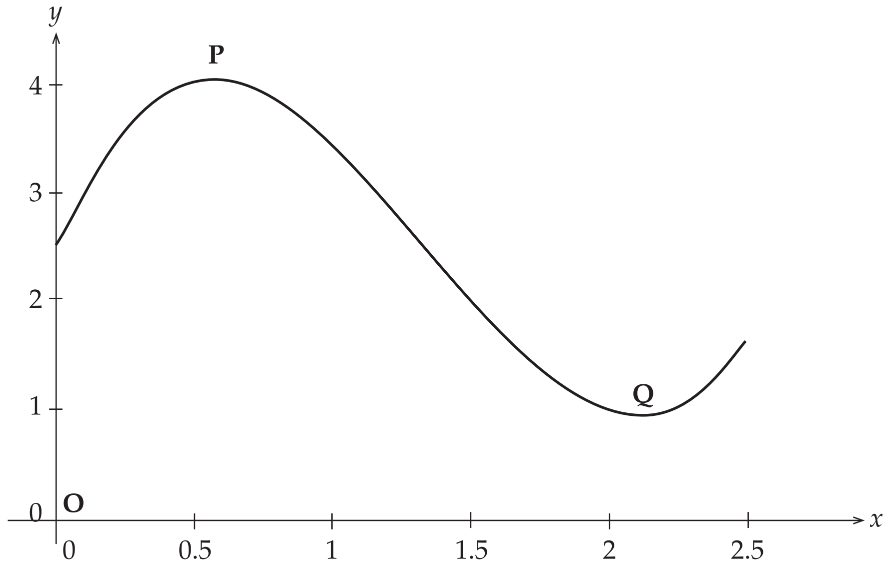
g If $a = 1$, use calculus to find the exact area enclosed by the graphs of $y = f(x)$ and its inverse.

4 marks

Total 14 marks

Question 3

The section of a mountain range could be modelled using the polynomial.
h: $[0, 2.5] \to R$, where $h(x) = 1.6875x^3 - 6.75x^2 + 6x + 2.5$ as shown in the graph. The height above sea level is h kilometres and the horizontal distance from O is x kilometres.



a Determine the coordinates of the stationary points, P and Q, and the two end points of the graph. Give answers correct to 2 decimal places.

4 marks

b A sine curve in the form $g(x) = A\sin(n\pi x) + B$ could also be used to model the mountain profile. The maximum and minimum points, S and T, on the function, $g(x)$, occur at (0.75, 4) and (2.25, 1) respectively. Find the values of A, n and B.

3 marks

c Sketch this curve on the axes above. 3 marks

d Write down an expression for the rate of change of $g(x)$ and hence find where this rate of change is greatest.

3 marks

e Calculate, correct to 3 decimal places, the cross-sectional area of the mountain range over the specified domain for

i $g(x)$

ii $h(x)$

1 + 1 = 2 marks

Total 15 marks

Question 4

A manufacturer produces artefacts that are packed for distribution in boxes of 10.

The buyer checks 2 items selected at random without replacement from a box and the box is accepted if neither item is damaged.

The box is rejected if both items are found to be damaged.

If only one item is damaged, then a third item is randomly selected and checked. If this item is also damaged the box is rejected. If it is not damaged the box is accepted.

Consider boxes that contain exactly 3 damaged items.

a. Draw a tree diagram that reflects this checking process.

3 marks

b Hence find the probability that a particular box is accepted.

2 marks

c Find the probability that an accepted box had only 2 items checked.

2 marks

d If 8 boxes are selected for testing, find, correct to 4 decimal places, the probability that 6 of these are accepted.

2 marks

The manufacturer changes her distribution procedures and no longer packages the items in boxes of ten. The proportion of damaged items distributed is reduced to 3%. A store orders 500 items that are randomly selected and dispatched.

e **i** Estimate the expected number of damaged items in this order.

ii Estimate, correct to 4 decimal places, the standard deviation of the number of damaged items in this order.

iii Calculate a Normal approximation to the probability that this order contains at least 6 and no more than 16 damaged items. Give your answer correct to 2 decimal places.

3 marks

Total 12 marks