2004 Mathematical Methods GA 2: Examination 1

GENERAL COMMENTS

The number of students who sat for the 2004 examination was 23 919, which was 395 fewer than the 24 314 who sat in 2003. Almost 18% scored 90% or more of the available marks, compared with 17% in 2003.

The overall quality of responses was similar to that of recent years. There were many very good responses and it was rewarding to see the substantial number of students who worked through to obtain full marks. However, the percentage who scored very few marks and who appeared to attempt little or nothing in Part II continues to be disappointing. There seems to be little evidence that failure to attempt Part II is due to lack of time. It was also noticeable that quite a few students only answered Question 1a (the normal distribution question) and Question 4 (the circular function question) of Part II, yet attempted little else in this section of the paper.

Students should be familiar with instructions that will appear in the examination booklet; in particular they need to be aware that the instruction to show working is applied rigorously in the marking of papers. Failure to show appropriate working when more then one mark is available will result in marks not being awarded if only an answer is given in response to the question. Similarly, a decimal approximation will not be accepted if an exact answer is required. Stating how a calculator was used to obtain the ‘exact’ answer to a number of decimal places is not an appropriate response.

As noted in all previous Assessment Reports, there continues to be difficulties associated with algebraic skills, setting out and the use of mathematical notation. This was especially evident this year in Questions 2, 3a and c, 5b and c and 6.

PART I Multiple-choice questions

This table indicates the number of students who chose each option. The correct answer is indicated by shading.

	A	B	C	D	E	No Answer
Question	**%**	**%**	**%**	**%**	**%**	**%**
1	7	4	84	2	4	0
2	4	14	78	4	1	0
3	13	9	12	21	44	1
4	3	6	79	10	2	0
5	8	70	3	13	5	0
6	11	78	7	2	2	0
7	12	65	11	10	3	0
8	7	8	9	65	11	0
9	15	15	4	16	49	0
10	9	65	5	11	10	0
11	2	5	73	6	13	0
12	6	3	8	79	4	0
13	8	85	4	1	2	0
14	7	12	64	11	5	0
15	2	3	4	88	3	0
16	7	4	76	3	10	0
17	11	8	17	4	60	0
18	2	6	5	3	85	0
19	87	2	3	6	2	0
20	57	4	19	7	12	0
21	15	13	7	59	6	1
22	7	57	5	11	19	0
23	65	4	17	9	4	0
24	8	11	11	64	5	1
25	2	15	77	2	3	0
26	5	8	13	68	6	0
27	62	10	19	4	5	1

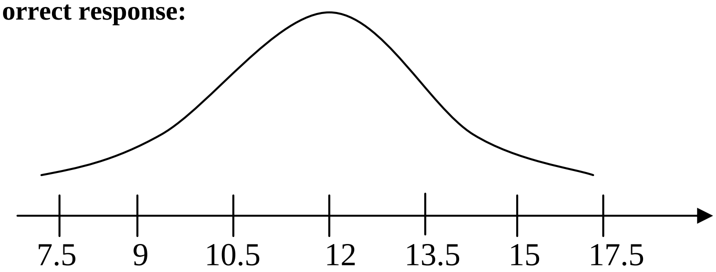
PART II

Question 1

a.

Marks	0	1	2	Average
%	10	36	54	**1.4**

Correct response:



It was surprising the number of students who did not show either axes or scale as part of their sketch. It was expected that all the information given in the stem of the question would be used; however, it was often the case that the mean was marked, with no markings to show some multiple of standard deviation.

b.

Marks	0	1	Average
%	41	59	**0.6**

Correct response: $k = 10.5$

This part was reasonably well done. Most students were quite proficient at using the inverse norm function on their graphics calculator. Use of the formula by others often resulted in errors. Often x or z was used in place of k.

Question 2

a.

Marks	0	1	Average
%	46	54	**0.5**

Correct response: $g(x) = (x - 3)^2$

Common incorrect responses were: $g(x) = x^2 \pm 3$ or $(x - 1)^2$

b.

Marks	0	1	Average
%	36	64	**0.6**

Correct response: $h(x) = (x - 3)^2 - 1$

Common incorrect responses were: $h(x) = (x + 1)^2 + 3$ or $(x - 1)^2 - 3$

c.

Marks	0	1	Average
%	82	18	**0.2**

Correct response: $k(x) = (2x - 3)^2 - 1$

Common incorrect responses were: $k(x) = 2(x - 3)^2 - 1$ or $0.5(x - 3)^2 - 1$ or $(2x - 6)^2 - 1$

Overall, this question was not well answered. Some students did not read the question and did not continue from the part before in finding the next answer. Common responses then given were $g(x) = (x - 3)^2$ then $h(x) = x^2 - 1$ and $k(x) = 2x^2$. Question 2c tended to be a strong indicator of overall performance on Part II of the paper.

Question 3

a.

Marks	0	1	Average
%	30	70	**0.7**

Correct response: Domain $(1, \infty)$
Range R

Common incorrect responses were:

Domain R^+ , $R^+\backslash\{1\}$, $[1,\infty)$, $[1, \infty]$
Range $R\backslash\{1\}$, $[-\infty, \infty]$, $(-R , R)$

Some students did not appear to understand function notation, and failed to realise that the domain was given to them. A lot of confusion was evident in the use of notation; use of square and round brackets and R^+.

b.

Marks	0	1	Average
%	31	69	**0.7**

Correct response: *f* is a one-to-one function / horizontal line test will do since it is already a function

This part was answered quite well. However, some students insisted on giving too much information and contradicted themselves, while others only stated that the graph passed the vertical line test. The word *it* was used a lot without it being clear whether this referred to the original function or its inverse.

c.

Marks	0	1	2	Average
%	18	20	62	**1.4**

Correct response:

$$x = 0.5 \log_e(y - 1)$$
$$2x = \log_e(y - 1)$$
$$e^{2x} = y - 1$$
$$y = e^{2x} + 1$$
$$f^{-1}(x) = e^{2x} + 1$$

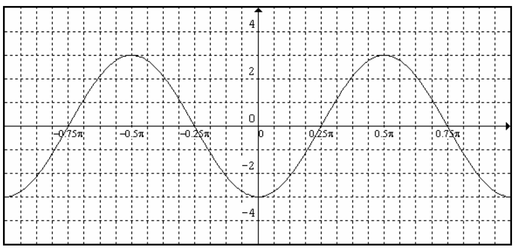
The majority of students realised the need to interchange *x* and *y*. Many were then able to go on and find the correct rule. However it was not uncommon to see $\frac{x}{0.5}$ unsimplified as the exponent. Common errors included exponents of $2x$ or $\frac{x}{2}$ or –1 in place of +1. It was pleasing to see that only a few students were unable to cope with the algebra of logarithms and exponentials, as this has often not been the case in past years.

Question 4

a.

Marks	0	1	2	Average
%	9	14	77	**1.7**

Correct response:



Clearly the best question in terms of the accuracy of student responses. Some very well sketched graphs were presented. Errors occurred in each of domain, phase, amplitude and period. However, some students produced graphs which had variable periods and/or amplitude, or were not properly curved and looked more like 'wedges'. Some students used their graphics calculator in degree mode resulting in virtually a straight line for the graph.

b.

Marks	0	1	Average
%	29	71	**0.7**

Correct response: 4 solutions

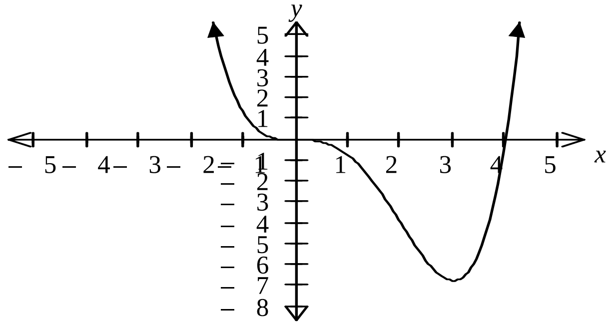
Quite a few students were confused with what was required in this part of the question, and gave a number of solutions that was inconsistent with the graph. Some students gave the correct answer of four, only to go on and give solutions to the equation $f(t) = 0$ rather than 1. Others unnecessarily solved the equation to give the four solutions. Students need to read questions carefully and give only the information that is required.

Question 5

a.

Marks	0	1	2	Average
%	39	12	48	**1.1**

Correct response:



Many students were able to draw a quartic polynomial graph; however the stationary point of inflection was often not well drawn. Some students did not use all the information in drawing their graph – the *x*-axis intercept of 4, the minimum turning point at $x = 3$ and the point of inflection were sometimes missing.

b.

Marks	0	1	2	Average
%	57	8	35	**0.8**

Correct response: $f(x) = ax^3(x-b)$

(0,0) and (4,0) imply x and $(x-4)$ are factors of $f(x)$ so $b = 4$

$f(2) = -4$ implies $-4 = 8a(2-4)$ hence $a = \frac{1}{4}$

Too many students were unable to use the information given in part a to assist in answering part b. Often students did not associate the *x*-axis intercept of 4 with the required value of *b*. This led to many and varied unsuccessful attempts to solve simultaneous equations to find *a* and *b*. The algebra required to find the correct solutions proved beyond many.

c.

Marks	0	1	2	3	Average
%	49	10	14	27	**1.2**

Correct response: $\frac{dy}{dx} = x^2(x-3)$

at $x = 4$, $\frac{dy}{dx} = 16$

so the equation of the tangent is $y - 0 = 16(x - 4)$

$\Rightarrow y = 16x - 64$

Quite a few students chose to differentiate their equation without expanding the brackets and assumed that the derivative of the product was the product of the derivatives. A significant number of students who correctly found the derivative at $x = 4$ stopped and did not go on to find the equation of the tangent. This may have been either a failure to read the question properly, or not understanding the relationship between gradient and tangent.

Question 6

a.

Marks	**0**	**1**	**Average**
%	56	44	**0.4**

Correct response: $Pr(\text{one rainbow ball}) = {}^4C_1 \times p \times (1-p)^3$

A common error was to replace p with $\frac{p}{100}$, or to use ${}^{100}C_4$. Some students used q without defining it. Only a small number of students attempted to use the hypergeometric distribution.

b.

Marks	**0**	**1**	**2**	**Average**
%	75	10	15	**0.4**

Correct response:

$$P'(p) = 4\times(1-p)^3 - 12\times p\times(1-p)^2$$
$$= 4\times(1-p)^2(1-4p)$$

For max, $P'(p) = 0$, so $p = \frac{1}{4}$ (since $0 < p < 1$)

Students who were able to correctly differentiate the expression generally found the correct answer. Most attempted to use the product rule. Of those who attempted to expand the expression, many made algebraic errors. Sometimes the instruction to use calculus was not followed.

This question was clearly the most difficult on the paper and was not handled well by students. Students did not seem to be able to understand what was required – formulating a probability in terms of a variable p, then differentiating the resultant expression to find its maximum value.