2004 Assessment Report

2004 Mathematical Methods GA 3: Written examination 2

GENERAL COMMENTS

In 2004, 17 990 students presented for this examination, which was 396 more than in 2003. Although no students scored full marks, there were fewer students at the bottom of the range than in previous years.

There was some evidence of ineffective calculator use, which led to answers that were incorrect in the last decimal place. Teachers should discourage the use of the 'trace' button, since it is inaccurate and inefficient, and the use of a table to identify points, as it is slow and often misses required points (such as in Question 4c). Using 'zoom' to get an appropriate graphing window is much less efficient than explicitly setting the 'window'. It is important that students learn to use the calculator efficiently for examinations, as poor calculator use has been a problem for students over a number of years.

As with last year, students were required to show their working for any question worth more than one mark. Some students lost several marks over the paper because they did not do this. It is important that students are clear what is required in these cases.

In Questions 2ai., 3aii., 3aiii. and 3b, students were required to **show** a given result. It is important that students have practised this kind of question, but there were many instances where these questions were not as well handled as they might have been. See the Specific Information section below for extended answers to these questions.

Students lost marks needlessly because they did not answer the specific question, gave the wrong number of decimal places or gave a decimal answer when an exact answer was required. It is important that students are familiar with the examination cover page well before they are faced with the actual examination paper and that they are aware of the importance of reading each question very carefully.

SPECIFIC INFORMATION

Question 1

1a

Marks	0	1	2	Average
%	38	7	54	1.2

$u = 3$, $v = 5$

1b

Marks	0	1	2	Average
%	41	17	42	1.0

$a = 1$, $b = \frac{5}{3}$

1c

Marks	0	1	2	Average
%	88	4	8	0.2

$p > 1$ and $p < \frac{23}{27}$

1d

Marks	0	1	2	3	Average
%	40	37	10	14	1.0

0.083

1ei.

Marks	0	1	2	Average
%	26	33	41	1.2

Dilation by factor of 2 from y-axis, translation of 1 unit vertically downwards

1eii.

Marks	0	1	Average
%	74	26	**0.3**

(2,0), (4,0)

1eiii.

Marks	0	1	Average
%	86	14	**0.2**

0.17

1f

Marks	0	1	2	Average
%	88	9	3	**0.2**

$1 \le h < 2$

In part a, a surprising number of students were unable to accurately differentiate by using the product rule or by first multiplying out the brackets then differentiating (or by using the expanded expression given in part d).

In part b, too many students were unable to find a and b correctly, although the use of a graphics calculator at least gave a clear picture of what was required. A decimal answer was accepted only if it was the correct recurring one. In part c, although a calculator quickly showed what was required and 1b gave the numbers needed, the question was too difficult for many. Some attempts to use the discriminant were evident.

A sketch graph was helpful at the start of part d, but rarely seen. It was expected that most students would successfully antidifferentiate the polynomial to get the first mark, but it was disappointing to see that few students went further. There were a lot of incorrect terminals on the integral; in some cases this was due to poor use of windows on the calculator, so that the cubic appeared to have a stationary point of inflection instead of maximum and minimum points. A significant number of students got the incorrect terminals as they tried to find the area bound by both axes and the curve, instead of the x-axis and the curve. Students lost a mark if their answer to part f was negative or if they did not show working to a positive answer via correct statements. For example, 'Area = – 0.083 = 0.083' did not gain a mark. The exact answer of $\frac{1}{12}$ was not correct to three decimal places and was not awarded a mark.

Some students did not clearly indicate the transformations they wanted in part ei.; only unambiguous descriptions of transformations gained marks. Students should practise this type of question so that they feel confident in this area. A dilation of ½ was a frequent incorrect response.

The result of a transformation such as a dilation was not sufficiently well understood by many students in parts eii. and eiii.; therefore, what should have been an easy question became much harder. There was limited appreciation of the connections between the parts of the question, and $f\left(\frac{x}{2}\right)$ frequently became $\frac{f(x)}{2}$ with subsequent disastrous results. The answer to part eiii. was awarded a mark only if it was consistent with earlier work.

Few students had any idea of how to answer part f. The correct working is shown below.
$f(x+h)=1$ implies $(x-1+h)^2(x-2+h)=0$ which has roots $x=1-h$ and $x=2-h$

When $1 \le h < 2$, these roots are of opposite sign; that is **only one** of them can be positive.

Question 2
2ai.

Marks	0	1	Average
%	40	60	**0.6**

Since the sum of the probabilities must be 1, $(4k^2 + 4k) + (5k^2 + 2k) + (k^2 + k) + 2k = 1$
Therefore, $10k^2 + 9k - 1 = 0$.

2aii.

Marks	0	1	Average
%	38	62	**0.7**

k = 0.1

2bi.

Marks	0	1	Average
%	25	75	**0.8**

1.26

2bii.

Marks	0	1	2	Average
%	25	10	65	**1.5**

0.245

2biii.

Marks	0	1	2	Average
%	55	11	34	**0.8**

16

2c

Marks	0	1	2	3	Average
%	41	11	4	43	**1.5**

18

2d

Marks	0	1	2	Average
%	22	20	58	**1.4**

0.637

2e

Marks	0	1	2	Average
%	57	14	29	**0.8**

0.751

Some students gave reasons for their answer to part aiii., saying that the 'probability must be less than 1'. This was not a good reason without further explanation, since none of the probabilities listed was in fact *k*; however, marks were not deducted for this because no reason was asked for. A significant number of students were unable to factorise the quadratic (although some solved it by other means); if a student is unable to do this, they should use the relevant program in their graphics calculator.

While part bi. was generally well done, some students incorrectly rounded their answer from 1.26 to 1.

Most students were able to identify the correct distribution in part bii. However, many students did not gain full marks due to carelessness with rounding (0.246 was a common incorrect response) or because they did not show their working.

Students familiar with 'smallest value' type questions were able to handle part biii. well, solving it through an indicial equation or by trial and error.

Part c was well answered by students with knowledge of the standard normal distribution.

Parts d and e were fairly well done, but a number of responses to part d considered only one defective instead of at least one, and there were a number of students who tried to apply conditional probability to part e.

Question 3

3ai.

Marks	0	1	2	Average
%	67	24	9	0.4

$t \le \frac{-a}{b}$

3aii.

Marks	0	1	2	Average
%	47	6	47	1.0

When $t = 0$, $P = P_0$, so $P_0 = P_0 e^{G(0)}$

$\Rightarrow e^{G(0)} = 1$

$\Rightarrow G(0) = 0$

3aiii.

Marks	0	1	2	Average
%	80	4	16	0.4

$G(t) = \int (a + bt)\, dt = at + \frac{1}{2}bt^2 + c$

$G(0) = 0 + 0 + c = 0$ (from i.)

$\Rightarrow c = 0$

$\Rightarrow G(t) = at + \frac{1}{2}bt^2$

3bi.

Marks	0	1	Average
%	50	50	0.5

Substituting for a and b,

$$P(t) = 3e^{\left(0.02t + \frac{1}{2}(-0.0002)t^2\right)}$$

$$= 3e^{\left(0.02t - 0.0001t^2\right)}$$

3bii.

Marks	0	1	Average
%	70	30	0.3

100

3biii.

Marks	0	1	Average
%	67	33	0.3

$$\frac{dP}{dt} = 3\left(0.02 - 0.0002t\right)e^{\left(0.02t - 0.0001t^2\right)}$$

3biv.

Marks	0	1	Average
%	72	29	0.3

0.069

3ci.

Marks	0	1	2	Average
%	38	11	52	**1.2**

$t = 100\log_e\left(\frac{P}{3}\right)$

3cii.

Marks	0	1	Average
%	69	31	**0.3**

2059

Surprisingly, there were a number of poor efforts at most parts of this question. Many students failed to understand what was required for part ai., and few were good at expressing themselves in aii. and aiii. A significant number of students omitted the integration constant required in aiii.

Many attempts at part bi. failed to clearly show that a substitution had to be made. Many students missed out on marks for the remaining parts of 3b through careless algebra (often caused by a lack of bracket use), differentiation or substitution. Attempts to use $\frac{\text{rise}}{\text{run}}$ were seen too often in part biii.

Most unsuccessful attempts at part ci. indicated that the student thought that an inverse was required, while in cii., many did not gain the mark because they answered the question incorrectly and gave '69' as their answer. Others gave the answer as 2060, rounding incorrectly.

Question 4

4a

Marks	0	1	Average
%	16	84	**0.9**

122 m

4b

Marks	0	1	Average
%	15	85	**0.9**

2 m

4c

Marks	0	1	Average
%	52	48	**0.5**

1:48 pm

4di.

Marks	0	1	2	Average
%	23	33	44	**1.2**

1:16 pm

4dii.

Marks	0	1	Average
%	57	43	**0.5**

16

4ei.

Marks	0	1	Average
%	61	39	**0.4**

$150\pi\cos\left(\frac{(5t-1)\pi}{2}\right)$

4eii.

Marks	0	1	Average
%	62	38	**0.4**

471.2

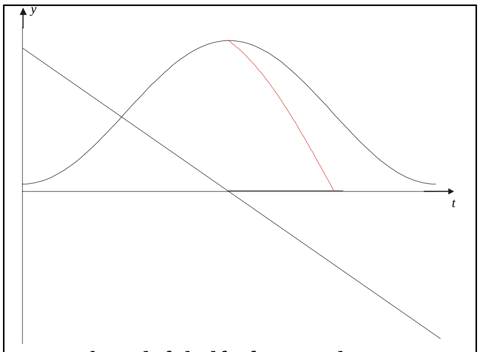
4fi.

Marks	0	1	Average
%	50	50	**0.5**

(0.4, 122) marked on graph

4fii.

Marks	0	1	2	Average
%	61	39	0	**0.4**



Must show left half of original curve as part of the answer.

4fiii.

Marks	0	1	2	Average
%	69	21	10	**0.4**

12 minutes

All parts of Question 4, except parts ei. and f, could be easily handled with a graphics calculator, so the total number of marks obtained by many students was disappointing. The early parts of the question were generally answered well by using a calculator, although some students offered ridiculously high answers for part c. This appeared to have been due to the use of a table to read off the values of t, but this only showed whole numbers of hours. Other students could not convert $t = \frac{4}{15}$ into a time after 1 pm.

There was the expected error of the omission of π from answers to part ei. In part eii., the correct answer could be found with a calculator, irrespective of a correct derivative in ei. A number of students used part ei. and gave an exact answer of 150π which was not awarded the mark.

Some students' graphs in answer to part fii. showed spiders leaping higher than the Ferris wheel or burrowing underground! A number of students used a calculator to find the graph sum of the two functions but failed to apply it to the correct domain.