VCE
2005 Mathematical Methods
Trial Examination 1

Suggested Solutions

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Question 1 C

$\Pr(X=1)+\Pr(X=4)=0.1+0.3=0.4$

Question 2 D

$2\Pr(X>a)=2[1-\Pr(X<a)]=2-2\Pr(X<a)$

Question 3 B

$\mathrm{E}(X)=n\frac{D}{N}=6\times\frac{4}{12}=2$

Question 4 A

$\Pr=\text{BBBGG}=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{32}$

Question 5 A

$\Pr(YY)+\Pr(RY)=\frac{4}{10}\times\frac{3}{9}+\frac{6}{10}\times\frac{4}{9}=\frac{2}{5}$

Question 6 C

$2\cos(\frac{x}{2})=1$

$\cos(\frac{x}{2})=\frac{1}{2}$

$\frac{x}{2}=\frac{\pi}{3}$

$x=\frac{2\pi}{3}$

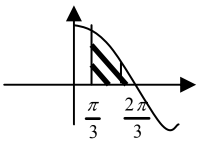
$2\cos(\frac{x}{2})=\sqrt{3}$

$\cos(\frac{x}{2})=\frac{\sqrt{3}}{2}$

$\frac{x}{2}=\frac{\pi}{6}$

$x=\frac{\pi}{3}$

$\{x:\frac{\pi}{3}\le x\le\frac{2\pi}{3}\}$



Question 7 E

Amplitude = 2

Translated 1 unit up $\therefore$ not A or B

Period = $\frac{2\pi}{n}=\frac{\pi}{3}\Rightarrow n=6$ $\therefore$ not C

When θ = 0, graph is a maximum $\therefore$ a cos graph

$\therefore$ E

Question 8 D

$2-2\cos^2\theta=3\cos\theta$

$2\cos^2\theta+3\cos\theta-2=0$

$(2\cos\theta-1)(\cos\theta+2)=0$

$\cos\theta=\frac{1}{2}$ or $\cos\theta=-2$

$-1\le\cos\theta\le1$

$\therefore\cos\theta=\frac{1}{2}$

$\Rightarrow\theta=\frac{\pi}{3},\frac{5\pi}{3}$

Sum of solutions $=\frac{\pi}{3}+\frac{5\pi}{3}=2\pi$

Question 9 E

$g(x)=f(x-1)=(x-1)^3-2(x-1)^2-8(x-1)-5$

$=x^3-3x^2+3x-1-2(x^2-2x+1)-8x+8-5$

$=x^3-5x^2-x$

Question 10 A

$y=a(x+1)^2+4$

$y=a(x-b)^2+c$

$\therefore b=-1, c=4$

When $x=0, y=2$

$\Rightarrow 2=a(0+1)^2+4$

$\Rightarrow a+4=2\Rightarrow a=-2$

Question 11 C

$$g(x) = -(e^x - 2) = -e^x + 2 = -f(x) + 2$$

Question 12 B

$$x = (y-1)^2 - 5$$

$$x + 5 = (y-1)^2$$

$$y - 1 = \pm\sqrt{(x+5)}$$

$$y = 1 \pm \sqrt{(x+5)} \quad y \geq 1, x \geq -5$$

$$\therefore y = 1 + \sqrt{(x+5)}$$

Question 13 E

To have an inverse, the graph must be one – to – one.
The only graph here, taking the domain of that graph into consideration, that is not one – to – one is E

Question 14 D

$$\log_a(xy) = \log_a(x) + \log_a(y)$$

$$\log_a(xy^4) = \log_a(x) + \log_a(y^4)$$

$$\log_a(xy^4) = \log_a(x) + 4\log_a(y) = p(1)$$

$$\log_a(\frac{y^3}{x}) = \log_a(y^3) - \log_a(x)$$

$$\log_a(\frac{y^3}{x}) = 3\log_a\overset{y}{(y)} - \log_a(x) = q(2)$$

$$(1) + (2) \rightarrow 7\log_a(y) = p + q$$

$$\rightarrow \log_a(y) = \frac{1}{7}(p+q)$$

$$4 \times (2) - 3 \times (1) \rightarrow -7\log_a(x) = 4q - 3p$$

$$\log_a(x) = \frac{1}{7}(3p - 4q)$$

$$\log_a(x) + \log_a(y) = \frac{1}{7}(3p - 4q + p + q)$$

$$\log_a(x) + \log_a(y) = \frac{4p - 3q}{7}$$

Question 15 B

$$x = 8^t$$

$$\log_2 x = \log_2 8^t$$

$$\log_2 x = t\log_2 2^3$$

$$\log_2 x = 3t\log_2 2$$

$$\log_2 x = 3t$$

Question 16 D

When $x = -\frac{1}{2}, y = 0$ When $x = 1, y = 0$

$f(x) = (2x+1)(x-1)\ \therefore$ can be written as the product of two real linear factors.

Completing the square gives

$$f(x) = 2(x^2 - \frac{1}{2}x - \frac{1}{2}) = 2(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{2} - \frac{1}{16})$$

$$f(x) = 2[(x - \frac{1}{4})^2 - \frac{9}{16}] = 2(x - \frac{1}{4})^2 - \frac{9}{8}$$

Between $x = -\frac{1}{2}$ and $x = 1$, where this graph cuts the X axis, the graph is negative.

$\therefore$ D is not true.

Question 17 E

By dividing $x+1$ into $2x+5$ we get

$$\frac{2x+5}{x+1} = 2 + \frac{3}{x+1}$$

This gives the graph of $\frac{3}{x}$ translated 1 unit to the left and 2 units up. This means a vertical asymptote with equation $x = -1$ and a horizontal asymptote with equation $y = 2$

Question 18 B

$$\frac{dy}{dx} = -2e^{5-2x} = -8$$

$$e^{5-2x} = 4$$

$$5 - 2x = \log_e 4$$

$$2x = 5 - \log_e 4$$

$$x = \frac{5}{2} - \frac{1}{2}\log_e 4$$

$$x = \frac{5}{2} - \log_e 4^{\frac{1}{2}}$$

$$x = \frac{5}{2} - \log_e 2$$

Question 19 A

$$\frac{dy}{dx} = \frac{1}{\cos(2x)} \times -2\sin(2x) = -2 \times \frac{\sin(2x)}{\cos(2x)}$$

$$= -2\tan(2x)$$

Question 20 C

$$\frac{dy}{dx} = (x+4)(-2e^{-2x}) + e^{-2x} \times 1$$

$$\frac{dy}{dx} = e^{-2x}(-2x - 8 + 1)$$

$$\frac{dy}{dx} = e^{-2x}(-2x - 7)$$

Increasing function when $e^{-2x}(-2x - 7) > 0$

e^{-2x} is always > 0

$\therefore$ Increasing function when $(-2x - 7) > 0$

$\Rightarrow -2x > 7$

$\Rightarrow x < -3\frac{1}{2}$

Question 21 E

$$f\,'(x) = \frac{1}{2}(3x^2 - 4x + 18)^{-\frac{1}{2}} \times (6x - 4)$$

$$= \frac{3x - 2}{\sqrt{3x^2 - 4x + 18}}$$

Question 22 B

$$f(1 - 0.02) \approx f(1) - 0.02 f'(1)$$

Question 23 C

The given graph starts with a positive gradient, then
has a gradient of zero, then a negative gradient then
a zero gradient, and then a positive gradient.
In the gradient graphs drawn for the solutions, a positive
gradient lies above the X axis and a negative gradient lies below the X axis and a gradient of zero lies on the X axis.
So the solution will start above the X axis, then go below the X axis and then above the X axis.
$\therefore$ C

Question 24 E

$$A = \int_a^b f(x)dx - \int_b^c f(x)dx + \int_c^d f(x)dx$$

$$A = \int_a^b f(x)dx + \int_c^b f(x)dx + \int_c^d f(x)dx$$

$\therefore$ E

Question 25 A

$$\sin(1-x) = 0$$

$$1 - x = -\pi, 0, \pi, \ldots$$

$$x = \pi + 1, 1, 1 - \pi, \ldots$$

But $0 < a < \dfrac{\pi}{2}$

$\therefore a = 1$

$$A = \int_0^1 \sin(1-x)dx$$

$$= -\cos(1-x) \times -1\Big]_0^1$$

$$= \cos(1-x)\Big]_0^1$$

$$= \cos(0) - \cos(1)$$

$$= 1 - \cos(1)$$

Question 26 E

$$A = \int_0^2 \frac{kx}{1+x^2}dx$$

$$A = k\int_0^2 \frac{x}{1+x^2}dx$$

$$A = \frac{k}{2}\int_0^2 \frac{2x}{1+x^2}dx$$

$$A = \frac{k}{2}\log_e(1+x^2)\Big]_0^2$$

$$A = \frac{k}{2}[\log_e(5) - \log_e(1)]$$

$$A = \frac{k}{2}\log_e(5)$$

Question 27 A

$$\int_0^3 [f(x) - 2]dx + \int_3^6 [f(x) + 1]dx = \int_0^3 f(x)dx - \int_0^3 2dx + \int_3^6 f(x)dx + \int_3^6 1dx$$

$$= \int_0^6 f(x)dx - \int_0^3 2dx + \int_3^6 1dx$$

$$= 5 - \big[2x\big]_0^3 + \big[x\big]_3^6$$

$$= 5 - 6 + 6 - 3 = 2$$

Question 1

a.

$$\begin{array}{r} 1 \\ x-3\overline{)x+1} \\ \underline{x-3} \\ 4 \end{array}$$

$f(x) = 1 + \dfrac{4}{x-3}$

$A = 1, B = 4, C = -3$ (1 mark)

b.

$1 + \dfrac{4}{x-3} = 0$

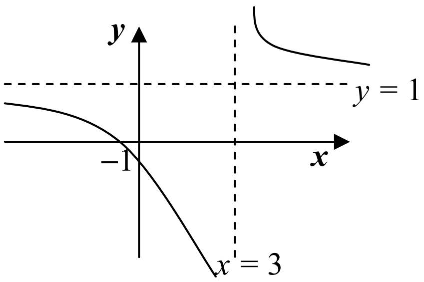
$\dfrac{4}{x-3} = -1$

$-x + 3 = 4$

$-x = 1$

$x = -1$ (1 mark)

$\{x : x < -1\} \cup \{x : x > 3\}$ (1 mark)



Question 2

$\Pr(X < 50) = 0.35$

$\Pr(Z < -a) = 0.35$

$\Pr(Z > a) = 0.35$

$\Pr(Z < a) = 1 - 0.35 = 0.65$

$a = 0.385$

$-a = -0.385$ (1 mark)

$-0.385 = \dfrac{50-60}{\sigma}$

$-0.385\sigma = -10$

$\sigma = 26$ to the nearest whole number (1 mark)

Question 3

a.

Let $e^x = A$

$A^2 + 4A - 12 = 0$

$(A+6)(A-2) = 0$

$A = -6$, or $A = 2$ (1 mark)

$\Rightarrow e^x = -6$, or $e^x = 2$

But $e^x > 0$

$\therefore e^x = 2 \Rightarrow x = \log_e(2)$ (1 mark)

Question 3

b.

$\log_e(y^2) = \log_e \dfrac{2x-8}{6}$

$(y^2) = \dfrac{2x-8}{6}$ (1 mark)

$(y^2) = \dfrac{x-4}{3}$

$y = \pm\sqrt{\dfrac{x-4}{3}}$

But $y > 0$

$y = \sqrt{\dfrac{x-4}{3}}$ (1 mark)

c.

$x - 4 > 0$

$\therefore x > 4$ (1 mark)

Question 4

a.

$y = mx + c$

$m = \dfrac{dy}{dx} = 2x - 5$

When $x = 3, m = 1$ (1 mark)

$y = x + c$

When $x = 3, y = 0$

$\therefore 0 = 3 + c$

$\Rightarrow c = -3$

$\therefore y = x - 3$ (1 mark)

b.

$y = -x + c$

$0 = -3 + c$

$3 = c$

$y = -x + 3$ (1 mark)

c.

At point of interscn. $x^2 - 5x + 6 = -x + 3$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$, or $x = 1$

A is point where $x = 1$

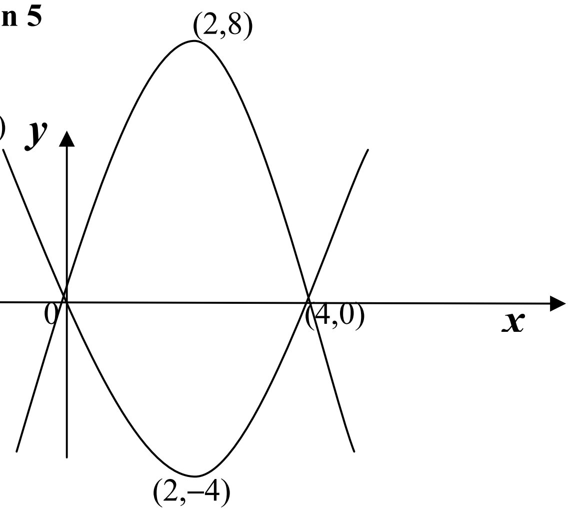
When $x = 1, y = 2$

$(1,2)$ (1 mark)

Question 5

a.

(1 mark)



b.

$g(x) = ax(x - 4)$

When $x = 2, g(x) = 8$

$8 = a \times 2 \times -2$

$-4a = 8$

$a = -2$

$g(x) = -2x^2 + 8x$ (1 mark)

c.

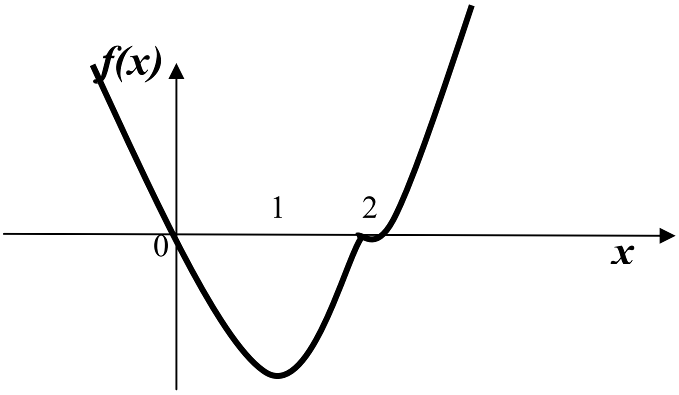
Translate $g(x)$ 2 units to left parallel to X axis and 8 units down parallel to Y axis. (2 marks)

Reflect in X axis (1 mark)

Dilate by a factor of $\frac{1}{2}$ in the Y direction (1 mark)

Question 6

- 1 mark for shape, with turning point at (1,0)
- 1 mark for (0.0)
- 1 mark for point of inflexion at (2,0)



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