Year 2005

VCE

Mathematical Methods

Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING **PO BOX 2227** **KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com

ABN 47 065 111 373

STUDENT NUMBER									Letter
Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2005

MATHEMATICAL METHODS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	55

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or whiteout liquid/tape.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e x + c, \text{for } x > 0$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \qquad \int \sin ax\, dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \qquad \int \cos ax\, dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}(\tan ax) = \frac{a}{\cos^2 ax} = a\sec^2 ax$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0, 1)$.		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Question 1

Consider the function $f : R \to R, f(x) = 2x^2 - 12x + 23$

a. Write $f(x)$ in the form $A(x - B)^2 + C$

2 marks

b. If the domain of $f(x)$ is $[a, \infty)$, find the smallest value of a so that $f^{-1}(x)$ exists.

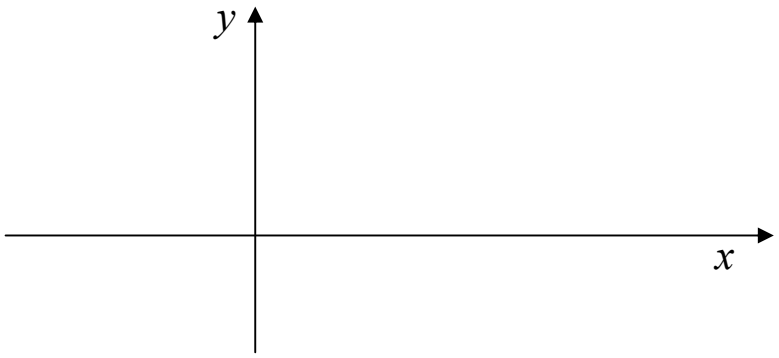
1 mark

c. Find $f^{-1}(x)$, stating its domain and range.

3 marks

Question 1 (continued)

d. Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the axes below.



2 marks

e. i. Find the gradient of $f(x)$ at the point where $x = 4$

e. ii. Find the gradient of $f^{-1}(x)$ at the point where $f^{-1}(x) = 4$

1 + 3 = 4 marks

Question 1 (continued)

f. A tangent is drawn to touch $f(x)$ at the point where $x = 4$, and another tangent is drawn to touch $f^{-1}(x)$ at the point where $f^{-1}(x) = 4$.
Find the point of intersection of these two tangents.

2 marks

g. Find the equation of all points $x, x \in R$, formed by the intersection of tangents to $f(x)$ at x and tangents to $f^{-1}(x)$ where $f^{-1}(x) = x$?

1 mark

(Total = 15 marks)

Question 2

In a certain country there are three types of schools, Academies, Branch Schools and Cultural Schools. University entrance records show that 35% of all year 12 students in this country continue on to university.

a. If there are 18,000 year 12 students in this country in a particular year, how many students would be expected to continue on to university?

_____

1 mark

b. The probability that a student from each of the different types of schools will enter university is given in the table below.

Type of School	Probability
Academy	$4k^2 + k$
Branch	$6k^2$
Cultural	$4k - 14k^2$

Find the value of k

_____

_____

_____

_____

_____

_____

_____

2 marks

c. What is the probability that a student from an Academy or Branch School will enter university?

_____

_____

_____

1 mark

Question 2 (continued)

d. At the university, it is found that 70% of all students who pass first year are from an Academy or a Branch school.

i. What is the probability that in a sample of 10 university students who passed first year, one of them is from a cultural school? Give your answer to three decimal places.

1 mark

ii. What is the probability that in a sample of 10 university students who passed first year, at least two of them are from a cultural school? Give your answer to three decimal places.

2 marks

e. i. The pass mark for third year mathematics at this university is 50. The results are normally distributed with a standard deviation of 10. If 93.32 % of the students pass, what is the mean mark?

2 marks

Question 2 (continued)

e. ii. Students with a mark higher than 82 are given first class honours. What is the probability that Stephanie, who is one of the students who passed the mathematics exam, gained first class honours? Give your answer to two decimal places.

2 marks

f. The mathematics department at this university has 30 staff, ten of whom favour raising the pass mark in first year mathematics. What is the probability that a randomly selected subcommittee of 11 staff will contain exactly 4 who are in favour of raising the pass mark? Give your answer to three decimal places.

2 marks

(Total = 13 marks)

Question 3

The number of hours of daylight, h, in Coora, can be modelled by the equation

$h(t) = 10 + 3\cos\left[\dfrac{2\pi(t-100.5)}{365}\right]$ where $t = 1$ for January 1, 2 for January 2, ...365 for December 31

(Ignore leap years.)

a. What is the maximum number of hours of daylight on any day of the year?

_____

1 mark

b. What is the minimum number of hours of daylight on any day of the year?

_____

1 mark

c. What is the period?

_____

1 mark

d. How many days from the beginning of the year does the minimum number of daylight hours occur? Show all working.

_____

_____

_____

_____

_____

_____

_____

_____

_____

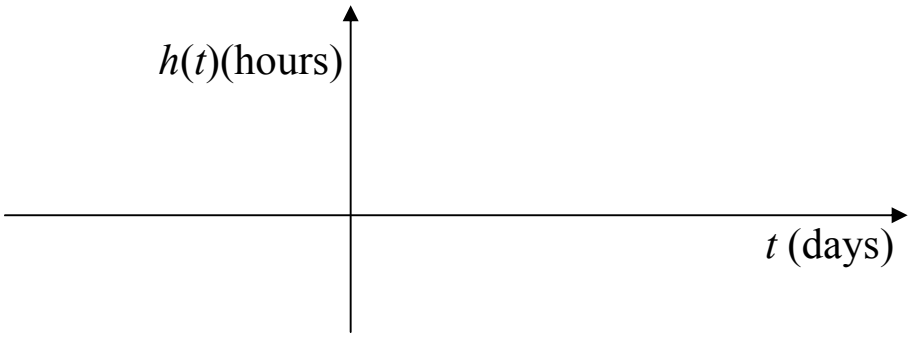
3 marks

Question 3 (continued)

e. On what dates of the year is there exactly 12 hours of daylight?
Give your answer to the closest day.

3 marks

f. Sketch the graph of $h(t)$ on the axes below, showing one complete cycle, and showing any intercepts with the axes correct to one decimal place.



1 mark

g. For how many days of the year were there more than 12 hours of daylight?

1 mark

Question 3 (continued)

h. At what rate is the number of minutes of daylight increasing on January 30? Give your answer to the nearest minute/day.

_____

_____

_____

_____

_____

2 marks

(Total = 13 marks)

Question 4.
Consider the function, $f(x) = xe^{kx} - 1$ where k is a constant.

a Find $f(0)$

_____

1 mark

b Find $f'(x)$

_____

1 mark

c. If the turning point occurs when $x = -\frac{1}{2}$, find the value of k

_____

_____

_____

_____

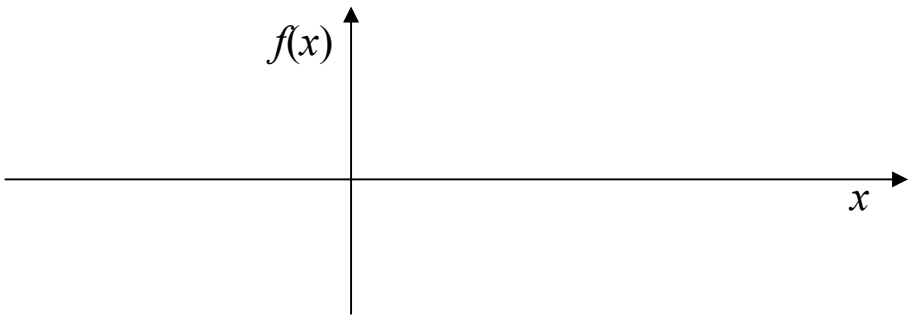
_____

_____

_____

2 marks

d. Sketch the graph of $f(x)$ on the axes below, giving the **exact** coordinates of all turning points. Show the intercepts with the axes, giving the x intercept to two decimal places. Give the equation of any asymptotes.



3 marks

Question 3 (continued)

e. Use your answer to part **b.** to find $\int xe^{5x}dx$

2 marks

f. Use calculus to find the area between the graph you sketched in part **d.**, the x axis and the ordinates $x = 0$ and $x = 1$. Give your answer to two decimal places.

3 marks

Question 3 (continued)

g. Find the point of intersection of the graph $g(x) = xe^{5x}$ and its inverse.

_____

_____

_____

_____

_____

_____

2 marks

(Total = 14 marks)

END OF QUESTION AND ANSWER BOOK

KILBAHA MULTIMEDIA PUBLISHING **PO BOX 2227** **KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com