UNIT 4 MATHEMATICAL METHODS 2005
TRIAL EXAMINATION SOLUTIONS

WRITTEN EXAMINATION 1

FACTS, SKILLS AND APPLICATION TASKS

PART I – MULTIPLE CHOICE QUESTIONS

	Answer		Answer
Question 1	C	Question 16	C
Question 2	D	Question 17	B
Question 3	B	Question 18	D
Question 4	B	Question 19	A
Question 5	C	Question 20	A
Question 6	D	Question 21	C
Question 7	D	Question 22	E
Question 8	A	Question 23	D
Question 9	B	Question 24	B
Question 10	A	Question 25	E
Question 11	E	Question 26	D
Question 12	C	Question 27	C
Question 13	D		
Question 14	D		
Question 15	B		

QUESTION 1 Answer is C

For tangent functions: $T = \dfrac{\pi}{magnitude\ of\ \ coefficient\ of\ \ x} = \dfrac{\pi}{2}$

QUESTION 2 Answer is D

$y = a(\sin x - 1) = a\sin x - a$

$Max\ value = a + -a = 0$

$Min\ value = -a + -a = -2a$

Range is $[-2a,\ 0]$ = D

QUESTION 3 Answer is B

$a\cos(x - b) = c$

$\cos(x - b) = \dfrac{c}{a}$

A solution exists if $-1 \le \dfrac{c}{a} \le 1$.

If $\dfrac{c}{a} < -1$, no solution exists i.e. $c < -a$ OR

If $\dfrac{c}{a} > 1$, no solution exists i.e. $c > a$

QUESTION 4 Answer is B

$\log_a 3 + 2\log_a (2x+1) = \log_a 48$

$\log_a 3 + \log_a (2x+1)^2 = \log_a 48$

$\log_a 3(2x+1)^2 = \log_a 48$

$3(2x+1)^2 = 48$

$(2x+1)^2 = 16$

$(2x+1) = \pm 4$

$x = -\dfrac{5}{2},\ \dfrac{3}{2}$ Check validity of solutions: $x \neq -\dfrac{5}{2}$

QUESTION 5 Answer is C

$(x+a)(x^2+b)(x-c)^2=0$

$(x+a)=0$ or $(x^2+b)=0$ or $(x-c)^2=0$

$x=-a$ No solution $x=c$

Two real number solutions will be obtained.

QUESTION 6 Answer is D

QUESTION 7 Answer is D

$4-at>0$
$-at>-4$
$at<4$
$\therefore t=\frac{4}{a}$

QUESTION 8 Answer is A

Sign of graph $=(+)\times(+)\times(-)^2=+$. Therefore, options A, B or C.

X intercept occurs at $(x+b)=0 \quad \therefore x=-b$.
Curve cuts at this point as power on brackets is 1.

X intercept occurs at $(a-x)^2=0 \quad \therefore x=a$.
Curve touches at this point as power on brackets is 2.

QUESTION 9 Answer is B

QUESTION 10 Answer is A

QUESTION 11 Answer is E

Negative hyperbola, therefore, coefficient of x must be negative. Answer is B, D or E.
Y asymptote is negative $(-b)$, therefore, vertical translation is b units in the negative direction. There must be a -1 separate from the fraction. Answer is D or E.
X asymptote is positive (a). The denominator of the fraction, when made equal to 0 must result in an answer of $x=a$. Answer is E.

QUESTION 12 Answer is C

Turning point is $(-a,-b)$ therefore, maximal domain is based on the value $x=-a$, and can be either greater than or less than this value.

QUESTION 13 Answer is D

$$f(x) = 2x^3 - x + \sqrt{x} = 2x^3 - x + x^{1/2}$$
$$f'(x) = 6x^2 - 1 + \frac{1}{2\sqrt{x}}$$

$$f'(a) = 6a^2 - 1 + \frac{1}{2\sqrt{a}}$$

QUESTION 14 Answer is D

$$y = \cos^3(2x) = \left(\cos(2x)\right)^3$$
$$\frac{dy}{dx} = 3 \times -2\sin(2x) \times \cos^2(2x) = -6\sin(2x)\cos^2(2x)$$

QUESTION 15 Answer is B

$y = \dfrac{\cos x}{e^{2x}}$ Apply Quotient Rule

$$\frac{dy}{dx} = \frac{\left(e^{2x} \times -\sin x\right) - (\cos x \times 2e^{2x})}{(e^{2x})^2} = \frac{-e^{2x}\sin x - 2\cos x e^{2x}}{(e^{2x})^2}$$

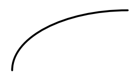
$$= \frac{-e^{2x}(\sin x + 2\cos x)}{(e^{2x})^2} = \frac{-(\sin x + 2\cos x)}{e^{2x}}$$

QUESTION 16 Answer is C

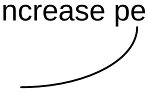
Container is filled from the bottom up.

Initial part of container – height of water increases by the same amount per unit time. Therefore h *vs* t graph is an oblique line. Answer is B or C.

Middle part of container is becoming wider. The height occupied by water will decrease per unit time. Gradient of curve is decreasing, therefore, shape of graph resembles:



Top part of container is becoming narrower. The height occupied by water will increase per unit time. Gradient of curve is increasing, therefore, shape of graph resembles:



QUESTION 17 Answer is B

QUESTION 18 Answer is D

$$f'(x) = 2\sin\left(2x - \frac{\pi}{2}\right)$$

As $f(x) = \int f'(x)dx$

$$\therefore\ f(x) = \int 2\sin\left(2x - \frac{\pi}{2}\right)dx = 2\int \sin\left(2x - \frac{\pi}{2}\right)dx = \frac{2 \times -\cos\left(2x - \frac{\pi}{2}\right)}{2} + c$$

$$\therefore\ f(x) = -\cos\left(2x - \frac{\pi}{2}\right) + c$$

When $x = \frac{\pi}{4}$, $y = 1$

$$\therefore 1 = -\cos\left(\frac{2\pi}{4} - \frac{\pi}{2}\right) + c$$

$$1 = -\cos(0) + c$$

$$\therefore 1 = -1 + c \qquad \therefore c = 2$$

$$\therefore\ f(x) = -\cos\left(2x - \frac{\pi}{2}\right) + 2$$

QUESTION 19 Answer is A

$$\int (5-3x)^3\,dx = \frac{(5-3x)^4}{4 \times -3} + c = -\frac{(5-3x)^4}{12} + c$$

QUESTION 20 Answer is A

$$\int_0^a \left(\frac{2}{4x+1}\right)dx = \frac{1}{2}\int_0^a \left(\frac{4}{4x+1}\right)dx = \frac{1}{2}\left[\log_e(4x+1)\right]_0^a = \frac{1}{2}\left[\log_e(4a+1) - \log_e 1\right]$$

$$= \frac{1}{2}\log_e(4a+1) = \log_e(4a+1)^{1/2}$$

$$\log_e(4a+1)^{1/2} = \log_e k$$

$$k = \sqrt{4a+1}$$

QUESTION 21 Answer is C

The antiderivative of $-ax$ results in $-\frac{ax^2}{2}+c$ i.e. a negative quadratic curve is required.

QUESTION 22 Answer is E

Note that: $\int_b^c [g(x)-f(x)]dx = -\int_b^c [f(x)-g(x)]dx$

QUESTION 23 Answer is D

General term $= {}^5C_r(4)^{5-r}(-ax)^r$
For x^3, $r=3$

${}^5C_3(4)^{5-3}(-ax)^3 = 10\times16\times-a^3x^3 = -160a^3x^3$
The coefficient of $x^3 = -160a^3 = -4320 \;\therefore a=3$

QUESTION 24 Answer is B

QUESTION 25 Answer is E

n is the same for both distributions.

A is positively skewed so $p<0.5$. B is negatively skewed so $p>0.5$.
As $\mu = np$, $\mu_B > \mu_A$. Answer is A or E.

As curves are mirror images about $X=10$, sum of p values will equal to 1. Therefore, σ's are equal. Answer is E.

QUESTION 26 Answer is D

As order is important the standard probability formula cannot be applied.

$$\Pr(bull's\ eye\ last\ 2\ rounds) = \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2$$

QUESTION 27 Answer is C

z values to the left of the mean are negative. $\therefore$ A is correct.

Normal curves are symmetrical about the mean, irrespective of whether z or X values are being used. $\therefore$ B is correct.

C cannot be the correct answer as a and b represent z values – not X.

PART II - SHORT ANSWER QUESTIONS

QUESTION 1

a. As $P(2) = 0$: $2(2)^3 + 3(2)^2 + 2a + b = 0$

$2a + b = -28$

As $P(-3) = 0$: $2(-3)^3 + 3(-3)^2 - 3a + b = 0$

$-3a + b = 27$

Solve equations simultaneously:

$$\begin{aligned} 2a + b &= -28 \\ -3a + b &= 27 \end{aligned} \quad -$$

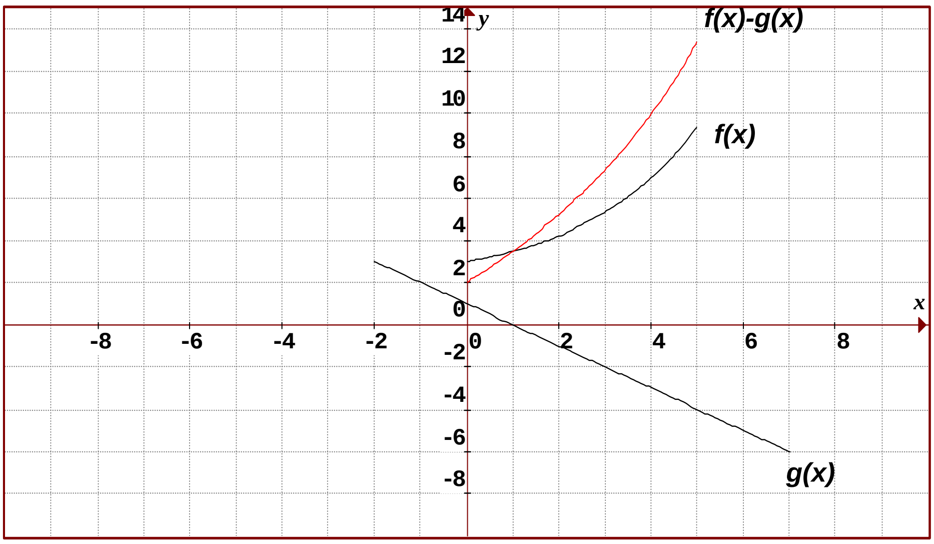
$\therefore a = -11$ and $b = -6$

b. To write $f(x)$ in the form $a + \dfrac{b}{x+2}$: Long Division is required.

$$\begin{array}{r|l} & \ \ 1 \\ \hline x+2 & x-2 \\ & x+2 \quad - \\ \hline & \ \ -4 \end{array}$$

Therefore, $f(x) = \dfrac{x-2}{x+2} = 1 - \dfrac{4}{x+2}$.

QUESTION 2



Note: Functions can only be added, subtracted (multiplied and divided) across the common domain.

QUESTION 3

$$\sin\left(2x+\frac{\pi}{3}\right)=-\sqrt{3}\cos\left(2x+\frac{\pi}{3}\right)$$

Divide both sides by $\cos\left(2x+\frac{\pi}{3}\right)$:

$$\frac{\sin\left(2x+\frac{\pi}{3}\right)}{\cos\left(2x+\frac{\pi}{3}\right)}=-\frac{\sqrt{3}\cos\left(2x+\frac{\pi}{3}\right)}{\cos\left(2x+\frac{\pi}{3}\right)}$$

$$\tan\left(2x+\frac{\pi}{3}\right)=-\sqrt{3}$$

1st Quadrant angle: $Tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$

Solutions to lie in Quadrants 2 and 4.

$$\left(2x+\frac{\pi}{3}\right)=\pi-\frac{\pi}{3} \quad \text{or} \quad \left(2x+\frac{\pi}{3}\right)=2\pi-\frac{\pi}{3}$$

$$x=\frac{\pi}{6},\frac{2\pi}{3}$$

QUESTION 4

a. $y=-3e^{2x}+1$

When $x=0$: $y=-3e^{0}+1=-2 \quad \therefore (0,-2)$

$\frac{dy}{dx}=-6e^{2x}$. When $x=0$: $\frac{dy}{dx}=-6e^{0}=-6$

$$m_{normal}=-\frac{1}{m_{\tan gent}}=\frac{1}{6}$$

Equation Normal:

$$y-y_1=m(x-x_1)$$

$$y+2=\frac{x}{6}$$

$$y=\frac{x}{6}-2$$

b. $y=-3e^{2x}+1$

X intercept, let $y=0$:

$$-3e^{2x}+1=0$$
$$e^{2x}=\frac{1}{3}$$
$$\log_e e^{2x}=\log_e\frac{1}{3}$$
$$2x=\log_e\frac{1}{3}$$
$$x=\frac{1}{2}\log_e\frac{1}{3}$$

Coordinates: $\left(\frac{1}{2}\log_e\frac{1}{3},\, 0\right)$

QUESTION 5

a. $\int_1^5\frac{f(x)}{2}\,dx=\frac{1}{2}\int_1^5 f(x)\,dx=\frac{1}{2}\times 20=10$

b. $\int_1^5\left[f(x)+1\right]dx=\int_1^5 f(x)\,dx+\int_1^5 1\,dx=20+\left[x\right]_1^5$

$$=20+(5-1)=24$$

c. $\int_5^1 f(x)\,dx+1=-\int_1^5 f(x)\,dx+1=-20+1=-19$

QUESTION 6

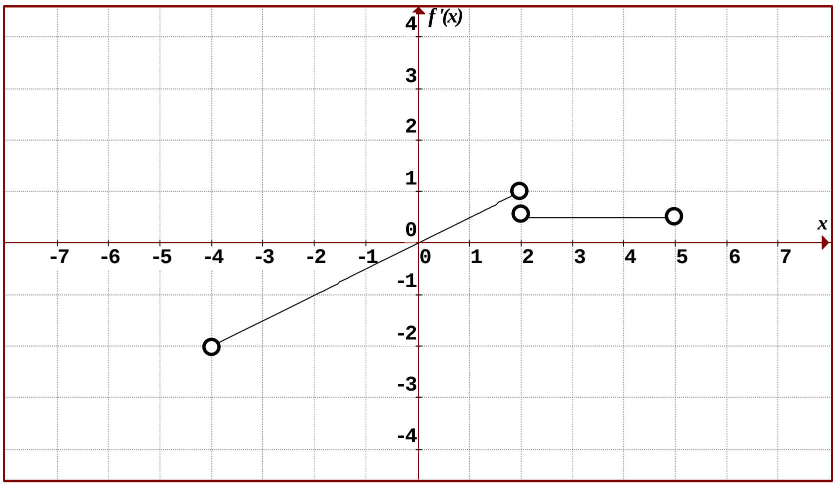
Gradient of line $= \dfrac{rise}{run} = \dfrac{1.5}{3} = 0.5$.

Draw tangents at $x = -4$ and $x = 2$, select 2 points that lie on each tangent and calculate the corresponding gradients.

At $x = -4$, $m \approx -2$.

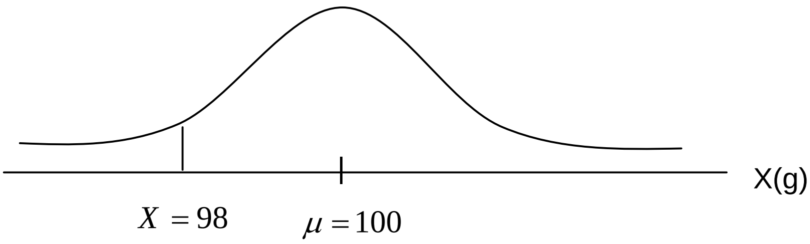
At $x = 2$, $m \approx 1$.

As a stationary point exists at $x = 0$ for $f(x)$, the graph of $f'(x)$ will cut the X axis at $x = 0$.



Note: Gradients are undefined at the end points of a domain.
Therefore, use open circles at the end points.

QUESTION 7



a. $\Pr(X < 98) = normalcdf\,(-1\times 10^{99}, 98, 100, 3) = 0.252$

Note: Variance is 9, therefore, standard deviation is 3.

b. Hypergeometric as we are sampling without replacement.

Let $X =$ number of underweight packets.

$$\Pr(X = 2) = \frac{\binom{6}{2}\binom{24}{0}}{\binom{30}{2}} = \frac{15\times 1}{435} = \frac{1}{29}$$

c. Box is rejected if:

(The 2 packets removed from the first sampling are underweight) AND
(1, 2 or 3 packets are underweight from the second sampling).

Second Sampling

$$= \frac{1}{29}\times\left[\Pr(X = 1) + \Pr(X = 2) + \Pr(X = 3)\right]$$

$$= \frac{1}{29}\times\left[1 - \Pr(X = 0)\right] = \frac{1}{29}\times\left[1 - \frac{\binom{4}{0}\binom{24}{3}}{\binom{28}{3}}\right]$$

$$= \frac{1}{29}\times\left(1 - \frac{2024}{3276}\right) = 0.013$$

Note: This part of the question requires hypergeometric manipulations as we are continuing to sample without replacement. However, a number of variables have changed in value. The sample size has decreased from 30 to 28 as 2 packets were removed in the first sampling. The number of underweight packets has decreased from 6 to 4 as 2 underweight packets were removed in the first sampling.