UNIT 4 MATHEMATICAL METHODS 2005

WRITTEN EXAMINATION 1

TRIAL EXAM 1: FACTS, SKILLS AND APPLICATION TASKS

Reading Time: 15 minutes
Writing Time: 90 minutes
Total Marks: 50 marks

QUESTION AND ANSWER BOOK

Structure of Book

Section	*Number of Questions*	*Number of Questions to be Answered*	*Number of Marks*	*Suggested Times (minutes)*
I	27	27	27	49
II	7	7	23	41
			Total 50	**Total 90**

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MULTIPLE CHOICE QUESTIONS - ANSWER SHEET

Please note that the format and requirements of this answer sheet are different to the answer sheet that will be issued in the VCAA examination. Copies of the actual examination answer sheet may be obtained at: www.vcaa.vic.edu.au

Choose the correct response or the response which best answers the question by shading the square corresponding to your response in the table below.

Question 1	A	B	C	D	E
Question 2	A	B	C	D	E
Question 3	A	B	C	D	E
Question 4	A	B	C	D	E
Question 5	A	B	C	D	E
Question 6	A	B	C	D	E
Question 7	A	B	C	D	E
Question 8	A	B	C	D	E
Question 9	A	B	C	D	E
Question 10	A	B	C	D	E
Question 11	A	B	C	D	E
Question 12	A	B	C	D	E
Question 13	A	B	C	D	E
Question 14	A	B	C	D	E
Question 15	A	B	C	D	E
Question 16	A	B	C	D	E
Question 17	A	B	C	D	E
Question 18	A	B	C	D	E
Question 19	A	B	C	D	E
Question 20	A	B	C	D	E
Question 21	A	B	C	D	E
Question 22	A	B	C	D	E
Question 23	A	B	C	D	E
Question 24	A	B	C	D	E
Question 25	A	B	C	D	E
Question 26	A	B	C	D	E
Question 27	A	B	C	D	E

PART I – MULTIPLE CHOICE QUESTIONS

Instructions For Part I

Answer **all** questions in the spaces provided.

Choose the answer that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

QUESTION 1

The period of the function $f(x) = \frac{1}{2}\tan\left(2x - \frac{\pi}{3}\right)$ is

A $\frac{\pi}{6}$

B $\frac{\pi}{3}$

C $\frac{\pi}{2}$

D π

E 2π

QUESTION 2

The function $f : R \to R,\ \ f(x) = a(\sin x - 1)$ where $a > 0$ has the range

A $[0,\ a]$

B $[-a,\ 0]$

C $[-a,\ a]$

D $[-2a,\ 0]$

E $[-a-1,\ a-1]$

QUESTION 3
In the following equation, a, b and c are positive constants. The equation $a\cos(x-b)=c$ is guaranteed to have **no solutions** in the interval $0 \le x \le 2\pi$ provided only that

A $c < a$
B $c > a$
C $b > \frac{\pi}{2}$
D $b < \frac{\pi}{2}$
E $c > 1$

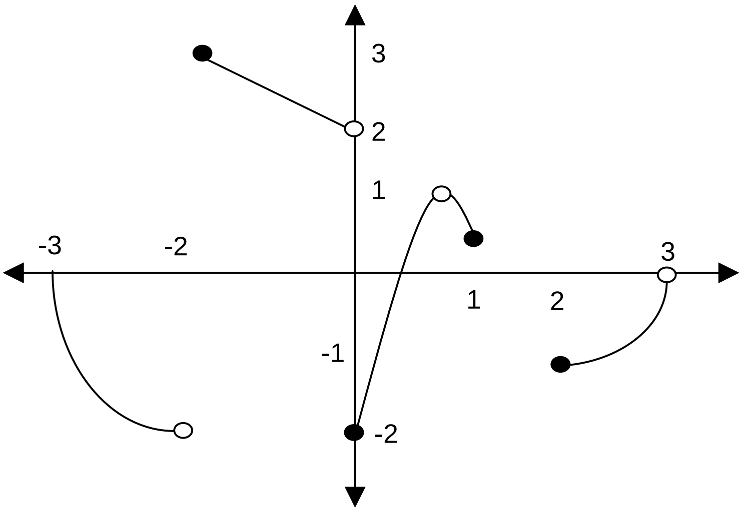
QUESTION 4
Given that $\log_a 3 + 2\log_a(2x+1) = \log_a 48$ then the value(s) of x which satisfy this equation are

A $-\frac{5}{2}$
B $\frac{3}{2}$
C $\frac{5}{2}$
D $\frac{5}{2}, -\frac{3}{2}$
E $-\frac{5}{2}, \frac{3}{2}$

QUESTION 5
Given that $p(x)=(x+a)(x^2+b)(x-c)^2$, where a, b, c represent different positive real numbers, the number of real solutions to the equation $p(x)=0$ is

A 0
B 1
C 2
D 3
E 4

QUESTION 6
The range of the function shown is:



A $[-2, 3]$
B $[-3, 3)$
C $[-3, 1) \cup [2, 3)$
D $[-2, 1) \cup (2, 3]$
E $[-2, 1] \cup (2, 3]$

QUESTION 7
Consider the function $f:\{t : t < a\} \to R,\ \ f(t) = -5\log_e(4 - at)$, where a represents a positive real number value. The largest value of t for which $f(t)$ is defined is

A 4

B $\frac{a}{4}$

C $-\frac{a}{4}$

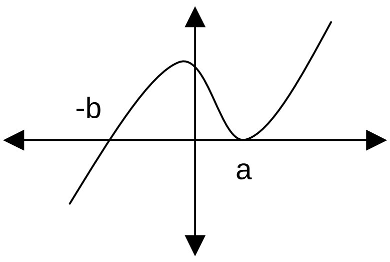
D $\frac{4}{a}$

E $-\frac{4}{a}$

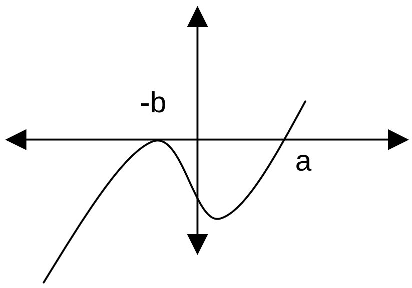
QUESTION 8

Which of the following curves best represents the graph of $y = (x+b)(a-x)^2$ where $a, b > 0$?

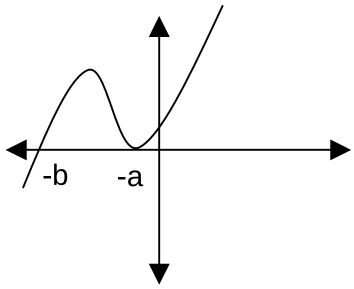
A



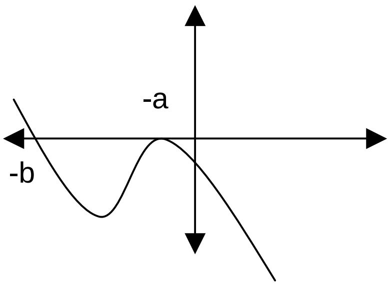
B



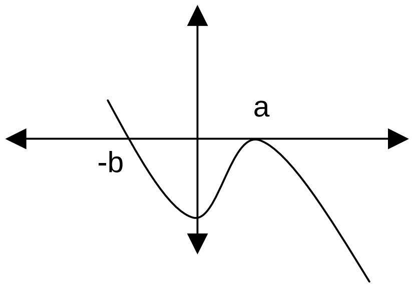
C



D



E



QUESTION 9

x	1	2	3	4	5	6	7	8	9
y	1	1.4	1.7	2	2.2	2.4	2.6	2.8	3

The data in the above table would be best modelled by a

A linear function
B power function
C circular function
D exponential function
E logarithmic function

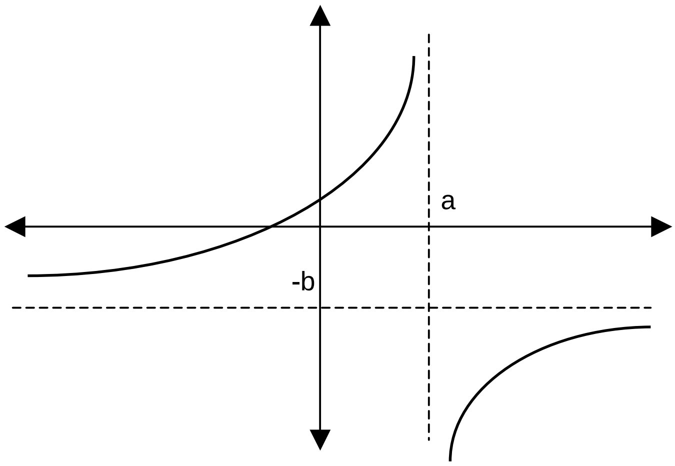
QUESTION 10

The function $f(x) = e^x$ is reflected in the X axis and dilated by a scale factor $\frac{1}{3}$ from the Y axis. The equation of the transformed curve is

A $f(x) = -e^{(3x)}$
B $f(x) = e^{(-3x)}$
C $f(x) = -e^{(x/3)}$
D $f(x) = e^{(-x/3)}$
E $f(x) = -e^{(-x/3)}$

QUESTION 11

Given that a and b represent positive constants, the equation of the curve shown alongside is best described by



A $y = \dfrac{1}{x-a} + b$

B $y = \dfrac{1}{a-x} + b$

C $y = \dfrac{1}{x+a} - b$

D $y = \dfrac{1}{-(x+a)} - b$

E $y = \dfrac{1}{a-x} - b$

QUESTION 12

The function f is defined by the rule $f(x) = -(x+a)^2 - b$, where $a, b > 0$. The largest domain that $f(x)$ can assume so that the inverse function $f^{-1}(x)$ exists is

A R
B $\{x : x \geq -b\}$
C $\{x : x \leq -a\}$
D $\{x : x \leq a\}$
E $\{x : x \geq a\}$

QUESTION 13

If $f(x) = 2x^3 - x + \sqrt{x}$ then $f'(a)$ is equal to

A $6a^2 - 1 - \dfrac{1}{2\sqrt{a}}$

B $2a^2 - 1 - \dfrac{1}{2\sqrt{a}}$

C $\dfrac{a^4}{2} - \dfrac{a^2}{2} + \dfrac{2a^{3/2}}{3}$

D $6a^2 - 1 + \dfrac{1}{2\sqrt{a}}$

E $2a^2 - 1 + \dfrac{1}{2\sqrt{a}}$

QUESTION 14

If $y = \cos^3(2x)$ then the gradient function is

A $-6\sin^2(2x)$

B $-24x^2 \sin(2x)^3$

C $24x^2 \sin(2x)^3$

D $-6\sin 2x \cos^2(2x)$

E $6\sin 2x \cos^2(2x)$

QUESTION 15

If $y = \dfrac{\cos x}{e^{2x}}$ then $\dfrac{dy}{dx}$ equals:

A $\dfrac{-\sin x}{2e^{2x}}$

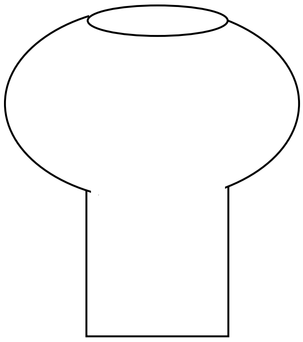
B $-\dfrac{(\sin x + 2\cos x)}{e^{2x}}$

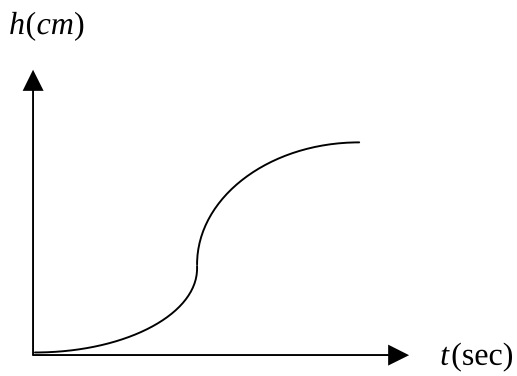
C $-\dfrac{(\sin x - 2\cos x)}{e^{2x}}$

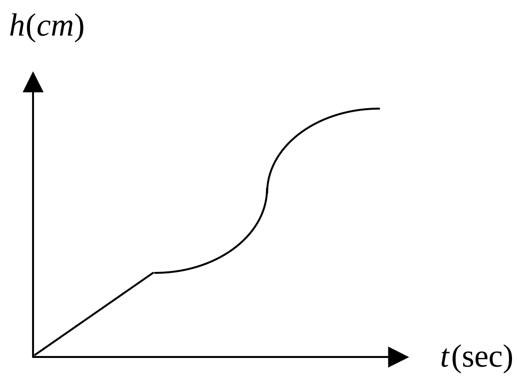
D $-\dfrac{(\sin x + 2\cos x)}{\left(e^{2x}\right)^2}$

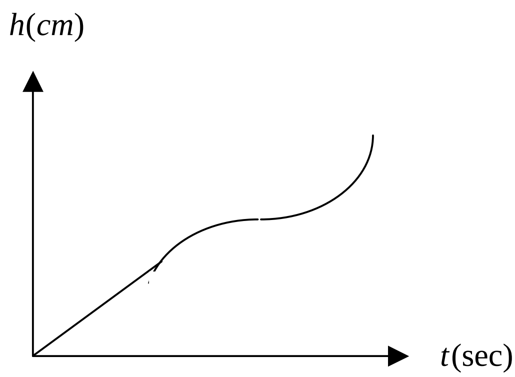
E $\dfrac{-e^{2x}\sin x - \cos x e^{2x-1}}{\left(e^{2x}\right)^2}$

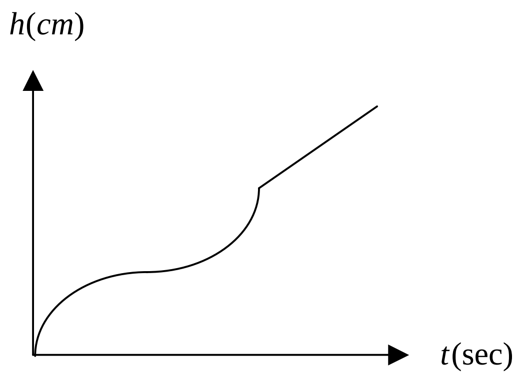
QUESTION 16
The container below is filled with water at a constant rate of $0.5\ ml/s$. Which of the following graphs **best** represents the change in the height of water as the container is being filled?

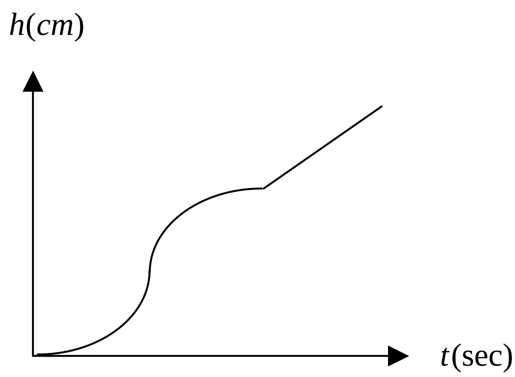


A


B


C


D


E


QUESTION 17

$\int (-2x^2 - 3e^{2x+1})\,dx$ is equal to

A $\quad -\dfrac{2x^3}{3} - 3(2x+1)e^{2x} + c$

B $\quad -\dfrac{2x^3}{3} - \dfrac{3e^{2x+1}}{2} + c$

C $\quad -\dfrac{2x^3}{3} - 3e^{2x} + c$

D $\quad -2x - 3e^{2x} + c$

E $\quad -\dfrac{2x^3}{3} - 6e^{2x+1} + c$

QUESTION 18

If $f\,'(x) = 2\sin\left(2x - \dfrac{\pi}{2}\right)$ and $f\left(\dfrac{\pi}{4}\right) = 1$, then $f(x)$ is equal to

A $\quad -2\cos\left(2x - \dfrac{\pi}{2}\right) + 3$

B $\quad 2\cos\left(2x - \dfrac{\pi}{2}\right) + 3$

C $\quad -2\cos\left(2x - \dfrac{\pi}{2}\right) - 1$

D $\quad -\cos\left(2x - \dfrac{\pi}{2}\right) + 2$

E $\quad \cos\left(2x - \dfrac{\pi}{2}\right) + 2$

QUESTION 19

$\int (5-3x)^3\,dx$ is equal to:

A $-\dfrac{(5-3x)^4}{12}+c$

B $\dfrac{(5-3x)^4}{12}+c$

C $-\dfrac{(5-3x)^2}{6}+c$

D $\dfrac{(5-3x)^2}{6}+c$

E $\dfrac{(5-3x)^4}{4}+c$

QUESTION 20

If $\int_0^a \left(\dfrac{2}{4x+1}\right)dx = \log_e k$ then k is equal to:

A $\sqrt{4a+1}$

B $(4a+1)^2$

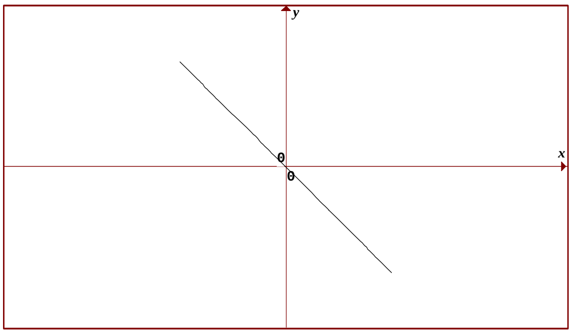
C $4a+1$

D $e^{\frac{8}{(4a+1)^2}}-8$

E $e^{\frac{2}{(4a+1)^2}}-2$

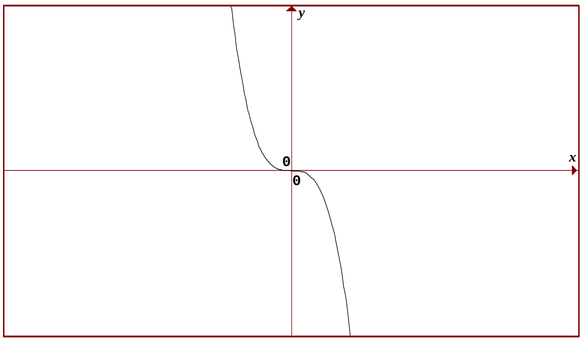
QUESTION 21

The graph of $y = f(x)$ is given below.

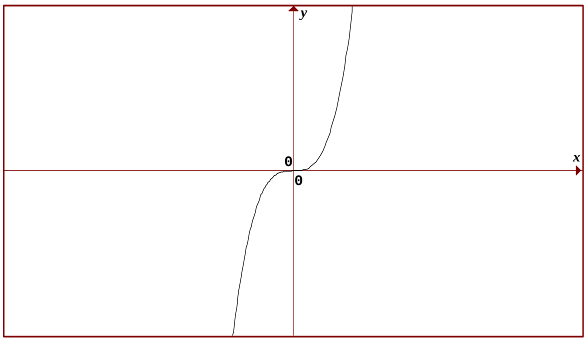


Which one of the following is most likely to be the graph of the area function $y = F(x)$?

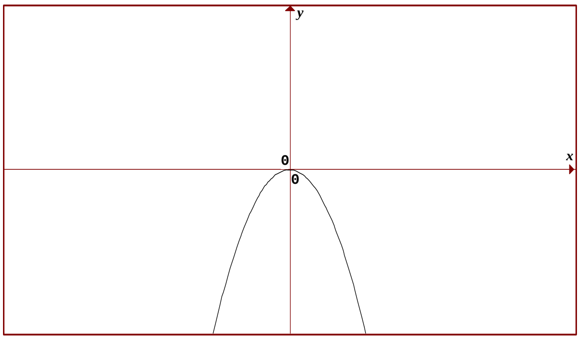
A



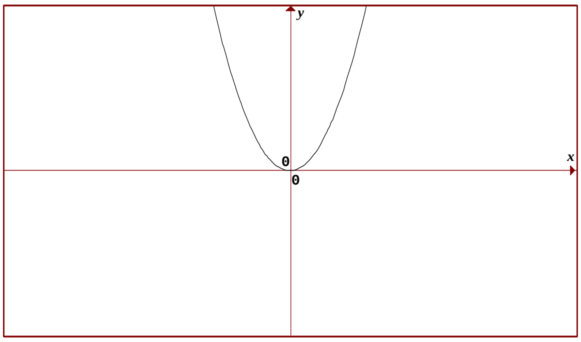
B



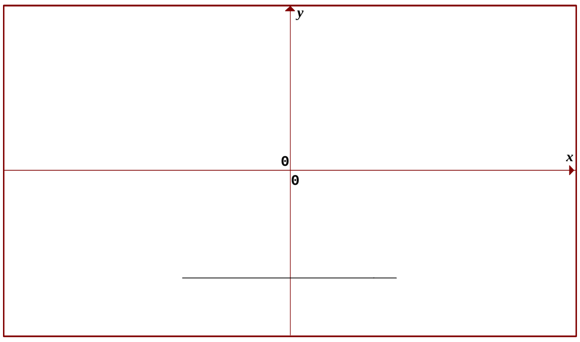
C



D

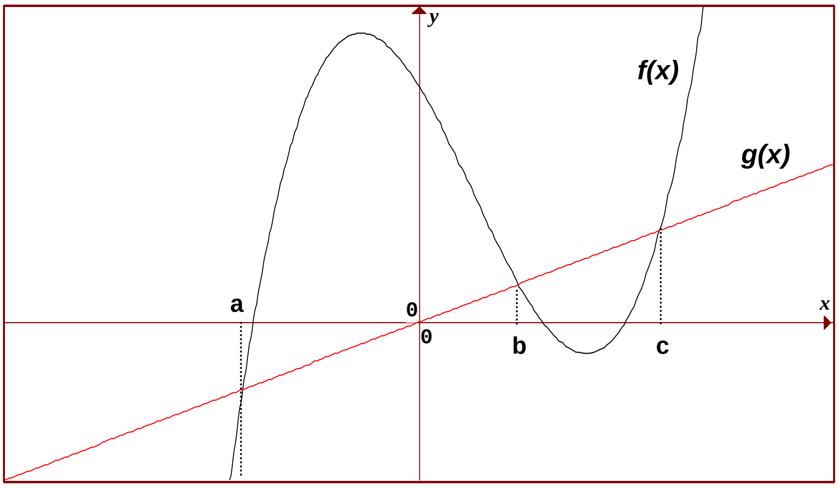


E



QUESTION 22

The graphs of $f(x)$ and $g(x)$ are given below.



The area enclosed by the graphs of $f(x)$ and $g(x)$ between $x = a$ and $x = c$ is best described by:

A $\int_a^c [g(x) - f(x)]dx$

B $\int_a^c [f(x) - g(x)]dx$

C $\int_a^b f(x)\, dx - \int_b^c g(x)\, dx$

D $\int_a^b [f(x) - g(x)]dx + \int_b^c [f(x) - g(x)]dx$

E $\int_a^b [f(x) - g(x)]dx - \int_b^c [f(x) - g(x)]dx$

QUESTION 23
The first seven lines of Pascal's Triangle are given below.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The coefficient of x^3 in the expansion $(4-ax)^5$ is equal to -4320. The value of a is equal to

A -3
B -1
C 1
D 3
E 12

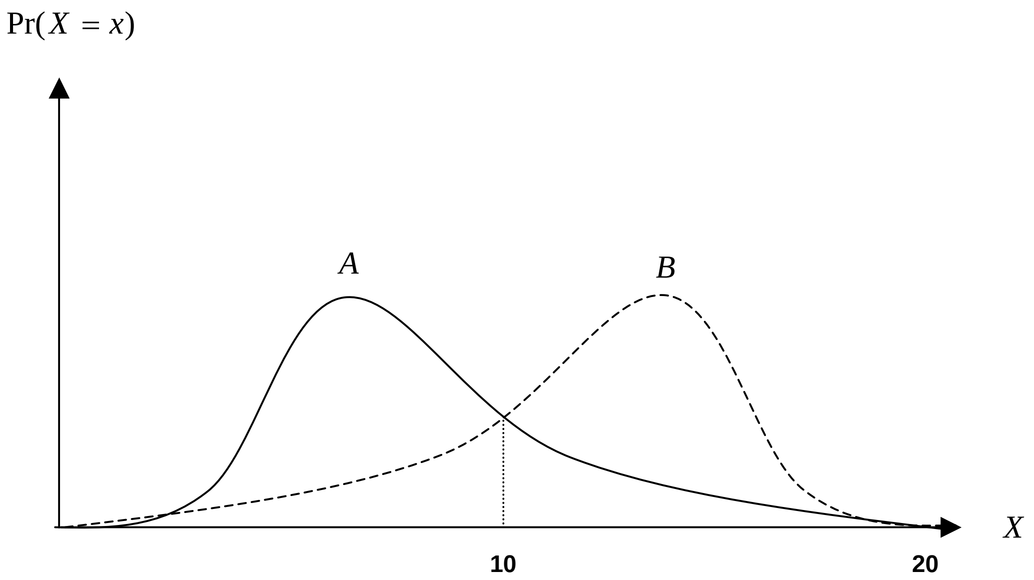
QUESTION 24
For a \$10 monthly fee, subscribers to Australia News receive a weekly magazine that addresses major issues concerning the nation. If a subscriber does not receive a magazine in any particular week, he/she is issued a \$5 refund for each magazine they did not receive. The probability that a subscriber does not receive a magazine across any month of the year is given below.

$X =$ Number of magazines not received across one month	0	1	2	3	4
$\Pr(X = x)$	0.70	0.20	0.10	0	0

Let X represent the number of magazines not received across any month and P represent the profit in \$. The expected monthly profit to Australia News is equal to

A $10-5X$
B $10-5E(X)$
C $(0\times 0.70)+(1\times 0.20)+(2\times 0.10)$
D $(0\times 0.70)+(1^2\times 0.20)+(2^2\times 0.10)$
E $10-(0\times 0.70)+(1\times 0.20)+(2\times 0.10)$

QUESTION 25
The diagram below shows the graphs of two binomial distribution curves, A and B.



Which one of the following statements is true?

A $\mu_A < \mu_B$ and $\sigma_A < \sigma_B$
B $\mu_A > \mu_B$ and $\sigma_A = \sigma_B$
C $\mu_A > \mu_B$ and $\sigma_A < \sigma_B$
D $\mu_A = \mu_B$ and $\sigma_A = \sigma_B$
E $\mu_A < \mu_B$ and $\sigma_A = \sigma_B$

QUESTION 26

A marksman finds that in the long run, he scores a bull's eye in 2 out of 3 occasions. He fires 5 rounds at a target. The probability that he scores a bull's eye in the last two rounds is

A $\binom{5}{2}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$

B $\binom{5}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3$

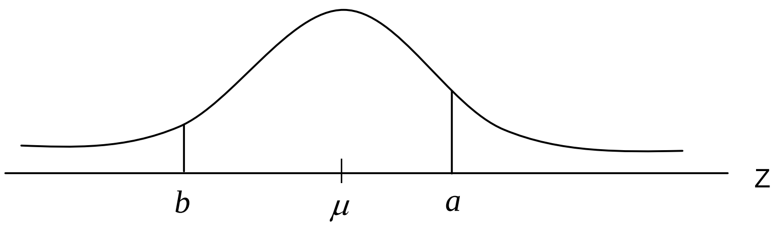
C $\binom{5}{1}\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^4$

D $\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3$

E $\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$

QUESTION 27

The random variable X is normally distributed with a mean μ and standard deviation σ.



Which one of the following statements regarding this distribution is **incorrect**?

A $b < 0$

B $\Pr(X > \mu) = 0.5$

C $\Pr(b < X < a) = \Pr(X < a) - \Pr(X < b)$

D $\Pr(b < z < a) = \Pr(z < a) - \Pr(z > -b)$

E $\Pr(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95$

PART II - SHORT ANSWER QUESTIONS

Instructions For Part II

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

Where an **exact** answer is required to a question, appropriate working must be shown.

In questions where more than 1 mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

QUESTION 1

a. Given that $(x-2)$ and $(x+3)$ are factors of $2x^3+3x^2+ax+b$, find the values of a and b.

2 marks

b. The function $f(x)$ is described by the rule: $\{x : x > -2\} \to R, \quad f(x) = \dfrac{x-2}{x+2}$.

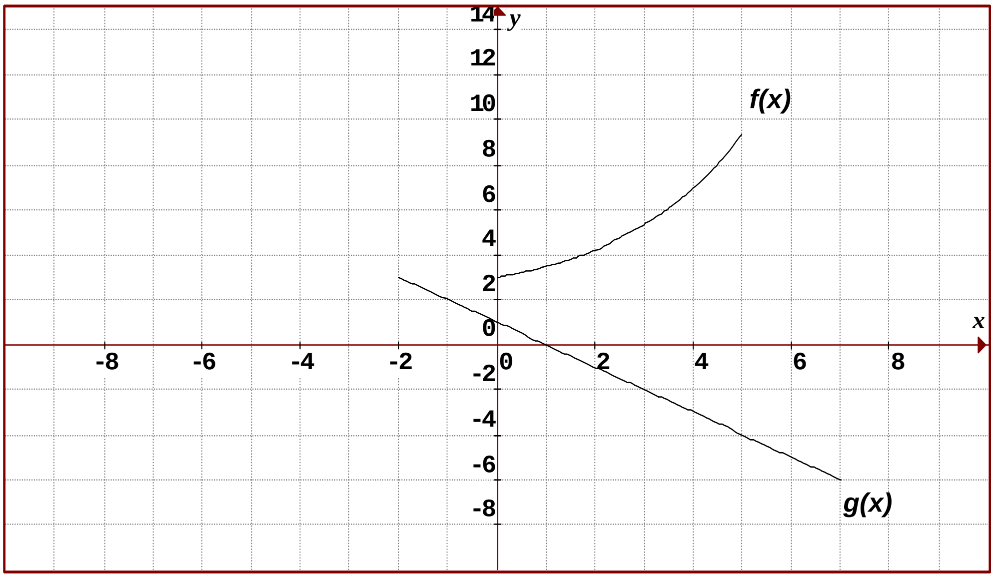
Use algebra to write $f(x)$ in the form $a + \dfrac{b}{x+2}$.

1 mark

Total = 3 marks

QUESTION 2

The graphs of $f(x)$ and $g(x)$ are illustrated below. On the same set of axes, sketch the graph of $f(x)-g(x)$.



Total = 2 marks

QUESTION 3

Find the exact solutions of the equation $\sin\left(2x+\frac{\pi}{3}\right)=-\sqrt{3}\cos\left(2x+\frac{\pi}{3}\right)$, $0 \le x \le \pi$.

3 marks

QUESTION 4

a. Find the equation of the normal at the point where $y = -3e^{2x} + 1$ crosses the Y axis.

2 marks

b. Find the exact coordinates where $y = -3e^{2x} + 1$ crosses the X axis.

2 marks

Total = 4 marks

QUESTION 5

If $\int_1^5 f(x)\,dx = 20$, find:

a. $\int_1^5 \frac{f(x)}{2}\,dx$

1 mark

b. $\int_1^5 [f(x)+1]\,dx$

1 mark

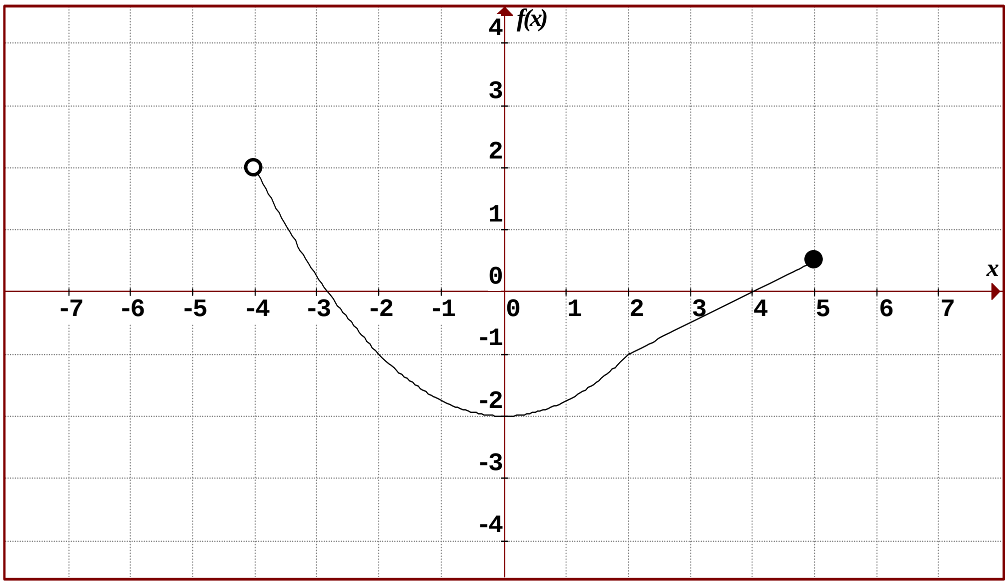
c. $\int_5^1 f(x)\,dx + 1$

1 mark

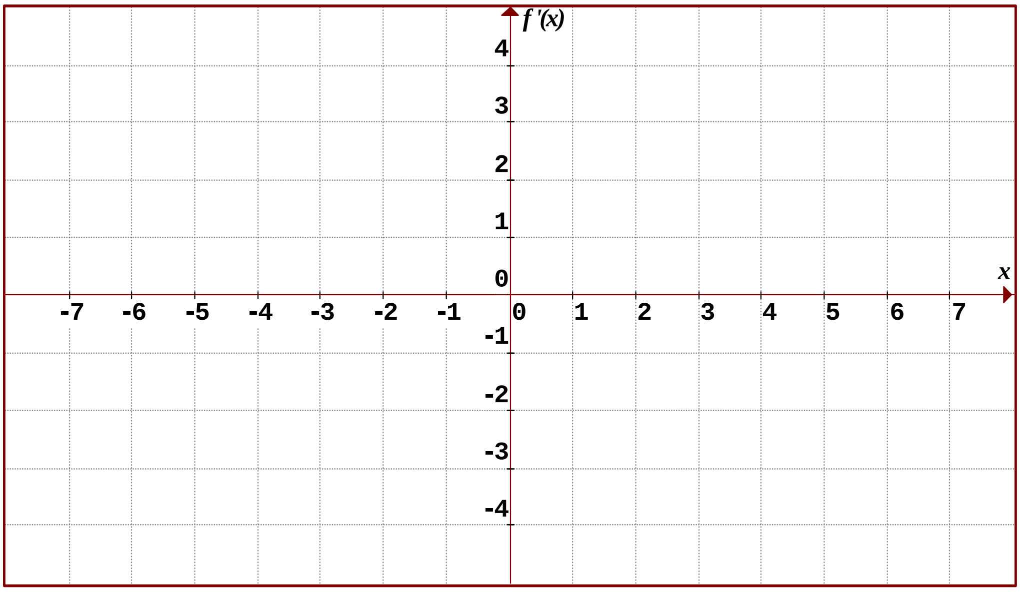
Total = 3 marks

QUESTION 6

The graph of the function $f:(-4, 5) \to R$ is shown below.



On the set of axes below, sketch the graph of the derived function $f'(x)$. Do not attempt to find the exact value of f' at $x = -4$ or $x = 5$.



Total = 2 marks

QUESTION 7
Twisted Noodles Pty Ltd produces packets of instant noodles. Each packet of noodles is labelled as weighing 100 grams. There is some variation in the weights of packets of noodles. It is found that the weights (W) of packets of noodles follows a normal distribution with a mean of 102 grams and a variance of 9 grams.

a. Find the probability that a packet removed at random weighs less than 98 grams. State your answer to 3 decimal places.

2 marks

Noodles are packed in boxes of 30. Each packet is supposed to weigh at least 98 grams. A quality control technician randomly removes a box from the production line, and a sample of 2 packets is withdrawn.

b. Find the probability that both packets withdrawn weigh less than 98 grams, if it is known that a box contains 6 underweight packets. State your answer to 3 decimal places.

2 marks

c. If the two packets initially selected weigh less than 98 grams, then a further sample of 3 packets is removed from a box. If any of these packets is underweight, the box is rejected. Otherwise, the box is accepted. Find the probability correct to three decimal places that a box is rejected.

2 marks

Total = 6 marks

End of Paper