UNIT 4 MATHEMATICAL METHODS 2005

WRITTEN EXAMINATION 2

TRIAL EXAM 2 - ANALYSIS TASK

Reading Time: 15 minutes
Writing Time: 90 minutes
Total Marks: 58 marks

QUESTION AND ANSWER BOOK

Structure of Booklet

Number of Questions	Number of Questions to be Answered
4	4

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

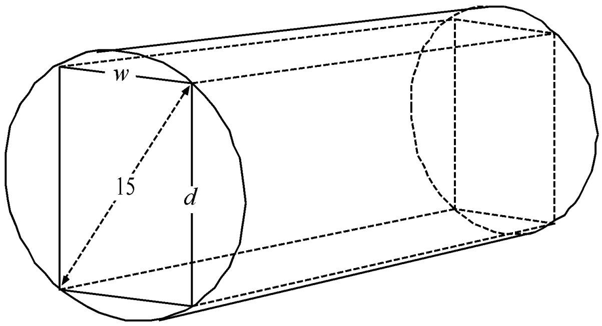
In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to use **calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

QUESTION 1

A rectangular beam of width w cm and depth d cm is cut from a cylindrical **pine** log as illustrated below.



The diameter of the cross-section of the log (and hence the diagonal of the cross-section of the beam) is 15 cm.

The strength S of the beam is proportional to the product of its width and the square of its depth, so that $S = kd^2w$, where $k > 0$.

a. (i) Show that $S = k(225w - w^3)$.

1 mark

(ii) Determine the interval for which the value of w is defined.

2 marks

b. The strength S of the beam is dependent on the value of k, a positive constant that is dependent on the width, w, of the beam, as well as the nature of the timber used to construct the beam.

The area enclosed by the graph of S and the lines $w = 0$ and $w = w$ provides an accurate measure of the value of k for a beam of width $w\ cm$.

(i) If the area enclosed by the graph of S, the lines $w = 0$ and $w = 5$ is $425\ units^2$, use calculus to show that the value of k for a **pine** beam of width $5\ cm$ is $\frac{4}{25}$.

3 marks

(ii) The frame of a house is to be constructed using beams from **pine** logs. The builder requires that the width of the beams, w, does not exceed $7\ cm$.

Find the exact dimensions that will give each **pine** beam maximum strength.

2 marks

c. Beams are cut from logs using machinery that may be set to produce beams of different widths (w). The cost expended each time the machinery is re-set for different values of w is equal to 20 cents per cm^2 of the cross-sectional area of the beam.

(i) Show that the cost (C$) of re-setting the machine is equal to $C = 0.2w\sqrt{225 - w^2}$.

1 mark

(ii) Use calculus to find the maximum set up cost that would be incurred for a beam of width $w\ cm$, correct to the nearest cent.

4 marks

Total = 13 Marks

QUESTION 2

Two fishing trawlers return to Port Campbell after a weekend of fishing, jointly catching a total of 5000 fish. The number of fish caught by variety and trawler on this particular weekend is given below.

	Number of Tuna	Number of Salmon	Total Catch
Boat 1	1800	900	3000
Boat 2	800	300	2000

a. (i) A fish is selected at random from Boat 1. What is the probability that it is a tuna?

1 mark

(ii) 500 fish are selected at random from Boat 1. What is the expected number of tuna in this sample?

1 mark

b. (i) A fish is selected at random from all of the fish that were caught by both trawlers, and was found to be a tuna. What is the exact probability that this fish was caught by Boat 1?

1 mark

(ii) One fish is randomly selected from each boat. What is the probability that one of these fish is a tuna? Give your answer correct to three decimal places.

2 marks

c. As each boat comes in, the catch is inspected and sorted according to type.
All tuna are sorted into one bin and weighed. The salmon are sorted into their own bin, and the other varieties of fish are placed into a common bin.

The fishermen are paid $10 per kilo for each kilogram of tuna caught. Salmon are paid at a rate of $12 per kilo, whereas the other varieties of fish are paid at a rate of $5 per kilogram.

(i) Construct a probability distribution graph for the price paid per variety of fish to the fishermen on Boat 1.

2 marks

(ii) Hence calculate the mean price paid per kilogram to the fishermen on Boat 1. Give your answer to the nearest cent.

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1 mark

d. (i) The weight of tuna delivered to Port Campbell is normally distributed with a mean of 35 kg. The fish are classified as acceptable for sale to John West Pty Ltd if the weight of a tuna lies within a units either side of the mean. The probability that a tuna from Port Campbell is accepted by John West is 0.70. Find an expression for a in terms of σ, correct to 4 decimal places.

3 marks

(ii) A sample of 5 tuna is randomly selected from Boat 1. Find the probability that more than two fish will be accepted by John West. Give your answer correct to 3 decimal places.

2 marks

e. On another weekend, John West decided to inspect the tuna from every boat and trawler that docked at a rival port in Western Australia. The total number of tuna that had been caught that weekend was very large. John West randomly selected a total of 5 tuna from the entire weekend's catch at that port.

Let p represent the proportion of tuna in the pool of fish caught over the weekend that fall within John West's acceptable weight range.

(i) Write down an expression for the probability that two of the 5 fish selected are within John West's acceptable weight range.

1 mark

(ii) Use calculus to find the exact value of p for which this probability will be a maximum.

2 marks

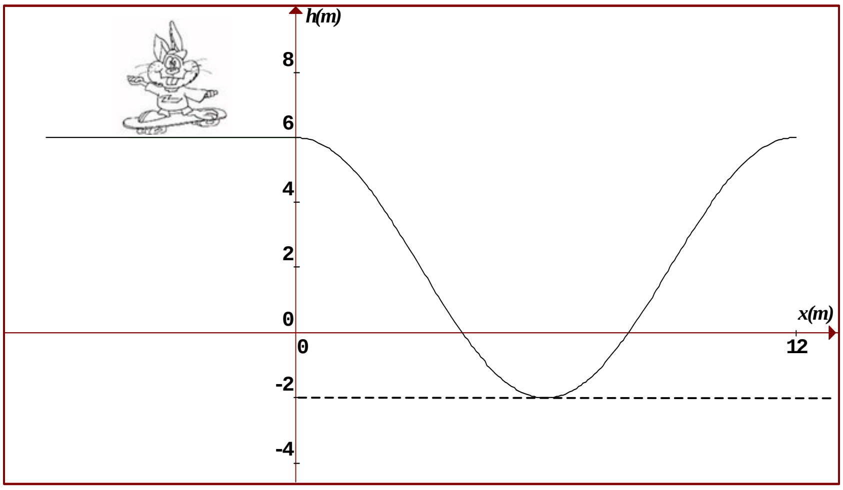
Total = 16 marks

QUESTION 3

A skateboard ramp is to be constructed for the local youth in Skate -Town. Most of this ramp is to be constructed above ground level, with a portion of the ramp to be built under ground. The height of the ramp relative to ground level is modelled by the equation

$$h(x) = a\cos(bx) + c\ ,\ 0 \le x(m) \le 12$$

h represents the height of the ramp (in metres) in relation to ground level, and x represents the length of the ramp (in metres) along ground level.



a. (i) Show that $a = 4$.

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1 mark

(ii) Show that $b = \dfrac{\pi}{6}$.

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1 mark

(iii) **Hence** show that $c = 2$.

1 mark

b. Find the exact length(s) of the ramp when the ramp height is $2(\sqrt{2}+1)$ metres above ground level.

3 marks

c. Using calculus, find the approximate change in the height of the ramp as x changes from 2.9 to 3.0, given that $h(3) = 2$. State your answer to 3 decimal places.

3 marks

d. A alternate ramp design involves joining the ramp modelled by the function $h(x)$ with another ramp which is modelled by the function $g(x) = \frac{2}{9}(x-6)^2 - 2$, $6 \le x \le 12$.

(i) Using calculus, show that the join linking $h(x)$ and $g(x)$ is smooth.

1 mark

(ii) Using calculus, find the region enclosed by $h(x)$ and $g(x)$ for $6 \le x \le 12$. State your answer to 3 decimal places.

3 marks

Total marks = 13

QUESTION 4

The intensity (y units) of light emitted from a tungsten light globe is given by

$$y = A\log_e(t+b)$$

where t is the time in microseconds (μ sec). The initial globe intensity is 0 units and the intensity at $8\,\mu$ sec is 50 units.

a. (i) Show that $b=1$.

1 mark

(ii) Show that $A=\dfrac{50}{\log_e 9}$.

1 mark

b. (i) Find the exact intensity of light at 15 μ sec.

1 mark

(ii) Find the exact time at which the intensity of the globe is $\dfrac{1}{\log_e 9}$ units.

2 marks

c. (i) Let $y = f(t)$. Find the equation describing the inverse function, f^{-1}.

2 marks

(ii) State the range of f^{-1}.

1 mark

d. The graph of $y = A\log_e(t+b)$ undergoes a transformation to produce the curve with equation $Y = A\log_e(t)$, where $A = \dfrac{50}{\log_e 9}$.

(i) State the transformation required to convert $y(t)$ to $Y(t)$.

1 mark

(ii) Using three rectangles, find the sum of the left rectangles between $t = 1$ and $t = 7$. State your answer correct to 3 decimal places.

2 marks

e. (i) Find the derivative of $t \log_e t$ with respect to t.

1 mark

(ii) Hence find an antiderivative of $\log_e t$.

2 marks

(ii) Hence find the actual area enclosed by $Y(t)$ and the lines $t = 1$ and $t = 7$. State your answer correct to 3 decimal places.

2 marks

Total marks = 16

End of Paper

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