C

Free Exam

for 2006-15 VCE study design

Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Practice Exam Solutions

Stop!

Don’t look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Multiple-choice questions

Question 1

The correct answer is C.

For there to be infinitely many solutions, the two lines must be equal.

They must be equal in gradient:

$\frac{-3}{k} = -(k+2)$

$k^2 + 2k - 3 = 0$

$k = -3, 1$

If k=-3, the lines are of two different equations, therefore they must be parallel. If k=1, the lines are the same, hence there are infinitely many solutions.

Question 2

The correct answer is D.

Pr(A) x Pr(B) = Pr(A∩B)

Pr(A) x Pr(B)=$\frac{1}{4}$ x $\frac{1}{5}$ =$\frac{1}{20}$, Pr(A∩B)=$\frac{1}{20}$

Question 3

The correct answer is A.

$log_k(l) + log_l(m) + log_m(n) = \frac{log_l(l)}{log_l(k)} + \frac{log_m(m)}{log_m(l)} + \frac{log_n(n)}{log_n(m)} = \frac{1}{log_l(k)} + \frac{1}{log_m(l)} + \frac{1}{log_n(m)}$

Question 4

The correct answer is C.

$\sqrt{3}\tan(2\mathrm{x}) = 1 \Rightarrow \tan(2x) = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{12}, k \in \mathbb{Z}$

Question 5

The correct answer is A.

Domain: $x^2 - 4 > 0 \Rightarrow x^2 > 4 \ \Rightarrow x > 2 \ and \ x < -2$ Range: there are no implied restrictions on range

Question 6

The correct answer is E.

Question 7

The correct answer is D.

y=$\binom{5}{0}(3x)^5 + \binom{5}{1}(3x)^4(5) + \binom{5}{2}(3x)^3(5)^2 + \binom{5}{3}(3x)^2(5)^3 + \binom{5}{4}(3x)(5)^4 + \binom{5}{5}(5)^5$

3rd term is: $\binom{5}{2}(3x)^3(5)^2 = \binom{5}{2}(27x^3)(25) = \binom{5}{2}675x^3$

Question 8

The correct answer is B.

Average value of function $\frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{6-1}\int_1^6 \frac{1}{\mathrm{x}} + x^2\, dx.$

Question 9

The correct answer is C.

$y = \frac{2x+6}{x-1} = 2 + \frac{8}{x-1} \Rightarrow x \neq 1, y \neq 2$

Question 10
The correct answer is D.
$\Pr(S \cap B) = 0.3$, $Pr(B|S) = 0.5$. $\Rightarrow \Pr(S) = \frac{\Pr(S \cap B)}{Pr(B|S)} = 0.6$

Question 11
The correct answer is C.
This is given by the formula for linear approximation, and it will be an underestimate of the true value due to increasing gradient so tangent being below the graph.

Question 12
The correct answer is B.
$f'(x) = \frac{x^2+4x+3}{(x+2)^2} = 0 \; when \; x = -1, f(-1) = -2 \; and \; x = -3, f(-3) = -6$

Question 13
The correct answer is A.
$\{x: \sin^2(2x) + 2\sin(2x) = 0\} = \{x: \sin(2x)(\sin(2x) + 2) = 0\} \Rightarrow \sin(2x) + 2 > 0$

Question 14
The correct answer is A.
Sub in $x = 2$ to get point (2,6). M= $f'(2) = -3 \Rightarrow y - 6 = -3(x - 2) \Rightarrow y + 3x = 12$

Question 15
The correct answer is E.
$x - 2 + \frac{1}{x-2} = \frac{(x-2)^2}{x-2} + \frac{1}{x-2} = \frac{x^2-4x+5}{x-2}$.

Question 16
The correct answer is E.
This follows from the general probability mass function of binomially distributed random variables. Odds are 9-2 therefore a $\frac{2}{11}$ chance of winning.

Question 17
The correct answer is D.
$$\lim_{\partial x_i \to 0} \sum_{i=1}^{n} (x_i^2 + 1\partial x) = \int_1^4 x^2 + 1dx = 24$$

Question 18
The correct answer is B.
a is a vertical asymptote, restricting it such that it is one to one$\Rightarrow a = 2$. Reasons for taking only the positive root in our rule can be seen by reflecting our restricted f in the line $y = x$.

Question 19
The correct answer is E.
Differentiating gives a local maximum at $x = \frac{\pi}{2}$ which is higher than the endpoint at $x = 1$. The minimum is found at the left endpoint.

Question 20
The correct answer is B.

Question 21

The correct answer is C.

$$\begin{bmatrix} \Pr(savoury\ then\ savoury) & \Pr(sweet\ then\ savoury) \\ \Pr(savoury\ then\ sweet) & \Pr(sweet\ then\ sweet) \end{bmatrix}^{50} \begin{bmatrix} number\ of\ savouy\ first\ day \\ number\ of\ sweet\ first\ day \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}^{50} \begin{bmatrix} 25 \\ 100 \end{bmatrix}$$

$\begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 0.6 \end{bmatrix}^{50} \begin{bmatrix} 100 \\ 25 \end{bmatrix}$ would also give the steady state numbers but is not an option

Question 22

The correct answer is B.

g must be restricted such that its range is a subset of the domain of f.

Section B – Short-answer questions

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$15\cos\left(\frac{\pi}{2}((20)-8)\right)+21=36$ Must meet at point (20,36) [1]

$36=a(20)+91$

$a=-\frac{11}{4}$ [1]

Question 1b

Only employees in the store at minimums in graph

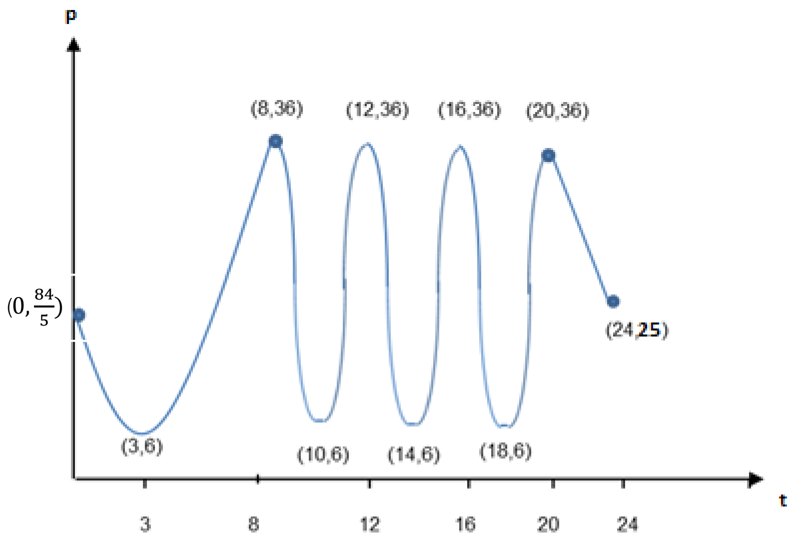
$\Rightarrow \frac{dp}{dt}=0$ [1]

$p=6$ [1]

Question 1c

Correct shape of graph [1]
Correctly labelled maximum and minimums [1]
Correctly labelled intercepts and endpoints [1]
Appropriate scale [1]

Question 1d



$$p'(t)=\begin{cases}\frac{12}{5}(t-3) & 0<t<8\\ -\frac{15\pi}{2}\sin\left(\frac{\pi}{2}(t-8)\right) & 8<t<20\\ -\frac{11}{4} & 20<t<24\end{cases}$$

Correct differentiation of the hybrid function [3]
changing endpoints to be non-inclusive [1]

Question 1e

$\frac{df}{dt} = 0.1\,\frac{dp}{dt}$

$\frac{dp}{dt} = \frac{36}{5}$[1]

$\frac{df}{dt} = \frac{36}{5} * \frac{1}{10}$ [1]

$\frac{df}{dt} = \frac{18}{25} kg/hour$ [1]

Question 2a

For inverse $\Leftrightarrow y$, $x = \frac{1}{3} log_e\left(\frac{y+2}{2}\right) + 1$ [1]

$e^{3(x-1)} = \frac{y+2}{2} \Rightarrow y = 2e^{3(x-1)} - 2$ [1]

$f^{-1}(x) = 2e^{3(x-1)} - 2$

Domain of $f^{-1}(x)$= range of $f(x) = R \rightarrow R$

Question 2b

Existence requires $ran(f^{-1}) \subseteq dom(f)$ ie. $(-2, \infty) \subseteq (-2, \infty)$

$f[f^{-1}(x)] = \frac{1}{3} log_e\left(\frac{(2e^{3(x-1)}-2)+2}{2}\right) + 1 = x$ [1]

Rule is $h: (-2, \infty) \rightarrow R, h(x) = x$ [1]

Question 2c

Existence requires $ran(g) \subseteq dom(f)$. $[0, \infty) \subseteq R\backslash\{-2\}$

$f \circ g: [0, \infty) \rightarrow R, f\big(g(x)\big) = \frac{2}{\sqrt{x+3}+2} - 1$ [1]

Question 2d

$\begin{bmatrix} x^1 \\ y^1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$[1]

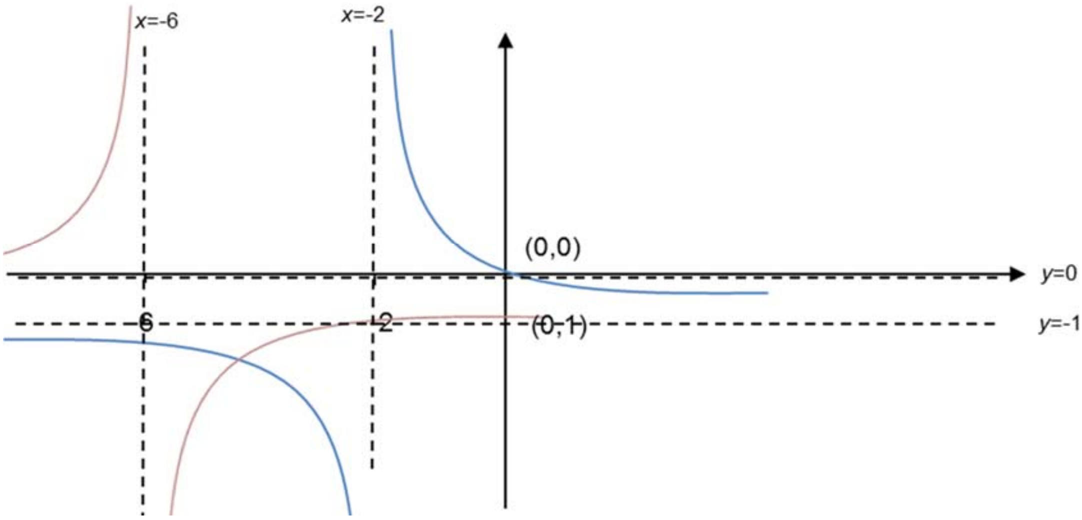
$x^1 = 3\,x \;\Rightarrow x = \frac{1}{3}x^1$

$y^1 = -y - 1 \Rightarrow y = -\,(y^1 + 1)$ [1]

$y = \frac{2}{x+2} - 1$ sub in for $-\,(y^1 + 1) = \frac{2}{\frac{1}{3}x^1+2} - 1$

$d(x) = -\frac{2}{\frac{1}{3}x^1+2}$ [1]

Question 2e



[1]

Correct shape [1]
Correct asymptotes [2]
Correct axial intercepts [1]

Question 3a

$0 = a(2)^3 - b(2)^2 + c$ [1]

$\frac{dy}{dx} = 3ax^2 - 2bx \Rightarrow -8 = 3a(2)^2 - 2b(2)$ [1]

$0 = 3a\left(\frac{8}{3}\right)^2 - 2b\left(\frac{8}{3}\right)$ [1]

$a = 2, b = 8$ and $c = 16$ [2]

Question 3b

$y = 2\left(\frac{8}{3}\right)^3 - 8\left(\frac{8}{3}\right)^2 + 16 = -\frac{80}{27}\ pt\ \left(\frac{8}{3}, -\frac{80}{27}\right)$[1]

$\frac{dy}{dx} = 2x(3x - 8) = 0 \Rightarrow x = 0$ pt$(0{,}16)$ [1]

$\Rightarrow m = \frac{16-\left(-\frac{80}{27}\right)}{0-\frac{8}{3}} = -\frac{64}{9}$ [1]

$\Rightarrow y = -\frac{64}{9}x + c$, sub in point (0,16)

$\Rightarrow c = 16$

$y = -\frac{64}{9}x + 16$ [1]

Question 3c

$Solve\ -\frac{64}{9}x + 16 = 2x^3 - 8x^2 + 16 \Rightarrow x\left(2x^2 - 8x + \frac{64}{9}\right) = 0, \frac{4}{3}, \frac{8}{3}$[1]

$\Rightarrow$ intersect at $x = 0, \frac{4}{3}, \frac{8}{3}$ [1]

$A = \int_0^{\frac{4}{3}}(2x^3 - 8x^2 + 16)dx - \int\left(-\frac{64}{9}x + 16\right)dx + \int_{\frac{4}{3}}^{\frac{8}{3}}\left(-\frac{64}{9}x + 16\right) - (2x^3 - 8x^2 + 16)dx$[1]

A= $3.16\ units^2$[1]

Question 4a

$0.3 \times 1 + 0.4 \times 2 + 0.1 \times 3 + 4b = 1.6$ [1]

$0.3 + 0.4 + 0.1 + a + b = 1$

$\Rightarrow b = 0.05,\ a = 0.15$ [1]

Question 4b

$\int_0^{10} msin\left(\frac{\pi t}{10}\right)\delta x = 1 \Rightarrow m = \frac{\pi}{20}$[1]

Var(T)=$\int_0^{10} t^2\left(\frac{\pi}{20}\sin\left(\frac{\pi t}{10}\right)\right)\delta t - \left(\int_0^{10} t\left(\frac{\pi}{20}\sin\left(\frac{\pi t}{10}\right)\right)\delta t\right)^2 = 4.736$ [1]

SD(T)=$\sqrt{Var(T)} = 2.176$ [1]

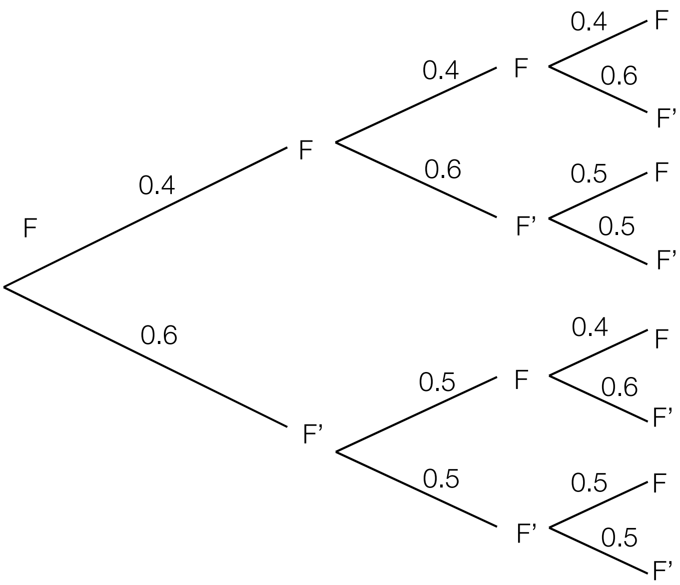
Var(T)=4.736 , SD(T)=2.176 [1]

Question 4c

$\Pr(t > 7.5) = \int_{7.5}^{10}\frac{\pi}{20}\sin(\frac{\pi t}{10})\delta x = 0.146$[1]

$\Pr(one\ fish \cap longer\ than\ 7.5\ minutes) = 0.3 \times 0.146 = 0.0438$ [1]

Question 4d



$\Pr(FFF) = (0.4)(0.4)(0.4)$

$\Pr(FFF') = (0.4)(0.4)(0.6)$

$\Pr(FF'F) = (0.4)(0.6)(0.5)$

$\Pr(FF'F') = (0.4)(0.6)(0.5)$

$\Pr(F'FF) = (0.6)(0.5)(0.4)$

$\Pr(F'FF') = (0.6)(0.5)(0.6)$

$\Pr(F'F'F) = (0.6)(0.5)(0.5)$

$\Pr(F'F'F') = (0.6)(0.5)(0.5)$

Showing all outcomes[1]

Correct assigning of probabilities [2]

Question 4e

For this question we shall use $invnorm(x)$ to denote the inverse standard normal of x

$invnorm(0.95) = \frac{6.7-\mu}{\sigma}$ and $invnorm(0.1) = \frac{1.3-\mu}{\sigma}$ [1]

$invnorm(0.95) - invnorm(0.1) = \frac{6.7-\mu}{\sigma} - \frac{1.3-\mu}{\sigma}$

$\sigma = \frac{6.7-1.3}{invnorm(0.95)-invnorm(0.1)} = 1.845\ (3DP)$ [1]

$\mu = 6.7 - \sigma \times invnorm(0.95) = 3.665\ (3DP)$ [1]

Question 4f

Pr(Freddie catches 2 fish) = 0.4 [1]

Pr(fish are of appropriate weight) = 1-(0.05+0.10) [1]

Pr(Freddie catches 2 fish of appropriate weight) = 0.4×0.85 [1]

= 0.34 [1]