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Free Exam
for 2006-15 VCE study design

Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Technology-enabled Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of marks
A	22	22	22
B	4	4	58
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory does not need to be cleared) and, if desired, one scientific calculator.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:
- This question and answer booklet of 20 pages, including a detachable formula sheet.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Section A – Multiple-choice questions

Instructions

Answer all questions by circling your choice.
Choose the response that is correct or that best answers the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Questions

Question 1

Consider the simultaneous equations containing the real constant k

$$(k+2)x + y = k$$
$$3x + ky = 1$$

The value of k for which there are infinite solutions:

A. $k \in R\backslash\{-3, 1\}$
B. $k = 3$
C. $k = 1$
D. $k = -3$
E. $k = -1$

Question 2

A number between 1 and 20 is picked; each number has an equal chance of being picked. Which of these are independent events:

A. $\{5,8,14,17,20\}, \{5,14\}$
B. $\{3,17\}, \{3,11\}$
C. $\{2,6,12,13\}, \{1,10,16\}$
D. $\{1,5,7,8\}, \{2,7,12,19,20\}$
E. $\{5,6,7\}, \{8,9,10,11\}$

Question 3

Considering log laws, the equation $log_k(l) + log_l(m) + log_m(n)$ is equal to:

A. $\frac{1}{log_l(k)} + \frac{1}{log_m(l)} + \frac{1}{log_n(m)}$
B. $-\frac{1}{log_k(l)} - \frac{1}{log_l(m)} - \frac{1}{log_m(n)}$
C. $\frac{1}{log_k(-l)} + \frac{1}{log_l(-m)} + \frac{1}{log_m(-n)}$
D. $\frac{log_l(k)}{log_k(l)} + \frac{log_m(l)}{log_l(m)} + \frac{log_n(m)}{log_m(n)}$
E. $\frac{log_e(k)}{log_e(l)} + \frac{log_e(l)}{log_e(m)} + \frac{log_e(m)}{log_e(n)}$

Question 4

The general solution to the equation $\sqrt{3}\tan(2x) = 1$ is:

A. $x = k\pi + \frac{\pi}{6}, k \in \mathbb{Z}$
B. $x = k\pi + \frac{\pi}{12}, k \in \mathbb{Z}$
C. $x = \frac{k\pi}{2} + \frac{\pi}{12}, k \in \mathbb{Z}$
D. $x = \frac{\pi}{12} + k\pi \ or \ \frac{5\pi}{12} + k\pi, k \in \mathbb{Z}$
E. $x = \frac{\pi}{12} + \frac{k\pi}{2} \ or \ \frac{13\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z}$

Question 5

The maximum domain and range of $f(x) = 3log_e(x^2 - 4)$ are:

A. Domain: $x \in (-\infty, -2) \cup (2, \infty)$, Range: $y \in R$
B. Domain: $x \in R$, Range: $y \in (-\infty, -2) \cup (2, \infty)$
C. Domain: $x \in R\backslash\{-2,2\}$, Range: $y \in R$
D. Domain: $x \in R$, Range: $y \in \backslash\{-2,2\}$
E. Domain: $x \in R\backslash\{4\}$, Range: $y \in R$

Question 6

A matrix equation equivalent to the system

$$3x + 2z = 1$$
$$-z = 9$$
$$2y - x = 5$$

is given by:

A. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 0 & 0 & -1 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \\ 5 \end{bmatrix}$
B. $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 0 & -1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
C. $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \\ 5 \end{bmatrix}$
D. $\begin{bmatrix} 3 & 2 & 0 \\ 0 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \\ 5 \end{bmatrix}$
E. $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 0 & -1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \\ 5 \end{bmatrix}$

Question 7

The third term of the equation$y = (3x + 5)^5$, when it has been expanded is:

A. $675x^3$
B. $\binom{5}{3}(3x)^3(5)^2$
C. $\binom{3}{5}(3x)^3(5)^2$
D. $\binom{5}{2}675x^3$
E. $\binom{2}{5}675x^3$

Question 8

The average value of the function $f: (1,6] \to R, f(x) = \frac{1}{x} + x^2$:

A. $\int_1^6 \frac{1}{x} + x^2 dx$
B. $\frac{1}{6-1}\int_1^6 \frac{1}{x} + x^2 dx$
C. $\frac{1}{2}\int_1^6 \frac{1}{x} + x^2 dx$
D. $\frac{f(6)-f(1)}{6-1}$
E. $\frac{f(6)+f(1)}{2}$

Question 9

The equations of the asymptotes for the $y = \frac{2x+6}{x-1}$ are:

A. $x = 1$
B. $x = 1, y = 0$
C. $x = 1, y = 2$
D. $x = 0, y = 1$
E. There are no asymptotes

Question 10

The probability that Suzie goes to both the supermarket and the butcher's on any given day is 0.3. When Suzie goes to the supermarket on a given day, the probability that she goes to the butcher's on the same day is 0.5. The probability that Suzie goes to the supermarket on any given day is:

A. 0.15
B. 0.3
C. 0.5
D. 0.6
E. 1

Question 11

Let $f(x) = x^3$. $(3.9)^3$may be approximated linearly by, and will be an overestimate or an underestimate:

A. $f(4) + 0.1f'(4)$, underestimate
B. $f(4) + 0.1f'(4)$, overestimate
C. $f(4) - 0.1f'(4)$, underestimate
D. $f(4) - 0.1f'(4)$, overestimate
E. $f(4) + 0.1f'(3.9)$, underestimate

Question 12

The function $f(x) = \frac{x^2-3}{x+2}$ has:

A. a local maximum at $(-1, -2)$ and a local minimum at $(-3, -6)$
B. a local maximum at $(-3, -6)$ and a local minimum at $(-1, -2)$
C. a local maximum at $(-3, -6)$ and point of inflection at $(-1, -2)$
D. a local maximum at $(-1, -2)$ and point of inflection at $(-3, -6)$
E. a point of inflection at $(-3, -6)$

Question 13
$\{x: \sin^2(2x) + 2\sin(2x) = 0\}$ is equal to:

A. $\{x: \sin(x) = 0\}$
B. $\{x: \sin(x) = \frac{1}{2}\}$
C. $\{x: \sin(x) = -\frac{1}{2}\}$
D. $\{x: \sin(x) = -\frac{1}{2}\} \cup \{x: \sin(x) = 0\}$
E. $\{x: \sin(x) = -\frac{1}{2}\} \cup \{x: \sin(x) = \frac{1}{2}\}$

Question 14
The equation of the tangent to the graph $f(x) = x^3 - 2x^2 - 7x + 20$, when $x = 2$ is:

A. $y + 3x = 12$
B. $y - 3x = 12$
C. $y - 6 = 3(x - 2)$
D. $y - 6 = -3x - 6$
E. $y = -3(x - 2)$

Question 15
The graph of the function $f: R \to R, f(x) = \frac{x^2-4x+5}{x-2}$ could be obtained by adding the ordinates of two graphs, g and h. The rules for g and h could be:

A. $g(x) = x + 1$ and $h(x) = \frac{x-5}{x-2}$
B. $g(x) = x - 5$ and $h(x) = \frac{x+1}{x-2}$
C. $g(x) = (x - 2)^2$ and $h(x) = \frac{1}{x-2}$
D. $g(x) = x - 4$ and $h(x) = \frac{1}{x-2}$
E. $g(x) = x - 2$ and $h(x) = \frac{1}{x-2}$

Question 16
The odds of a horse, Sooquik, winning are 9-2. Sooquik is entered into 6 races over the season, the probability that she wins exactly 4 races is:

A. $\left(\frac{2}{11}\right)^4 \left(\frac{9}{11}\right)^2$
B. $\left(\frac{2}{9}\right)^4 \left(\frac{7}{9}\right)^2$
C. $C_6^4 \left(\frac{2}{11}\right)^4 \left(\frac{9}{11}\right)^2$
D. $C_6^4 \left(\frac{2}{9}\right)^4 \left(\frac{7}{9}\right)^2$
E. $C_4^6 \left(\frac{2}{11}\right)^4 \left(\frac{9}{11}\right)^2$

Question 17

The interval [1,4] is divided into n equal subintervals by the points $x_0, x_1, \ldots, x_{n-1}, x_n$ where 1=$x_0 < x_1 < \ldots < x_{n-1} < x_n = 4$. Let $\partial x = x_i - x_{i-1}$ for $i = 1,2, \ldots n$. Then

$$\lim_{\partial x_i \to 0} \sum_{i=1}^{n} (x_i^2 + 1 \partial x)$$

Is equal to:

A. $\int_4^1 x^2 + 1 dx$
B. $\int_1^4 \frac{1}{3} x^3 + x dx$
C. $[x^2 + 1]_1^4$
D. 24
E. 66

Question 18

The smallest value of a such that the inverse of $f: [a, \infty) \to \mathbb{R}, f(x) = \frac{3}{5(x-2)^2} - 2$ exists, and the rule for f^{-1} are:

A. $a = 2 \; and \; f^{-1}(x) = 2 \pm \sqrt{\frac{3}{5(x+2)^2}}$
B. $a = 2 \; and \; f^{-1}(x) = 2 + \sqrt{\frac{3}{5(x+2)^2}}$
C. $a = -2 \; and \; f^{-1}(x) = 2 - \sqrt{\frac{3}{5(x+2)^2}}$
D. $a = -2 \; and \; f^{-1}(x) = 2 + \sqrt{\frac{3}{5(x+2)^2}}$
E. $a = -2 \; and \; f^{-1}(x) = 2 \pm \sqrt{\frac{3}{5(x+2)^2}}$

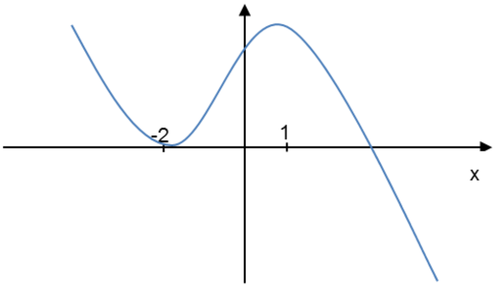
Question 19

The range of the function $f: [0, \pi] \to \mathbb{R}, f(x) = \cos(x) + x$ is:

A. $[0, \pi^2 - 1]$
B. $\left[0, \frac{\pi}{2}\right]$
C. $[1,1]$
D. $[1, \pi^2 - 1]$
E. $\left[1, \frac{\pi}{2}\right]$

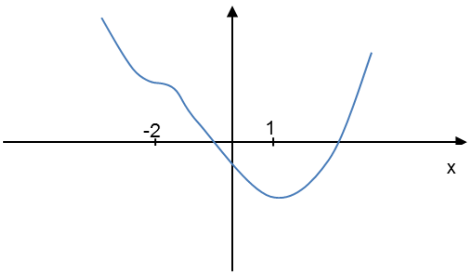
Question 20

When graph of $f'(x)$is:

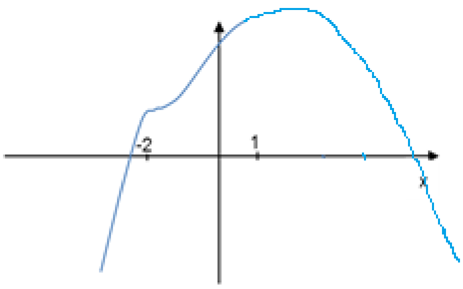


The graph of $f(x)$ could be:

A.



B.



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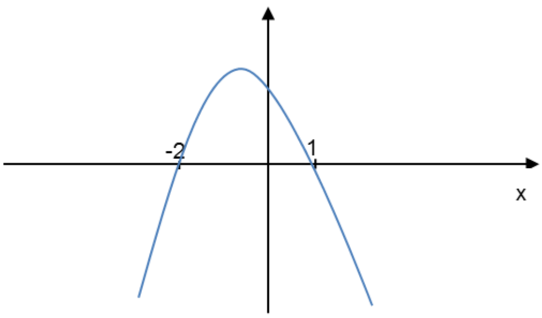

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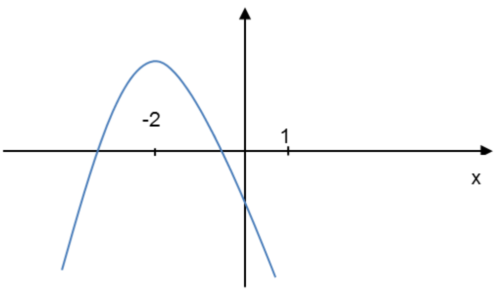
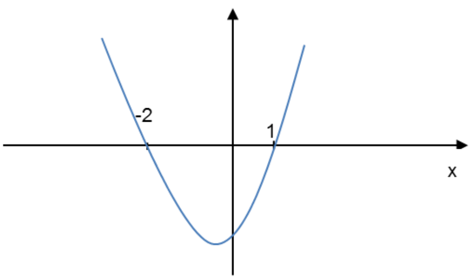


E.



Question 21

A bakery has both a sweet and savoury choice of muffin. If a customer buys a sweet muffin one day the probability that they will buy a savoury muffin the next day is 0.7, if they buy a savoury muffin one day the probability that they buy a sweet muffin the next day is 0.4. The first day the bakery sells the muffins 100 people buy sweet muffins and 25 muffins buy savoury muffins, assuming that the customers remain the same, the long term steady state is equal to:

A. $\begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}^{50} \begin{bmatrix} 25 \\ 100 \end{bmatrix}$

B. $\begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}^{50} \begin{bmatrix} 100 \\ 25 \end{bmatrix}$

C. $\begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}^{50} \begin{bmatrix} 25 \\ 100 \end{bmatrix}$

D. $\begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}^{50} \begin{bmatrix} 100 \\ 25 \end{bmatrix}$

E. $\begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}^{50} \begin{bmatrix} 25 \\ 100 \end{bmatrix}$

Question 22

The largest possible domain of $f(g(x))$, where $f(x) = \frac{1}{x+2}$ and $g(x) = \sqrt{x+1} - 2$ is:

A. $\mathbb{R}\backslash\{-1\}$

B. $(-1, \infty)$

C. $[-1, \infty)$

D. $[3, \infty)$

E. $[-1,3) \cap (3,\infty)$

Section B – Short-answer questions

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

Large Marge owns a Fast food store. She recognises that she can model the number of employees and customers in her store by the function:

$$p(t) = \begin{cases} \frac{6}{5}(t-3)^2 + 6 & 0 \le t \le 8 \\ 15\cos\left(\frac{\pi}{2}(t-8)\right) + 21 & 8 < t \le 20 \\ at + 91 & 20 < t \le 24 \end{cases}$$

Where *p* is the number of people in the store and *t* is the number of hours after 12 am.
(For all questions assume the number of employees in the store remains constant)

a. Find the value of *a* such that the function is continuous for all values of *t*.

2 marks

b. How many employees are working at any given time if it is known that there are times when there are no customers in the Large Marge's Fast food store?

2 marks

c. Sketch the graph of $p(t)$ against t on the axes below for $t \in [0,24]$. Label any stationary points.

4 marks

d. Find $p'(t)$.

4 marks

e. Large Marge finds that often they do not cook fries fast enough to keep up with the quantity being ordered. She knows that fries are ordered at a rate of 0.1 kg per person. When the time is 6 am at what rate with respect to time do they need cook fries so that they do not run out?

3 marks

Total: 15 marks

Question 2

Consider the function $f: (-2, \infty) \to R, f(x) = \frac{1}{3} log_e\left(\frac{x+2}{2}\right) + 1$

a. Show that the equation of the inverse for the function f is $f^{-1}: R \to R, f^{-1}(x) = 2e^{3(x-1)} - 2.$

2 marks

b. If it exists, find the equation for h, if $h = f[f^{-1}(x)]$ over its maximal domain.

2 marks

c. Consider the functions f and g, where $f(x) = \frac{2}{x+2} - 1$ and $g(x) = \sqrt{x+3}$. If it exists find the equation of $f[g(x)]$.

2 marks

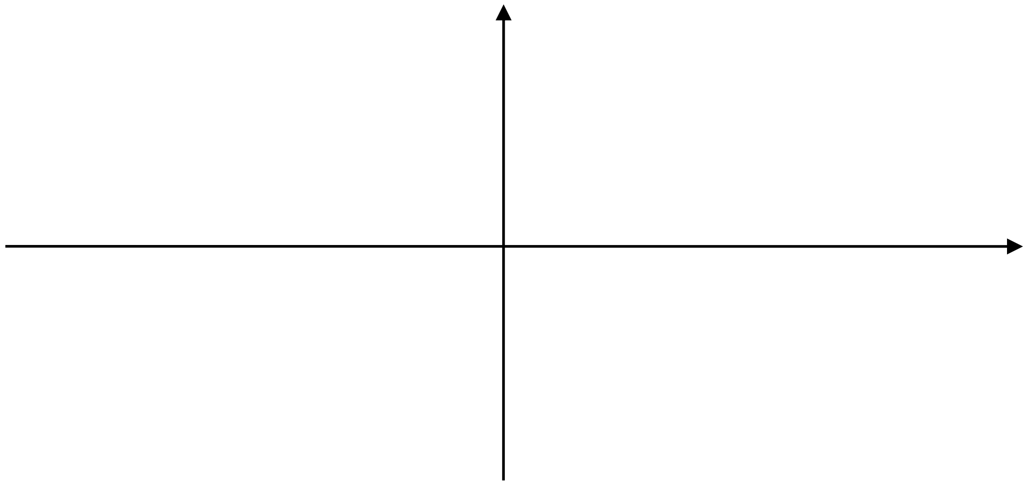
d. The graph $p(x) = \frac{2}{x+2} - 1$ under undergoes the following transformations to become the graph d(x).

- Reflection in the x-axis
- Dilation factor 3 from the y-axis
- Translation 1 unit in the negative y direction

What is the equation of $d(x)$?

3 marks

e. Sketch the graphs $p(x)$ and $d(x)$ on the same axis, labelling all axial intercepts and asymptotes.



4 marks

Total Marks: 13 marks

Question 3

A graph has the equation $y = ax^3 - bx^2 + c$. The graph intercepts the x-axis at $x = 2$, at this point the gradient is -8. The graph has a turning point at $x = \frac{8}{3}$.

a. Find the values of a, b and c using simultaneous equations.

5 marks

b. Find the equation of the line connecting the two turning points of the equation.

4 marks

c. Write down an expression for the area bounded by the curve and the line, and use your calculator to find this area correct to 2 decimal places.

4 marks

Total: 13 marks

Question 4

Freddie and Frankie go fishing every Sunday. Their families have been getting annoyed as they do not know whether to expect fish for dinner or to organise something else. Freddie and Frankie decided to record how often they catch fish, so they can give their families an indication of whether they will bring home fish.

a. Freddie found that the number of fish he caught could be displayed by the following discrete probability distribution.

x	0	1	2	3	4
Pr(X=x)	a	0.3	0.4	0.1	b

If the mean is 1.6 what are the values of *a* and *b*.

2 marks

Freddie also notices that the length of time it takes for him to get the fish in the boat once it has been hooked can be given by the probability distribution function.

$$f(t) = \begin{cases} m\sin\left(\frac{\pi t}{10}\right) & 0 \leq t \leq 10 \\ 0 & \text{elsewhere} \end{cases}$$

b. Find the value of *m* and, hence, find the variance and standard deviation, correct to 3 decimal places.

4 marks

c. What is the probability that Freddie catches one fish and it takes longer than 7.5 minutes from being hooking to getting in the boat, *given that the events are independent,* to three decimal places.

2 marks

Freddie loses interest in fishing after 3 hours, He knows if he catches a fish one week that probability that he catches a fish the following week is 0.4. If he does not catch a fish one week the probability of him catching a fish the next week is 0.5.

d. Given he catches a fish one week, draw a tree diagram to show the possible outcomes over the next three weeks assigning probabilities.

3 marks

There are restrictions on the weight of fish that Freddie and Frankie can take from the sea. To be able to take a fish from the sea it must weigh more than 1.3kg and less than 6.7kg. Statistical analysis shows that W is a normally distributed random variable, and that 10% of fish in the lake are underweight while 5% are overweight.

e. Find the mean and standard deviation of W, correct to 3 decimal places.

3 marks

The family needs two fish to make their favourite recipe, rainbow trout with salsa verde.

f. Using information from part a) and part e), calculate the probability that Freddie will catch two fish and that they will both be of an appropriate weight to take home.

4 marks

Total: 17 marks

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation $f(x+h) = f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices $S_n = T^n \times S_0$

mean $\mu = E(X)$

variance $var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

End of Booklet

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