Engage
Education
Foundation

# *Units 3 and 4 Maths Methods (CAS): Exam 1*

*Practice Exam Question and Answer Booklet*

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

| Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|
| 10 | 10 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 8 pages.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

## Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

## Questions

Question 1

a. If $y = x^3 e^{-3x}$, find $\frac{dy}{dx}$.

2 mark

b. If $y = \frac{x^2}{3x+4}$, find $\frac{dy}{dx}$ at $x = -1$.

2 marks

Total: 4 marks

## Question 2

a. The fraction $\frac{3x+2}{x+1}$ can be expressed in the form $+\frac{b}{x+1}$ . Find the values of a and b.

2 marks

b. Hence find an antiderivative of $\frac{3x+2}{x+1}$

2 marks

**Total: 4 marks**

### Question 3

Solve the equation $2\cos\left(3x + \frac{\pi}{6}\right) = -\sqrt{3}$ for $x \in [0, \pi]$.

3 marks

## Question 4

The number of red lights, X, that Bob stops at on his way to work is a random variable with probability distribution given by:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\Pr(X = x)$ | 0.2 | p | 0.6p | $p^2$ |

a. Show that p=0.4.

3 marks

b. Hence find the mean of X?

2 mark

c. What is the probability that Bob stops at less than two red lights for two days in a row?

1 marks

Total: 6 marks

### Question 5

Consider the function $f: R \to R, f(x) = 3 - e^{\frac{x-2}{3}}$

a. Find the equation for the inverse $f^{-1}(x)$

3 marks

b. State the maximal domain of $f^{-1}(x)$

1 mark

**Total: 4 marks**

**Question 6**

Find the solutions of $x^3 - 13x = -12$

3 marks

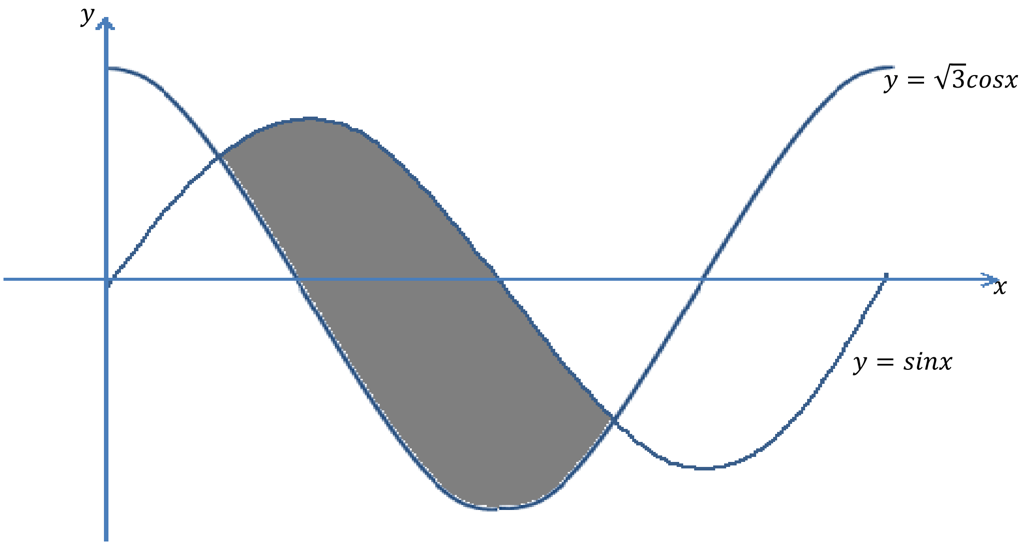
**Question 7**

Solve $\ln(x+4) - 2\ln(x+1) + \ln(x-1) = 0$,

3 marks

Question 8

Find the area enclosed by the curves $\boldsymbol{y = sinx}$ and $\boldsymbol{y = \sqrt{3}cosx}$ over one period.




5 marks

Question 9

If the line $y = -\frac{1}{3}x + c$ is a normal to the curve $y = (1 - x)^2$, show that $c = \frac{37}{12}$.

Question 4marks

Question 10

The function $\mathbf{f(x)} = \begin{cases} \boldsymbol{k\left(1 - \left|\frac{x}{6}\right|\right), \ -6 < x < 6} \\ \boldsymbol{0, \ elsewhere} \end{cases}$

is a probability density function for variable X.

a. Show that $\boldsymbol{k = \frac{1}{6}}$

.

2 marks

b. Hence find the value of q such that $\Pr(-q \leq X \leq q) = \frac{3}{4}$

2 mark

Total: 4 marks

# *Formula sheet*

## Mensuration

| | | | |
|---|---|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ | volume of a pyramid | $\frac{1}{3}Ah$ |
| curved surface area of a cylinder | $2\pi rh$ | volume of a sphere | $\frac{4}{3}\pi r^3$ |
| volume of a cylinder | $\pi r^2 h$ | area of a triangle | $\frac{1}{2}bc\sin A$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ | | |

## Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation $f(x+h) = f(x) + hf'(x)$

## Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices $S_n = T^n \times S_0$

mean $\mu = E(X)$

variance $var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

| probability distribution | | mean | variance |
|---|---|---|---|
| discrete | $\Pr(X = x) = p(x)$ | $\mu = \Sigma x p(x)$ | $\sigma^2 = \Sigma(x-\mu)^2 p(x)$ |
| continuous | $\Pr(a < X < b) = \int_a^b f(x)dx$ | $\mu = \int_{-\infty}^{\infty} xf(x)dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ |

End of Booklet

Looking for solutions? Visit www.engageeducation.org.au/practice-exams

To enrol in one of our Maths Methods lectures head to: http://engageeducation.org.au/lectures/