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Free Exam
for 2006-15 VCE study design

Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Technology-enabled Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of marks
A	22	22	22
B	4	4	58
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory does not need to be cleared) and, if desired, one scientific calculator.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:

- This question and answer booklet of 23 pages, including a detachable formula sheet.

Instructions:

- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Section A – Multiple-choice questions

Instructions

Answer all questions by circling your choice.
Choose the response that is correct or that best answers the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Questions

Question 1

If $f(x) = 3e^{(2x+3)}$ and $g(x) = sin(x)$, what is the range of the function $f(g(x))$

A. $[e, e^5]$
B. $(0, \infty)$
C. $[3e, 3e^5]$
D. $(0, 3e^5]$
E. R

Question 2

Simplifying the expression $\frac{4x^5y^3}{x^2z^4} \times \frac{3}{2}x^{-1}yz^3$ gives

A. $\frac{6y^4}{z}$
B. $6x^6y^4z^7$
C. $6x^2y^4z$
D. $\frac{6x^2y^4}{z}$
E. $\frac{3x^2y^3}{z^2}$

Question 3

$mx - (2 + \mathrm{m})y = 2\mathrm{m} - 4$
$(\mathrm{m} - 3)x + 4y = 4$

The simultaneous linear equations above will have an infinite number of solutions when m equals:

A. 6
B. $\frac{1}{2}$
C. 1
D. -1
E. -1, 6

Question 4

Solve the equation $e^{2x} - 3e^x + 2 = 0$

A. $x = \ln(2)$
B. $x = \ln(2), 0$
C. $x = e^2, e$
D. $x = 2, 1$
E. $no\ solutions$

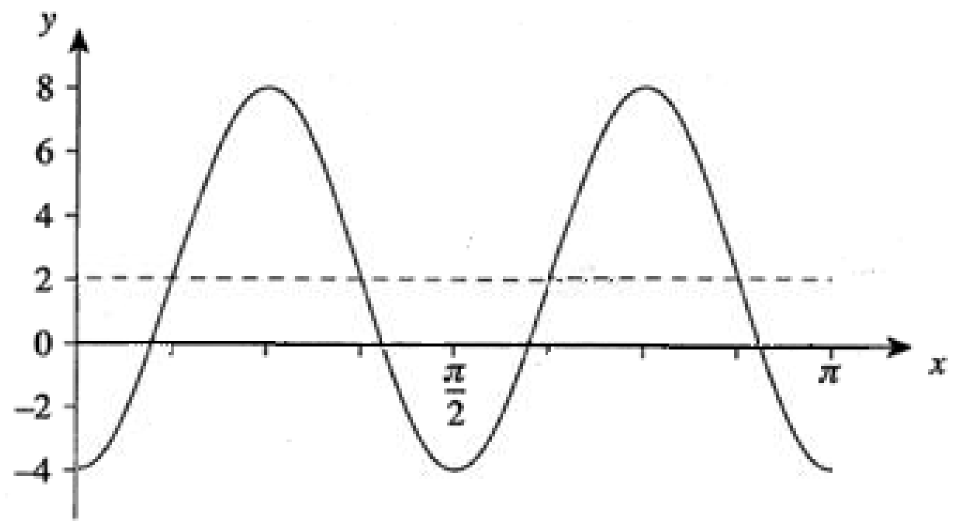
Question 5

Solve the equation $\ln(1-x) - \ln(x+2) = \ln(x+3)$

A. $x = 1$
B. $x = 1, 5$
C. $x = -1$
D. $x = -1, -5$
E. no solutions

Question 6

The following graph is in the form $y = a\cos(nx) + c$.



What are the values of a,b and c?

A. $a = 6,\ b = 4, c = 2$
B. $a = 6, b = \frac{1}{4}, c = 2$
C. $a = 3, b = 4, c = 2$
D. $a = 12, b = 2,\ c = 2$
E. $a = 3, b = 1, c = 0$

Question 7

If $\int_1^4 f(x)dx = 4$, what is the value of $\int_4^1 (2f(x) - 2x)dx$

A. 7
B. 23
C. -2
D. 22
E. 4

Question 8

Over what domain is the function $f(x) = -x^3 + \frac{3}{2}x^2 + 6x + 3$ strictly increasing?

A. [-1,2]
B. $(-\infty, -1) \cup (2, \infty)$
C. $[0,2)$
D. $(-1,2)$
E. $(-1,6)$

Question 9

The line $y = 3x + 2$ is tangent to the function $f(x) = e^{kx-6}$ +c at the point x=2. The values of k and c are:

A. $k = -\frac{1}{3}, c = 0$
B. $k = 1, c = 4$
C. $k = 3, c = 5$
D. $k = 3, c = 7$
E. $k = 3, c = 8$

Question 10

For the function$f(x) = \sqrt{x}$. An approximation for $f(1.1)$ could be

A. 6
B. $\frac{21}{20}$
C. $\frac{1}{20}$
D. $\frac{3}{2}$
E. $\frac{11}{10}$

Question 11

The function $y = -\ln(\tan(x))$ is undefined at

A. $x = \frac{\pi}{2} + \pi k, k \in Z$
B. $x = 0$
C. $x = \pi k, k \in Z$
D. $x = \frac{\pi}{2}k, k \in Z$
E. $x \leq 0$

Question 12

The function $y = x^3 - 3x^2 - 9x + k$ has exactly two x-intercepts when

A. $k = -1,3$
B. $k > 27$
C. $-5 < k < 27$
D. $k = -5,3$
E. $k = -5,27$

Question 13

The table below gives incomplete probabilities for the events A and B.

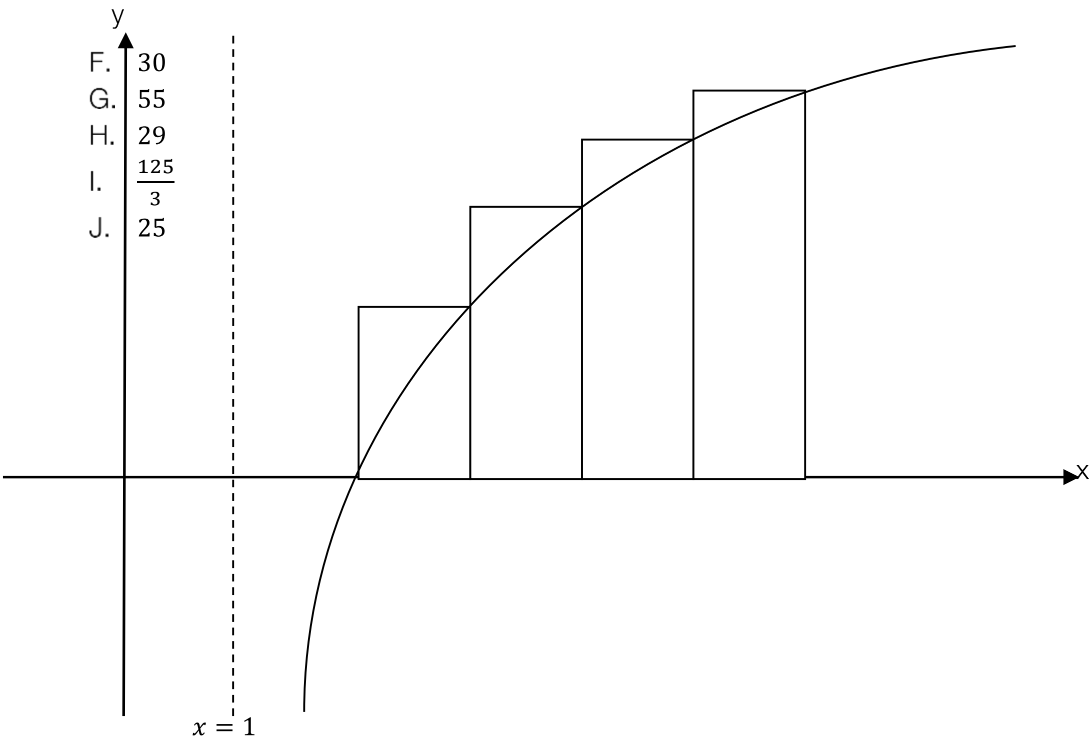
	A	A'	
B	1/8		1/5
B'		3/20	
			1

Hence, $Pr(B'|A)$ is:

A. $\frac{5}{31}$
B. $\frac{13}{20}$
C. $\frac{13}{16}$
D. $\frac{31}{40}$
E. $\frac{26}{31}$

Question 14

Find the right endpoint estimate of the area between the x-axis, the line $x = 6$ and the function $f(x) = \ln(x-1)$, using the rectangles shown below.



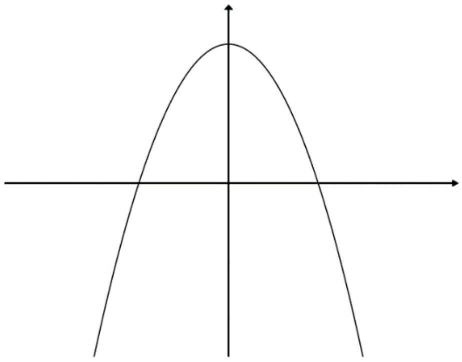
F. 30
G. 55
H. 29
I. $\frac{125}{3}$
J. 25

A. A. 2ln(30)
B. B. ln(14)
C. C. ln(24)
D. D. ln(120)
E. E. ln(720)

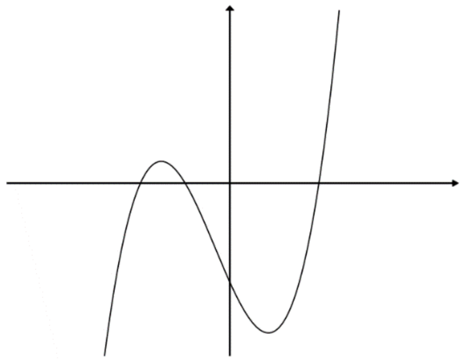
Question 15

$f'(x) = 3x^2 + 4x - 1$. $f(x)$ could be:

A.



B.



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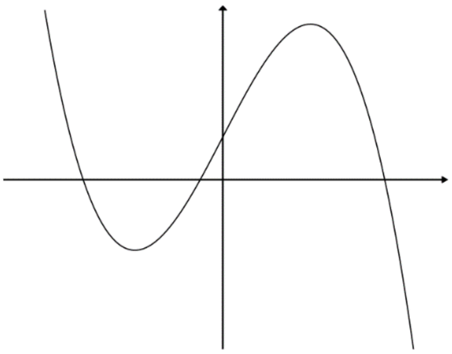

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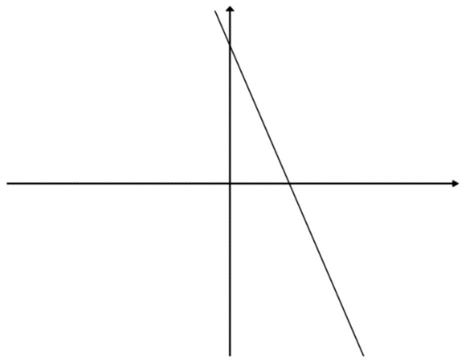

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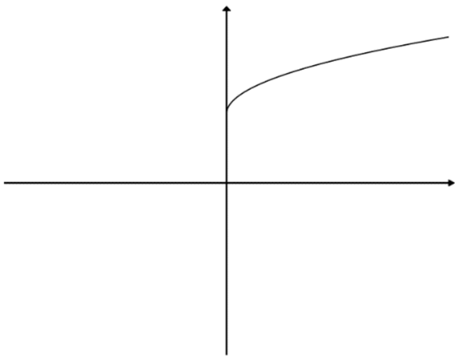


Question 16

The times at which 2000 people ran a marathon are normally distributed with mean of 3hr and 21min and a standard deviation of 11min. The probability that someone ran less than 3hr 15min given that they ran under 3hr 30min is

A. 0.5006
B. 0.2927
C. 0.7934
D. 0.5847
E. 0.3690

Question 17

A draw contains 6 red socks, 2 blue socks and 4 blue socks. What is the probability that if two socks are removed from the draw without replacement they are both the same colour?

A. $\frac{11}{36}$
B. $\frac{41}{132}$
C. $\frac{49}{144}$
D. $\frac{11}{12}$
E. $\frac{11}{48}$

Question 18

The transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ maps the function $y = (x-2)^2$ to the function

A. $y = \frac{3}{4}(x+2)^2 - 3$
B. $y = \frac{3}{4}x^2 - 3$
C. $y = 12(1-x)^2 - 3$
D. $y = \frac{3}{4}(x-6)^2 - 3$
E. $y = \frac{3}{4}(x+2)^2 + 3$

Question 19

Whether Tom rides to school or catches the bus is given by the following transition matrix

$$\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}$$

The probability that Tom will ride to school given that he rode to school yesterday is 0.8. What is the long run probability that Tom will catch the bus?

A. $\frac{5}{13}$
B. $\frac{3}{10}$
C. $\frac{2}{7}$
D. $\frac{5}{7}$
E. $\frac{1}{2}$

Question 20

Two events A and B have the probabilities, $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{3}$.

What is $\Pr(A \cap B')$ given that A and B are independent events.

A. $\frac{2}{5}$
B. $\frac{1}{5}$
C. $\frac{2}{15}$
D. $\frac{1}{3}$
E. $\frac{3}{5}$

Question 21

The variance of a variable X is $Var(X) = 2.1$ .Then $Var(2X + 3)$ is equal to

A. 1.05
B. 5.1
C. 7.2
D. 8.4
E. 11.4

Question 22

A probability density function is given by,

$$p(x) = \begin{cases} 0, & x \leq 0 \\ 3e^{-3x}, & x > 0 \end{cases}$$

The mean of X is

A. 3
B. $\frac{1}{3}$
C. $\frac{1}{9}$
D. 1
E. $\frac{2}{9}$

Section B – Short-answer questions

Instructions

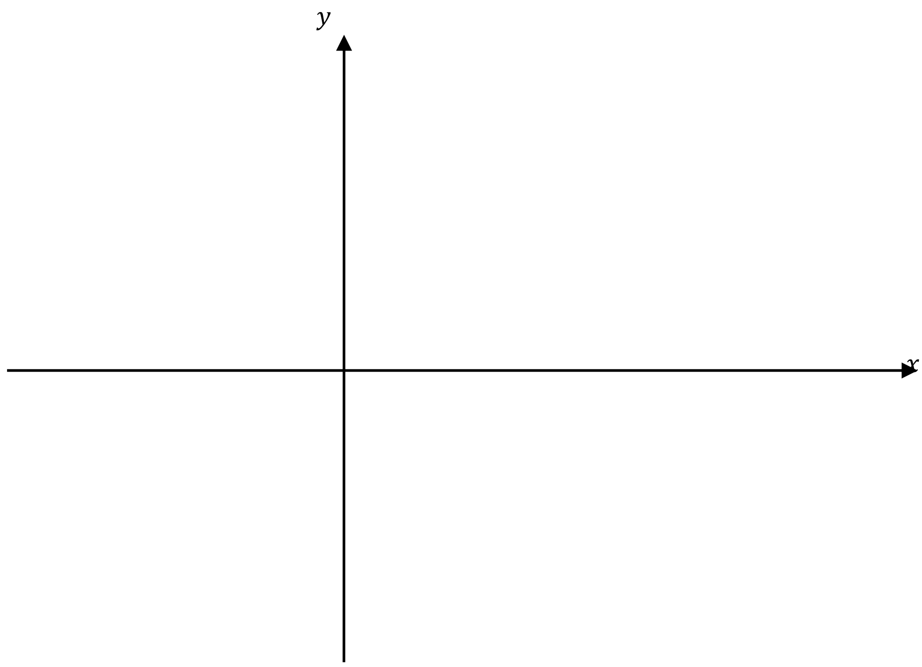
Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

$f(x) = \frac{1}{2}\ln(2 - x) - 3$

a. Sketch the graph of $f(x)$ on the axis provided, label the coordinates of all intercepts, asymptotes and stationary points.



3 marks

b. Find:

i. the range and domain of f

1 mark

ii. $f'(x)$

1 mark

c. The graph of f(x) undergoes the following transformations
- Reflection in the y-axis
- Dilation of 3 from the x-axis
- Translation of 2 units in the negative direction of the x-axis and 7 units in the positive direction of the y-axis

Find the function g(x) which is created by these transformations

3 marks

d. Consider the function $h: R \rightarrow R, h(x) = x^2 + 4x + c$

i. Find restrictions on the value of 'c' for which $f\left(h(x)\right)$ exists.

2 marks

ii. Let c = 3. Find $f(h(x))$ and state its range.

`

2 marks

iii. Explain whether $h(f(x))$ exists.

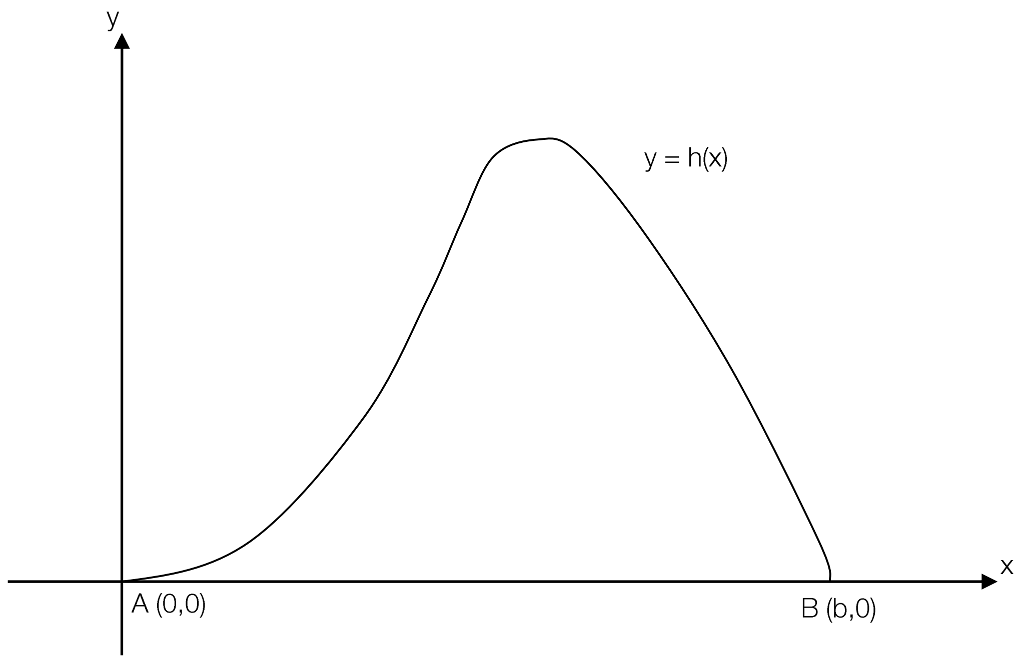
1 mark

Total: 14 marks

Question 2

William the Conqueror has just won a battle at town A and now has to cross over a mountain to reach town B where he plans to have his next battle. The height of the mountain can be given by the equation:

$$h(x) = e^{\frac{-(6-x)^2}{40}}(-3x^3 + 36x^2), \quad 0 \leq x \leq b$$



a.

i. Find the value of b, the horizontal distance from town A to town B.

1 mark

ii. State the derivative function of h(x).

1 mark

iii. Find the maximum height of the mountain between town A and town B correct to 2 decimal places, include units in your answer.

2 marks

b. William's army also has to transport catapults to town B in order to guarantee victory. The maximum gradient a catapult can be pushed up is 150 m/km.

i. Will William be able to get his catapult from town A to town B?

2 marks

ii. In order to make the climb easier for his army William decides to build a ramp up the mountain. This ramp starts at town A and has a constant rate of change of 100 m/km. What are the coordinates for the end position of the ramp correct to two decimal places?

3 marks

c. Through experience in transporting armies, William also knows that his soldiers will be too tired to fight if they have to climb above 600m.

i. Find the horizontal distances at which the height of the mountain is 600m correct to two decimal places

2 marks

ii. Write a definite integral that evaluates the area of the graph h(x) above h(x) = 600 (do not worry about the units used)

1 mark

d. Finally William decides the best way to avoid having to climb over the mountain would be to build a tunnel through it. The maximum length of tunnel that William can build is 2km. What is the minimum height at which William can build this tunnel correct to two decimal places?

2 marks

Total: 14 marks

Question 3

The residents of a town have the option to buy their groceries at one of two shops each week. They can either choose to shop at Bert's Shopping Plaza or at Wendy's Supermarket. If a customer shops at Bert's Shopping Plaza one week there is a 0.8 probability that they will shop there again next week. Whereas if a customer shops at Wendy's Supermarket there is a 0.6 probability they will shop there again next week.

a.

i. Construct a transition matrix that represents the probabilities of customers going to Bert's Shopping Plaza or Wendy's Supermarket in the next week.

2 marks

ii. Find the probability that a customer will shop at Bert's Shopping Plaza exactly two out of the next three weeks given that they shopped at Bert's Shopping Plaza this week.

2 marks

b. There is a sudden influx of residents into the town such that there are 540 people that shop for groceries each week. Initially 400 shop at Wendy's Supermarket and 140 shop at Bert's Shopping Plaza.

i. How many people are expected to shop at Wendy's Supermarket after 4 weeks?

2 marks

ii. How many people shop at Wendy's Supermarket in the long run?

1 mark

c. In this town there is also a casino where people can play a game called 'Lucky Die'. In this game a player rolls a six sided die and the number of sixes observed is recorded. The probability of rolling a six is 0.15.

i. If the die is rolled ten times what is the probability that more than 2 sixes are observed correct to 4 decimal places?

1 mark

ii. How many times would you need to roll the die if the probability of getting at least two sixes is more than 0.9?

1 mark

iii. What is the mean and variance of X if the die is rolled 20 times?

2 marks

Total: 12 marks

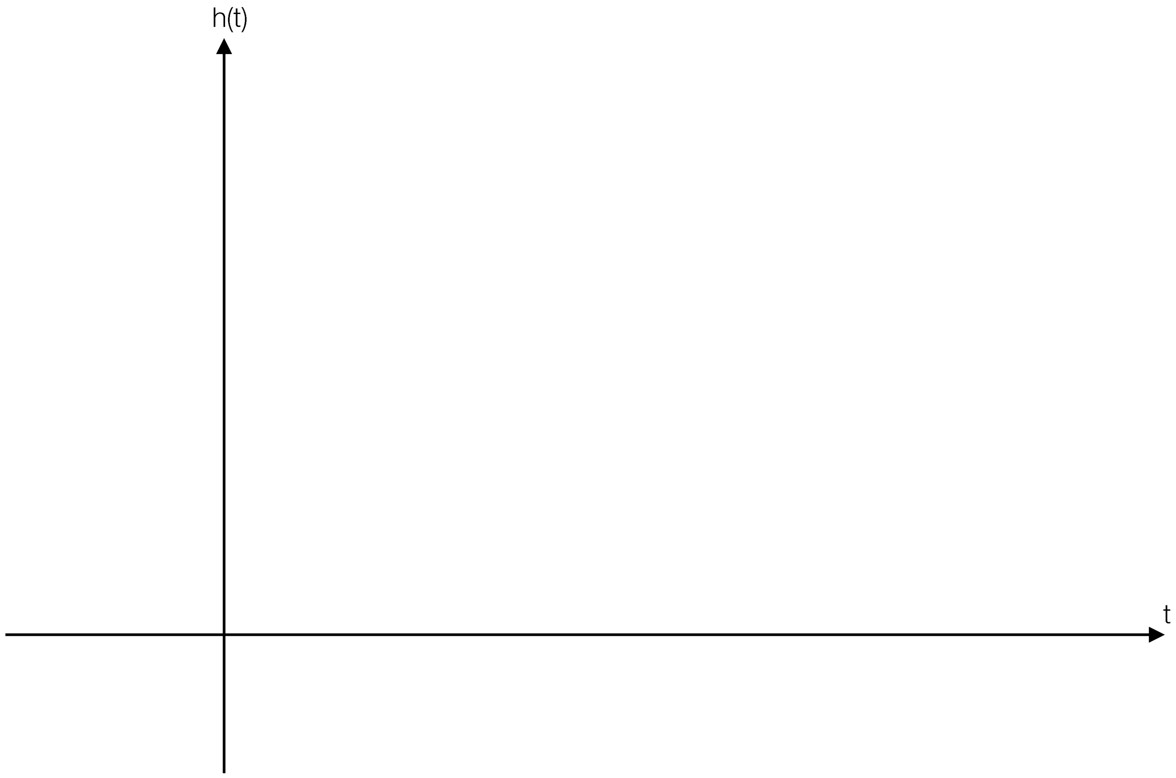
Question 4

Matt's favourite adventure park is Water World where he visits every Sunday. One of the rides at Water World, called Wave Rider, was deigned to mimic the feeling of floating up and down on a very large wave. The ride begins from a platform one metre above the ground. The height of a person, in metres, on this ride is given by the equation:

$$h(t) = 7 - 6\cos\left(\frac{\pi t}{9}\right) \quad 0 \leq t \leq 36$$

Where t is measured in seconds.

a. Draw a graph of h(t) on the axis below.



3 marks

b.

i. State the period of each wave.

____

____

1 mark

ii. State the maximum height reached and the average height of the ride.

1 mark

c.

i. Find the times at which the height was 11m correct to two decimal places.

1 mark

iii. Find the proportion of time spent above 11m during the ride to two decimal places.

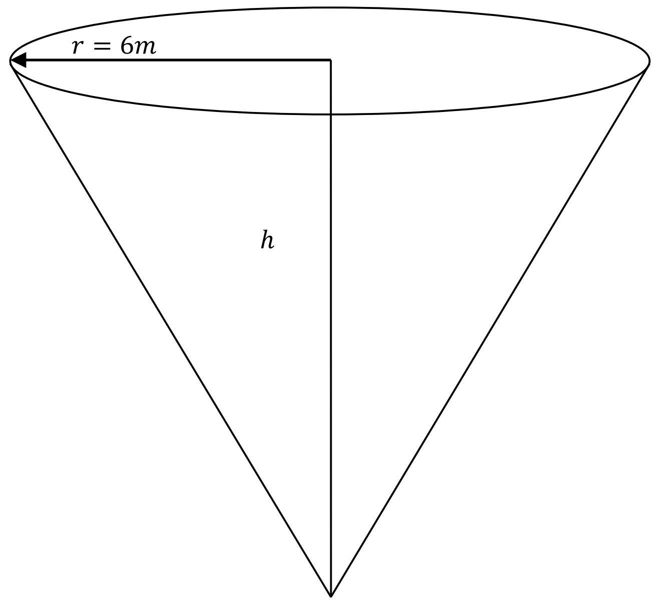
2 marks

Due to complaints about the ride being too boring the adventure park decided to change the ride so that the period of each wave was now 12 seconds. In addition they also increased the maximum height of the waves to 19m (the minimum height is still 1m where the passengers begin the ride).

d. Assuming that the ride still follows a cosine rule describe a set of transformations that describe the changes made to the ride.

2 marks

e. When Matt last visited Water World hi favourite ride, the Whirl Pool, was closed because it was being drained. The Whirl Pool consists of an upside down cone with radius of 6m.



The cone is being drained of water at a rate of $0.5\ m^3/second$

i. If the initial volume of water in the cone is 500 m^3, find the initial height of the water in exact terms if the initial radius is 6m.

1 mark

ii. Write an equation for the volume in the cone in terms of time t. State the amount of time taken to fully drain the cone in exact terms.

2 marks

iii. Find the exact rate of change of the height with respect to time when the radius is 3m.

3 marks

Total: 18 marks

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation $f(x+h) = f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices $S_n = T^n \times S_0$

mean $\mu = E(X)$

variance $var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

End of Booklet

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