THE HEFFERNAN GROUP

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MATHS METHODS 3 & 4 TRIAL EXAMINATION 1 SOLUTIONS 2006

Question 1

a. $0 \cdot 27 + 0 \cdot 13 + 0 \cdot 19 + 0 \cdot 34 + a = 1$

$a = 0 \cdot 07$

(1 mark)

b. $\Pr(X < 3) = \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)$

$= 0 \cdot 27 + 0 \cdot 13 + 0 \cdot 19$

$= 0 \cdot 59$

(1 mark)

c. mean of $X = E(X)$

$= 0 \times 0 \cdot 27 + 1 \times 0 \cdot 13 + 2 \times 0 \cdot 19 + 3 \times 0 \cdot 34 + 4 \times 0 \cdot 07$

$= 1 \cdot 81$

(1 mark)

If an incorrect value resulting from part **a**. is used and the resulting answer is correct the mark should be awarded.

d. The median is 2.
The median is the middle score. Half the scores are above it and half below.

Now, $\Pr(X \le 0) = 0 \cdot 27$

$\Pr(X \le 1) = 0 \cdot 27 + 0 \cdot 13$

$= 0 \cdot 4$

$\Pr(X \le 2) = 0 \cdot 27 + 0 \cdot 13 + 0 \cdot 19$

$= 0 \cdot 59$

So the median of X is 2 since half the scores are below it.

(1 mark)

Question 2

a. $\frac{d}{dx}\left(e^{2x}\tan(2x)\right)$

$= e^{2x} \times 2\sec^2(2x) + 2e^{2x}\tan(2x)$ **(1 mark)**

$= 2e^{2x}\left(\sec^2(2x) + \tan(2x)\right)$

(1 mark)

b. $\int (2x+3)^5 \, dx = \frac{1}{2(5+1)}(2x+3)^6 + c$

$= \frac{1}{12}(2x+3)^6 + c$

(1 mark)

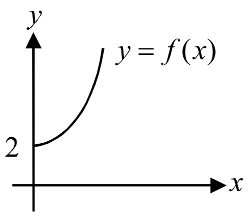
Question 3

a. $g(f(x))$ exists iff $r_f \subseteq d_g$

Now, $d_f = [0, \infty)$.

A quick sketch enables you to find r_f.

$r_f = [2, \infty)$ **(1 mark)**



Now, $d_g = R^+$

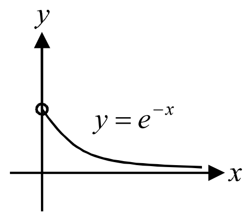
so, $[2, \infty) \subseteq R^+$

so $g(f(x))$ exists

(1 mark)

b. g^{-1} exists iff the graph of $y = g(x)$ is $1:1$.

Again, a quick sketch tells us whether g is a 1:1 function.



Since any horizontal line can be drawn on the graph so that it crosses the function only once, it is a 1:1 function.
So g^{-1} exists.

(1 mark)

c. Let $y = x^2 + 2$

Swap x and y $\quad x = y^2 + 2$

Rearrange $\quad y^2 = x - 2$

$y = \pm\sqrt{x-2}$ **(1 mark)**

Since $d_f = [0, \infty)$ and $r_f = [2, \infty)$ from part **a.**,

then $d_{f^{-1}} = [2, \infty)$ and $r_{f^{-1}} = [0, \infty)$

So the rule for f^{-1} is $y = \sqrt{x-2}$.

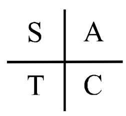
Note that we reject $y = -\sqrt{x+2}$ because the range of f^{-1} contains only positive values. **(1 mark)**

Also $d_{f^{-1}} = [2, \infty)$. **(1 mark)**

Question 4

$2\sin(2x) = -\sqrt{3}$ $\qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin(2x) = -\frac{\sqrt{3}}{2}$ $\qquad$ so $2x \in [-\pi, \pi]$ **(1 mark)**



$2x = -\frac{2\pi}{3}, -\frac{\pi}{3}$ **(1 mark)**

$x = -\frac{\pi}{3}, -\frac{\pi}{6}$

(1 mark)

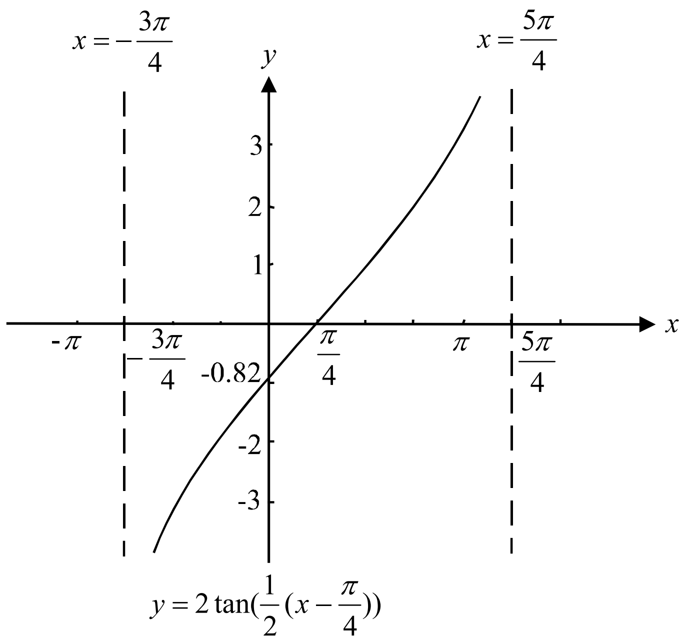
Question 5

a. $f(x) = 2\tan\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right)$

$$\begin{aligned}\text{period} &= \pi \div \frac{1}{2}\\ &= 2\pi\end{aligned}$$

(1 mark)

b.



(1 mark)

The period is 2π and the graph is translated $\frac{\pi}{4}$ units right.

The asymptotes therefore occur at $x = -\frac{3\pi}{4}$ and $x = \frac{5\pi}{4}$. **(1 mark)**

The x-intercept occurs at $\left(\frac{\pi}{4}, 0\right)$.

The y-intercept occurs when

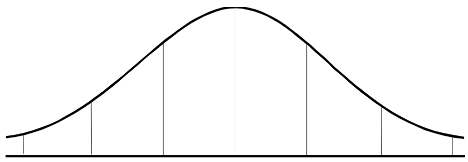
$$\begin{aligned}y &= 2\tan\left(\frac{1}{2}\left(0 - \frac{\pi}{4}\right)\right)\\ &= 2\tan\left(-\frac{\pi}{8}\right)\\ &= 2 \times -0 \cdot 41 \quad \text{(given in question)}\\ &= -0 \cdot 82\end{aligned}$$

The y-intercept occurs at $(0, -0 \cdot 82)$.

(1 mark) for intercepts

Question 6

Sketch a quick diagram.



a. The normal curve is symmetrical around the mean.

So $\Pr(X > (16+6)) = \Pr(X < (16-6))$

$\Pr(X > 22) = \Pr(X < 10)$

So $a = 10$

(1 mark)

b. Method 1

$\Pr(X < 7) = \Pr(Z > b)$

$\Pr(X < 7) = \Pr(X < (16-9))$

Since $\sigma = 4$, 9 below the mean represents $\frac{9}{4} = 2 \cdot 25$ standard deviations below the mean.

$\Pr(X < 7) = \Pr(Z < -2 \cdot 25)$

$= \Pr(Z > 2 \cdot 25)$

So $b = 2 \cdot 25$

(1 mark)

Method 2

$Z = \frac{X - \mu}{\sigma}$

$= \frac{7 - 16}{4}$

$= -2.25$

$\Pr(X < 7) = \Pr(Z < -2 \cdot 25)$

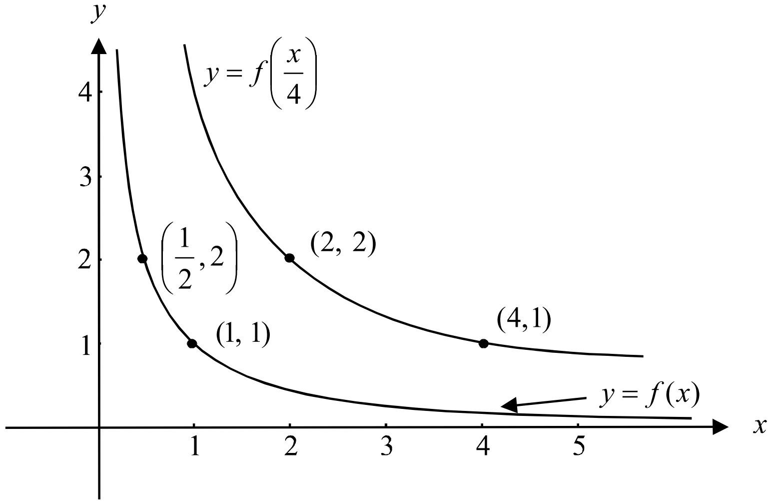
$= \Pr(Z > 2 \cdot 25)$

So $b = 2 \cdot 25$

(1 mark)

Question 7

a.



(1 mark) (correct shape) **(1 mark)** (showing coordinates of 2 correct points)

b. The graph of $y = f(x)$ has been dilated by a factor of 4 parallel to the x-axis or in the x direction or away from the y –axis. **(1 mark)**

c. $y = f\left(\frac{x}{4}\right)$

$= 1 \div \frac{x}{4}$

$y = \frac{4}{x}$ **(1 mark)**

Question 8

$y = -x^2 + 5x$

when $x = 1$

$y = -1 + 5$

$= 4$

Point of tangency is $(1, 4)$. **(1 mark)**

$\frac{dy}{dx} = -2x + 5$

At $x = 1$, $\frac{dy}{dx} = 3$ **(1 mark)**

The equation of the tangent at $(1,4)$ is

$y - 4 = 3(x - 1)$

$y = 3x - 3 + 4$ **(1 mark)**

$y = 3x + 1$

This tangent crosses the x-axis when $y = 0$

$0 = 3x + 1$

$x = -\frac{1}{3}$

So $a = -\frac{1}{3}$ **(1 mark)**

Question 9

a. The function f crosses the x-axis at $x = a$ so one linear factor is $(x - a)$.
The function f touches the x-axis at $x = c$ so there is a repeated linear factor of $(x - c)$.
The three linear factors are $x - a, x - c, x - c$.

(1 mark)

b. $f(x) > 0$ for $x \in (a, c) \cup (c, \infty)$ or $(a, \infty) \setminus \{c\}$

(1 mark)

c. $f'(x) < 0$ for $x \in (0, c)$

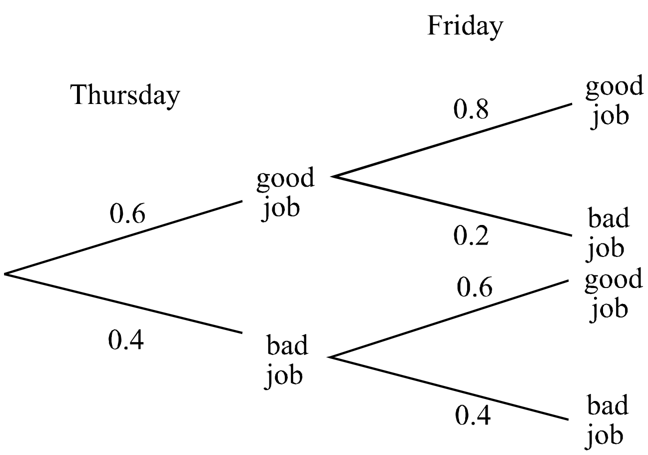
(1 mark)

Question 10

a. The probability that a team does a bad job when the previous day's team did a good job is $1 - 0 \cdot 8 = 0 \cdot 2$.

(1 mark)

b. This is a multi-stage event; that is, it involves what happens on Thursday and then on Friday.
A tree diagram is useful.



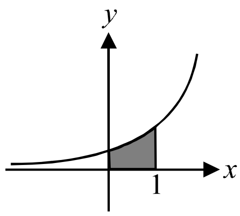
(1 mark)

$\Pr(\text{Friday's team does a good job})$
$= \Pr(\text{good job Thursday and good job Friday}) + \Pr(\text{bad job Thursday and good job Friday})$
$= 0 \cdot 6 \times 0 \cdot 8 + 0 \cdot 4 \times 0 \cdot 6$ **(1 mark)**
$= 0 \cdot 48 + 0 \cdot 24$
$= 0 \cdot 72$

(1 mark)

Question 11

Sketch the graph.



Area $= \int_0^1 e^{\frac{x}{2}}\, dx$ **(1 mark)**

$= \left[2e^{\frac{x}{2}}\right]_0^1$ **(1 mark)**

$= 2e^{\frac{1}{2}} - 2e^0$

$= 2\sqrt{e} - 2$

$= 2\left(\sqrt{e} - 1\right)$ square units

(1 mark)

Total 40 marks