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MATHEMATICAL METHODS UNITS 3 & 4

WRITTEN TRIAL EXAMINATION 2

2006

Reading Time: 15 minutes
Writing time: 2 hours

Instructions to students

This exam consists of Section 1 and Section 2.
Section 1 consists of 22 multiple-choice questions, which should be answered on the detachable answer sheet that can be found on page 25 of this exam.
Section 2 consists of 4 extended-answer questions that should be answered in the spaces provided.
Section 1 begins on page 2 of this exam and is worth 22 marks.
Section 2 begins on page 12 of this exam and is worth 58 marks.
There is a total of 80 marks available.
All questions in Section 1 and Section 2 should be answered.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
Where more than one mark is allocated to a question, appropriate working must be shown.
Students may bring one bound reference into the exam.
Students may bring an approved graphics or CAS calculator into the exam.
A formula sheet can be found on pages 24 of this exam.

SECTION 1

Question 1

The first three positive solutions of the equation $\sqrt{3}\tan(2x)=1$ are

A. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$

B. $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}$

C. $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}$

D. $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$

E. $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$

Question 2

$3\log_5(2)+\log_5(7)$ is closest to

A. 0.08
B. 0.22
C. 1.39
D. 2.32
E. 2.5

Question 3

The solution/s to the equation $2e^{2x}=e^x$ is/are

A. $-\log_e(2)$

B. 0

C. $0, -\log_e(2)$

D. $1, \log_e\left(\frac{1}{2}\right)$

E. $2e^2$

Question 4

Find the real values of x for which $|2x-1|<5$.

A. $x<3$
B. $x>-2$
C. $x<-2$ or $x>3$
D. $x>-2$ or $x>3$
E. $-2<x<3$

Question 5

The complete set of linear factors of the polynomial $x^4+4x^3-7x^2-10x$ are

A. $x-1, x+1, x+10$
B. $x-2, x+1, x+5$
C. $x, x-2, x+1, x+5$
D. $x, x-5, x+1, x+2$
E. $x, x-1, x+1, x+10$

Question 6

The inverse function of the function $f:(-1,\infty)\rightarrow R,\ f(x)=\log_e(x+1)-2$ is given by

A. $f^{-1}:(-1,\infty)\rightarrow R,\ f^{-1}(x)=\log_e(y+1)-2$
B. $f^{-1}:R\rightarrow R,\ f^{-1}(x)=e^{x-1}+2$
C. $f^{-1}:(-4,\infty)\rightarrow R,\ f^{-1}(x)=e^{x+2}-1$
D. $f^{-1}:(-1,\infty)\rightarrow R,\ f^{-1}(x)=e^{x+2}-1$
E. $f^{-1}:R\rightarrow R,\ f^{-1}(x)=e^{x+2}-1$

Question 7

Consider the functions

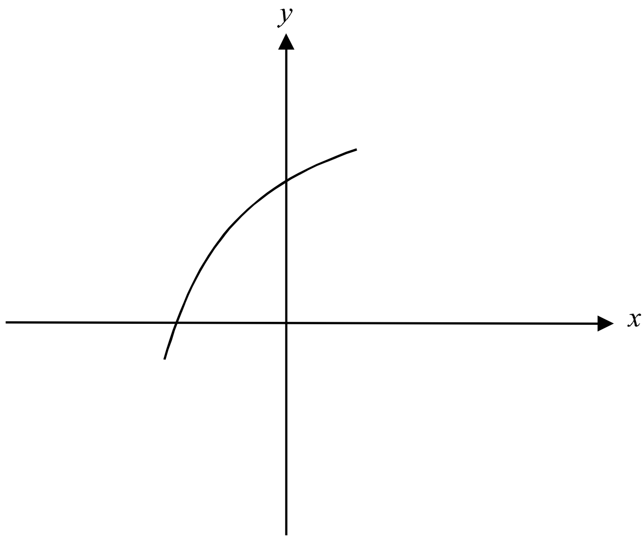
$$f:(0,\infty)\rightarrow R,\ f(x)=\frac{1}{x^2} \quad \text{and} \quad g:(a,\infty)\rightarrow R,\ g(x)=x+1$$

The least value of a such that the composite function $f(g(x))$ exists, is

A. $-\infty$
B. -1
C. 0
D. 1
E. 2

Question 8

Just part of the graph of a function is shown below.



Given that a, b, c and d are all constants, which one of the following rules could definitely **not** model that part of the function shown in the graph above?

A. $y = a\sin(b(x+c)) + d$

B. $y = ax^3 + bx^2 + cx + d$

C. $y = \log_e(x+a) + b$

D. $y = \dfrac{a}{x^2}$

E. $y = a(x-b)^2 + c$

Question 9

The graph of the function f with the rule $f(x) = \sin(x)$ is transformed to become the graph of the function g where the rule of g is given by $g(x) = 2\sin(x+\pi)$. The transformations that have taken place are

A. a dilation by a scale factor of $\frac{1}{2}$ from the x-axis and a translation of π units left.

B. a dilation by a scale factor of $\frac{1}{2}$ from the x-axis and a translation of π units right.

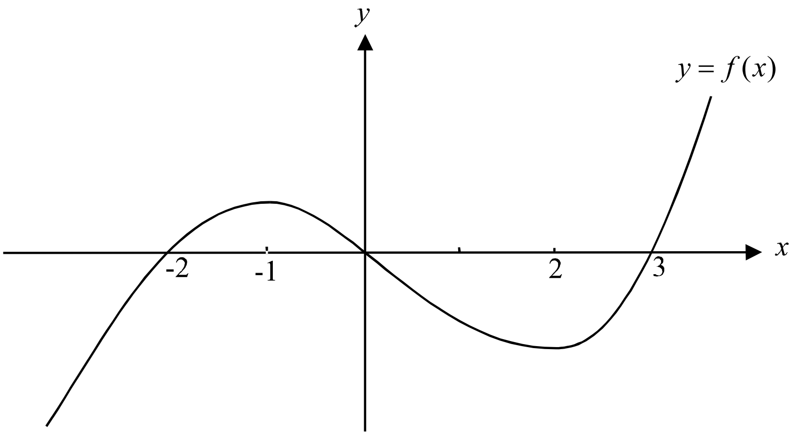
C. a dilation by a scale factor of 2 from the x-axis and a translation of π units left.

D. a dilation by a scale factor of $\frac{1}{2}$ from the y-axis and a translation of π units left.

E. a dilation by a scale factor of 2 from the y-axis and a translation of π units right.

Question 10

The graph of the function *f* is shown below. Stationary points occur at $x = -1$ and $x = 2$.



The rate of change of *f* with respect to *x* is positive for

A. $x \in (-\infty, 1)$
B. $x \in (-1, 2)$
C. $x \in (2, \infty)$
D. $x \in (-2, 0) \cup (3, \infty)$
E. $x \in (-\infty, -1) \cup (2, \infty)$

Question 11

The derivative of $\sin(2x^2 + 5x)$ is

A. $-(4x+5)\cos(2x^2+5x)$
B. $(4x+5)\cos(2x^2+5x)$
C. $-(2x^3+5x)\cos(2x^2+5x)$
D. $(2x^3+5x)\cos(2x^2+5x)$
E. $\dfrac{\cos(2x^2+5x)}{3x^3+5x^2}$

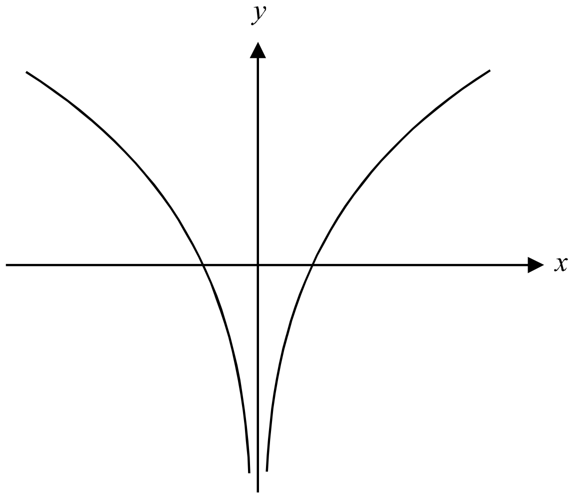
Question 12

The gradient function of the function $f(x) = e^{\sin(2x)}$ is given by

A. $-e^{2\cos(2x)}$
B. $e^{2\cos(2x)}$
C. $-2e^{\sin(2x)}\cos(2x)$
D. $-e^{\sin(2x)}\cos(2x)$
E. $2e^{\sin(2x)}\cos(2x)$

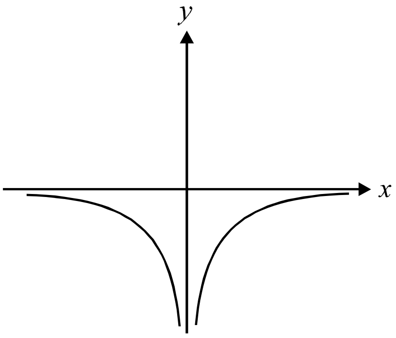
Question 13

The graph of the function $f : R \setminus \{0\} \rightarrow R$, with the rule $y = f(x)$ is shown below.

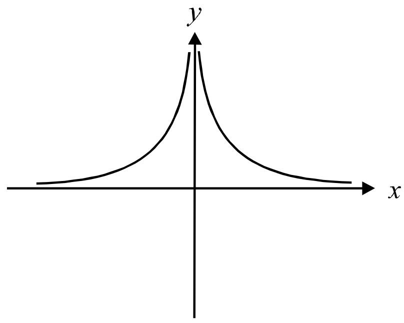


Which one of the following graphs could be the graph of $y = f'(x)$?

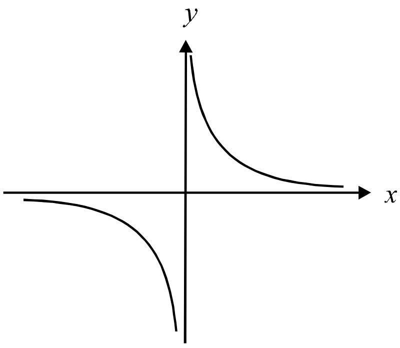
A.



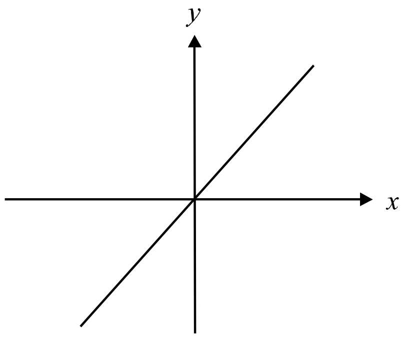
B.



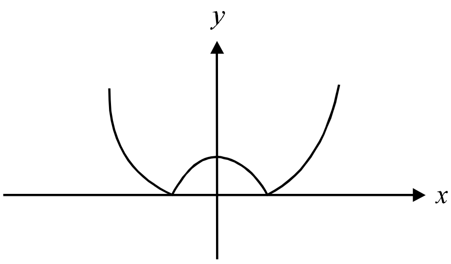
C.



D.



E.



Question 14

A circular puddle is expanding. Its radius is increasing at the rate of 0.5cm/s . The rate at which the area of the puddle is increasing when the radius is 10 cm is

A. $0\cdot 25\pi\ \text{cm}^2/\text{s}$

B. $\pi\ \text{cm}^2/\text{s}$

C. $10\pi\ \text{cm}^2/\text{s}$

D. $20\pi\ \text{cm}^2/\text{s}$

E. $100\pi\ \text{cm}^2/\text{s}$

Question 15

A continuous function, *f*, has the following properties:

$$f'(x) = 0 \text{ at } x = -3 \text{ and } x = 1$$
$$f'(x) < 0 \text{ for } x > 1$$
$$f'(x) > 0 \text{ for } x < -3 \text{ and } -3 < x < 1$$

Which one of the following is true?

A. The graph of $y = f(x)$ has a local minimum at $x = -3$.

B. The graph of $y = f(x)$ has a local maximum at $x = -3$.

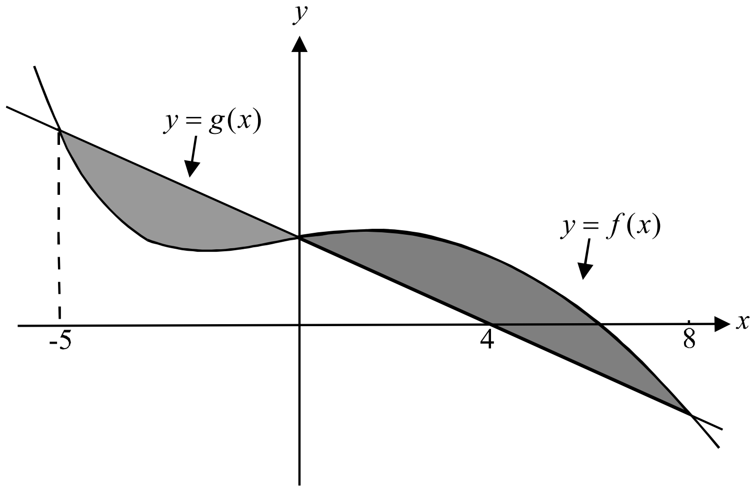
C. The graph of $y = f(x)$ has a local minimum at $x = 1$.

D. The graph of $y = f(x)$ has a stationary point of inflection at $x = -3$.

E. The graph of $y = f(x)$ has a stationary point of inflection at $x = 1$.

Question 16

The graphs of $y = g(x)$ and $y = f(x)$ are shown below.



The shaded region between these two graphs has an area in square units of

A. $\int_{-5}^{8} (f(x) - g(x))\, dx$

B. $\int_{-5}^{8} (g(x) - f(x))\, dx$

C. $\int_{-5}^{0} (g(x) - f(x))\, dx + \int_{0}^{8} (g(x) - f(x))dx$

D. $\int_{-5}^{0} (g(x) - f(x))\, dx + \int_{0}^{8} (f(x) - g(x))dx$

E. $\int_{-5}^{0} (g(x) - f(x))\, dx + \int_{0}^{4} (f(x) - g(x))dx - \int_{4}^{8} (f(x) - g(x))dx$

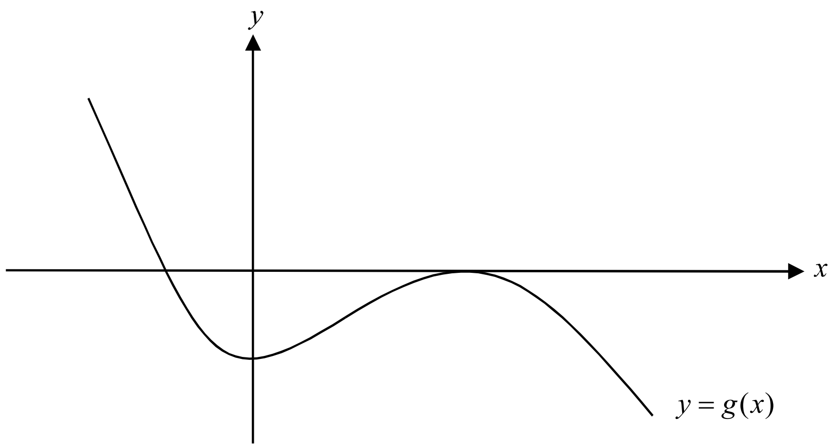
Question 17

The area enclosed by the graph $y = \log_e(x-1)$ the x-axis and the line $x = 5$ is approximated using the method of right rectangles with rectangles of width 1 unit.
The approximation in square units is

A. $\log_e(6)$
B. $\log_e(24)$
C. $\log_e(36)$
D. $\log_e(60)$
E. $\log_e(64)$

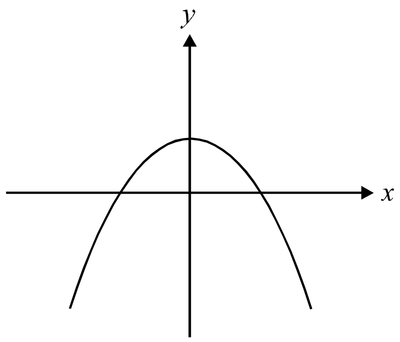
Question 18

The graph of the function g with rule $y = g(x)$ is shown below.

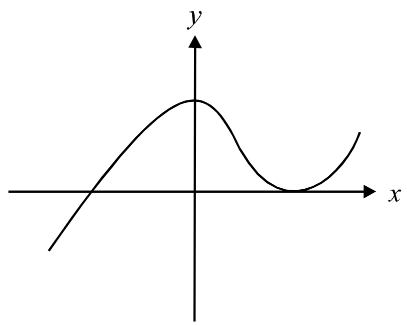


Which one of the following could be the graph of an antiderivative function of g?

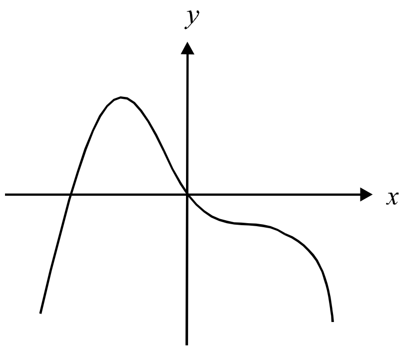
A.



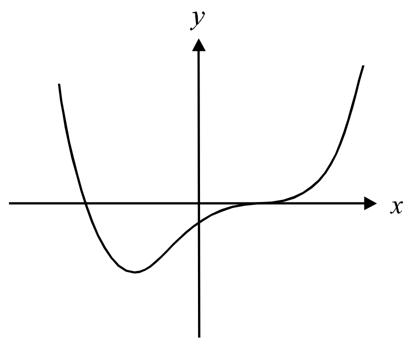
B.



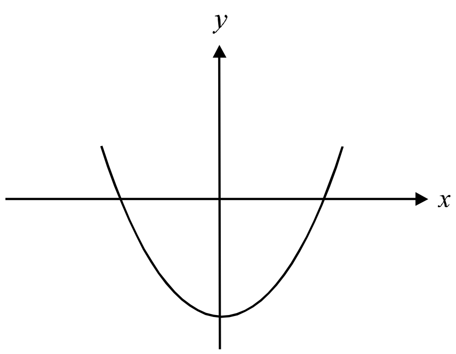
C.



D.



E.



Question 19

A random variable X has a normal distribution with a mean of 7 and a standard deviation of 1.5. The probability that X is greater than 3 is closest to

A. 0.0038
B. 0.7146
C. 0.8623
D. 0.9962
E. 0.9987

Question 20

The continuous variable X has a probability density function given by

$$f(x) = \begin{cases} e^{2x} \text{ if } 0 < x < \log_e\left(\sqrt{3}\right) \\ 0 \text{ for elsewhere} \end{cases}$$

$\Pr(X < 0 \cdot 2)$ is closest to

A. 0.2459
B. 0.3746
C. 0.4918
D. 0.6107
E. 0.7459

Question 21

In a game, Larry has a probability of x of landing a ball in a bucket. Each attempt to land the ball in the bucket is independent of any other attempt. The probability that he lands the ball in the bucket just once out of 3 attempts is given by the expression

A. $x^2 - x^3$
B. $3x^2 - 3x^3$
C. $x^3 - ax^2 + x$
D. $3x^3 - 6x^2 + 3x$
E. $-3x^4 + 9x^3 - 9x^2 + 3x$

Question 22

Claire throws a fair coin twice. Let X represent the number of heads that are thrown by Claire.
The variance of X is closest to

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. 1

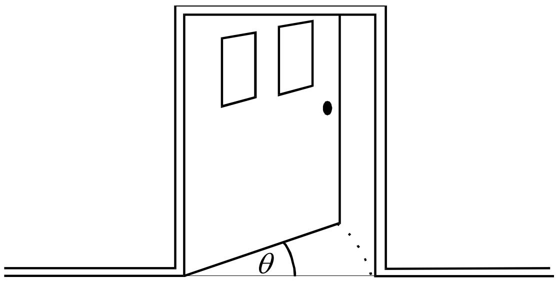
E. $1\frac{1}{2}$

SECTION 2

Answer all questions in this section.

Question 1

A door is in constant use and is left ajar after each use. The angle that is created when the door is left ajar is θ; where θ is a variable, and is indicated in the diagram below.



The angle that the door is left ajar on one occasion is independent of the angle that it is left ajar on another occasion.
The angle θ is a random variable with a probability density function given by

$$f(\theta)=\begin{cases}\sin(\theta) \text{ if } 0 \le \theta \le a \\ 0 \text{ otherwise}\end{cases}$$

a. Show that $a=\dfrac{\pi}{2}$.

2 marks

b. Show that the probability that the door is left ajar at an angle of less than $\dfrac{\pi}{4}$, is $0 \cdot 293$ (correct to 3 decimal places).

2 marks

c. What is the probability that the door is left ajar at an angle of less than $\frac{\pi}{4}$ on the next two occasions? Express your answer correct to 3 decimal places.

1 mark

d. What is the probability that the door is left ajar at an angle of less than $\frac{\pi}{4}$ on at least 4 out of the next 5 occasions it is used. Express your answer correct to 3 decimal places.

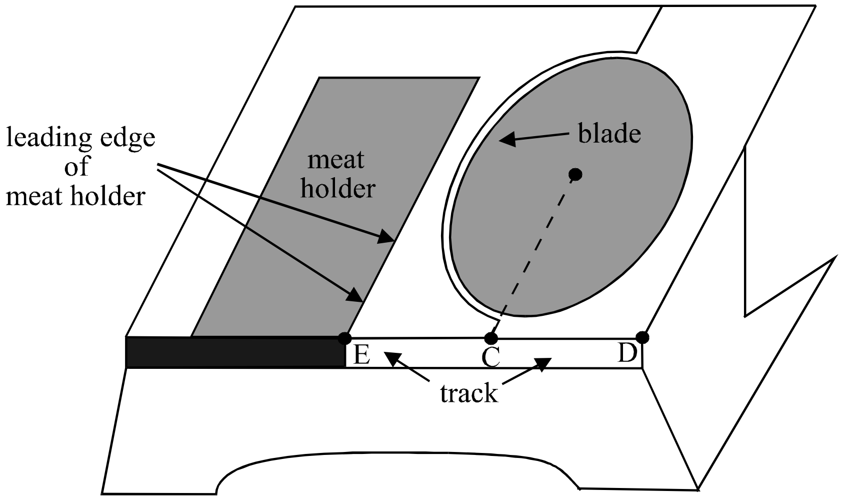
2 marks

e. What is the median value of θ?

4 marks
Total 11 marks

Question 2

A meat slicer that is used in a butchers shop is shown on the diagram below.



The main parts of the meat slicer are the blade, the meat holder and the track along which the meat holder runs indicated on the diagram as *ED*. The point *C* is in line with the centre of the circular blade and is halfway between points *E* and *D*. The point *E* lies on the track along which the meat holder slides and is in line with the leading edge of the meat holder when it is as far left as it can slide (the position shown in the diagram above).
The meat holder slides backwards and forwards along the track that runs from point *E* to point *D*. With every forward movement a slice of meat is cut by the rotating blade and falls to the underside of the machine. The meat slicer can be put on automatic so that the meat holder moves backwards and forwards continuously.
When on automatic, the position, p, of the leading edge of the meat holder along the track at time t seconds after the slicer is switched on, is given by

$$p(t) = -20\cos\left(\frac{2\pi t}{3}\right) \quad t \geq 0$$

where positive values of p indicate that the leading edge of the meat slicer is to the right of point *C* and negative values indicate that it is to the left of point *C*. The unit of measurement is the centimetre.
The meat slicer is switched on to automatic in the butchers shop at 10am on Tuesday.

a. What is the length of the track?

1 mark

b. According to the rule for $p(t)$, confirm whether or not the meat holder starts at the left end of the track.

1 mark

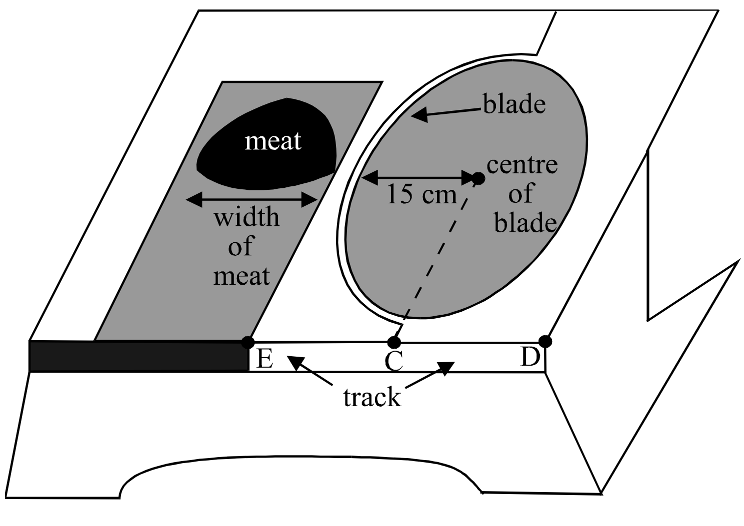
c. How long does it take for one complete forward and one complete backward movement of the meat holder?

1 mark

d. How many slices of meat have been cut by 10.15am on Tuesday?

1 mark

e. If the radius of the blade is 15cm and assuming that the edge of the meat to be sliced is on the leading edge of the meat holder in line with point *E* as indicated in the diagram, find the value of *t* when the meat first hits the rotating blade. Express your answer correct to 3 decimal places.



2 marks

f. The width of the meat that is to be sliced is 17cm.

i. What is the value of t when the first slice of meat has just been sliced off completely. Express your answer correct to 3 decimal places.

ii. How long does it take for the blade to cut one slice of meat? Express your answer correct to 3 decimal places.

$2 + 1 = 3$ marks

g. **i.** Find an expression for the rate of change of p with respect to time.

ii. Without using calculus find when the rate of change of p is a maximum.

iii. What is the position of the leading edge of the meat slicer when the rate of change of p is a maximum?

iv. At what rate (in cm/sec), is p changing at 10.30am on Tuesday?

1 + 1 + 1 + 1 = 4 marks
Total 13 marks

Question 3

Consider the function

$$f:[0,\infty)\to R,\ f(x)=e^{x}-x^{3}+1.$$

a. Write down the range of f, expressing your answer correct to 4 significant figures where appropriate.

2 marks

b. The function f_1 has the same rule as the function f.

The domain of f_1 is $x\in[a,3\cdot 7]$.

i. Find the value of a correct to 2 decimal places, such that both the following conditions are satisfied:

- the inverse function $f_1^{-1}(x)$ exists
- the domain of f_1 is a maximal domain

ii. Using your result to part **i**. write down the coordinates of the two endpoints of the graph of the inverse function $y=f_1^{-1}(x)$. Express your coordinates correct to 2 decimal places where appropriate.

iii. Find the coordinates of the point of intersection of the graphs of $y = f_1(x)$ and $y = f_1^{-1}(x)$. Express the coordinates correct to 2 decimal places.

$2 + 2 + 1 = 5$ marks

c. The graph of $y = g(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.

i. Write down a rule for the function g.

ii. Find the maximum value for $|f(x) - g(x)|$ for $x \in [0, 4 \cdot 5]$. Express your answer correct to 2 decimal places.

iii. Find the value of x, correct to 2 decimal places, for which $|f(x) - g(x)|$ for $x \in [0, 4 \cdot 5]$ is a maximum.

iv. Find the value of x, correct to 2 decimal places, for which $|f(x) - g(x)|$ for $x \in [0, 4 \cdot 5]$ is a minimum.

$1 + 2 + 1 + 1 = 5$ marks

The graph of $y = f(x)$ is transformed to become the graph of $y = f(x+h)$.

d. Describe the transformation to the graph of $y = f(x)$ that has taken place.

1 mark

e. **i.** Find the two solutions to $f(x) = 0$. Express them each correct to 2 decimal places.

ii. Using your answers to part **e.i.**, find the real values of *h* for which $f(x+h) = 0$ has two positive solutions.

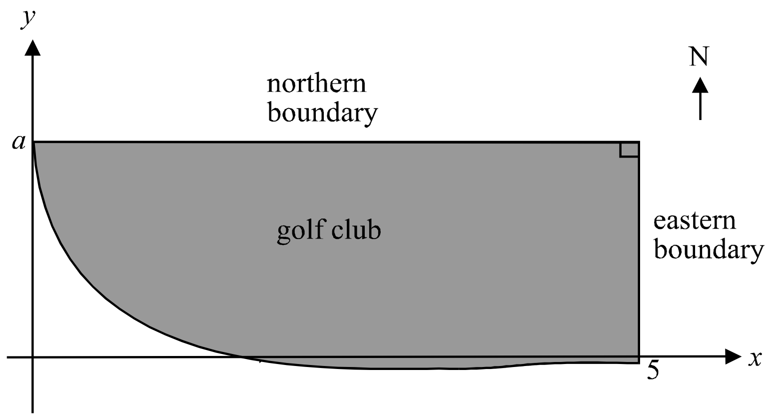
1 + 3 = 4 marks
Total 17 marks

Question 4

A golf club is bordered by two straight boundaries and a river.
The shaded area on the diagram below shows the golf club. The y-axis runs in a north-south line and in relation to the set of axes, the northern boundary of the golf club is described by the equation $y = a$. The eastern boundary is described by the equation $x = 5$.
The river boundary is described by the function

$$f(x) = \frac{1 - \log_e(x+1)}{(x+1)^2}, \; x \in [0, 5]$$

where 1 unit represents 1 kilometre. The north-west corner of the golf course is located at the point $(0, a)$.



a. Show that $a = 1$.

1 mark

b. Show that the function f crosses the x-axis at $x = e - 1$.

2 marks

c. Find the length of the eastern boundary of the golf course. Express your answer in kilometres correct to four decimal places.

2 marks

d. Find the coordinates of the southern most point on the golf club's boundary.
Express each coordinate correct to 2 decimal places.

2 marks

e. **i** Given that

$$\frac{d}{dx}\left(\frac{\log_e(x+1)}{x+1}\right)=\frac{a-b\log_e(x+1)}{(x+1)^n}$$

find the values of a, b and n.

2 marks

ii. Hence find $\int \frac{1-\log_e(x+1)}{(x+1)^2}\,dx$.

1 mark

f. **i.** Using your result to part **e. ii**., show that $\int_{e-1}^{5} f(x)\,dx = \frac{\log_e 6}{6} - \frac{1}{e}$.

ii. Explain why this value is negative.

iii. Hence find the area of the golf course. Express your answer as an exact value.

$2 + 1 + 4 = 7$ marks
Total 17 marks

END OF EXAM

Mathematical Methods Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$

$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$

$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$

$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$

$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$

$\int \frac{1}{x}dx=\log_e|x|+c$

$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$

$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$

$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$

$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$

$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

mean: $\mu=E(X)$

variance: $\text{var}(X)=\sigma^2=E((X-\mu)^2)=E(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\Sigma x\,p(x)$	$\sigma^2=\Sigma(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)dx$	$\mu=\int_{-\infty}^{\infty}x\,f(x)dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$

MATHEMATICAL METHODS

TRIAL EXAMINATION 2

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:...

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ▬ C D E

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	E	12.	A	B	C	D	E
2.	A	B	C	D	E	13.	A	B	C	D	E
3.	A	B	C	D	E	14.	A	B	C	D	E
4.	A	B	C	D	E	15.	A	B	C	D	E
5.	A	B	C	D	E	16.	A	B	C	D	E
6.	A	B	C	D	E	17.	A	B	C	D	E
7.	A	B	C	D	E	18.	A	B	C	D	E
8.	A	B	C	D	E	19.	A	B	C	D	E
9.	A	B	C	D	E	20.	A	B	C	D	E
10.	A	B	C	D	E	21.	A	B	C	D	E
11.	A	B	C	D	E	22.	A	B	C	D	E

