

VCE
2006 Mathematical Methods
Trial Examination 1

Suggested Solutions

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PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

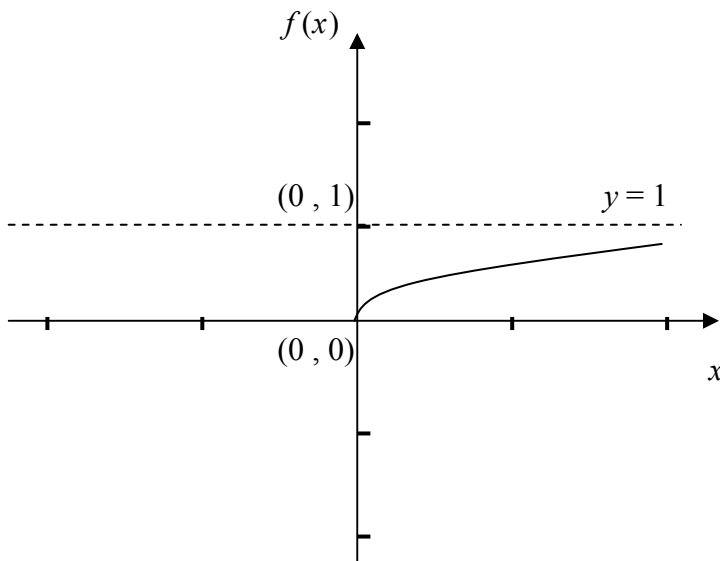
- knowledge of mathematical concepts
- skills in carrying out mathematical algorithms
- ability to apply concepts and skills in standard ways

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Question 1

a.



(1 mark) for point (0 , 0).
(1 mark) for the equation of the asymptote.
(1 mark) for the shape of the graph.

b.

Range is $[0, 1)$ (1 mark)

Question 2

a.

At the point of intersection:

$$e^x = 6e^{-x} - 1$$

Multiply both sides by e^x

$$\Rightarrow e^{2x} = 6 - e^x$$

$$\Rightarrow e^{2x} + e^x - 6 = 0$$

Let $y = e^x$

$$y^2 + y - 6 = 0$$

$$(y + 3)(y - 2) = 0$$

$$\Rightarrow y = -3 \text{ or } y = 2 \quad (1 \text{ mark})$$

$$\Rightarrow e^x = -3 \text{ or } e^x = 2$$

But $e^x > 0 \therefore e^x = 2$ only (1 mark)

$$y = 2, x = \log_e 2$$

$$(\log_e 2, 2) \quad (1 \text{ mark})$$

Question 3

<p>a.</p> $f'(x) = \frac{1}{x^2 + 5} \times 2x \quad (1 \text{ mark})$ $f'(x) = \frac{2x}{x^2 + 5} \quad (1 \text{ mark})$	<p>b.</p> $f(x) = \int (1 - 6 \sin 3x) dx$ $f(x) = x + 2 \cos 3x + c \quad (1 \text{ mark})$ $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 0 + c = \frac{\pi}{3}$ $\Rightarrow c = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ $f(x) = x + 2 \cos 3x + \frac{\pi}{6} \quad (1 \text{ mark})$
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Question 4

<p>a.</p> $x = \frac{1}{3y - 1}$ $\Rightarrow 3y - 1 = \frac{1}{x} \quad (1 \text{ mark})$ $\Rightarrow 3y = \frac{1}{x} + 1 = \frac{1 + x}{x}$ $\Rightarrow y = \frac{1 + x}{3x} \quad (1 \text{ mark})$	<p>b.</p> <p>Domain is $(0, \infty)$ (1 mark)</p> <p>Range is $\left(\frac{1}{3}, \infty\right)$ (1 mark)</p>
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Question 5

$2 \sin 3\left(\theta + \frac{\pi}{4}\right) - 1 = 0, \quad 0 \leq \theta \leq \frac{\pi}{2}$ $\Rightarrow 2 \sin 3\left(\theta + \frac{\pi}{4}\right) = 1, \quad \frac{3\pi}{4} \leq 3\left(\theta + \frac{\pi}{4}\right) \leq \frac{9\pi}{4} \quad (1 \text{ mark})$ $\Rightarrow \sin 3\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2}$ $\Rightarrow 3\left(\theta + \frac{\pi}{4}\right) = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$ $\Rightarrow 3\left(\theta + \frac{\pi}{4}\right) = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ $\Rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18} \quad (1 \text{ mark})$ $\Rightarrow \theta = \frac{\pi}{18} - \frac{\pi}{4}, \frac{5\pi}{18} - \frac{\pi}{4}, \frac{13\pi}{18} - \frac{\pi}{4} \Rightarrow \theta = \frac{-7\pi}{36}, \frac{\pi}{36}, \frac{17\pi}{36}$ <p>But $0 \leq \theta \leq \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{36}, \frac{17\pi}{36} \quad (1 \text{ mark})$</p>	
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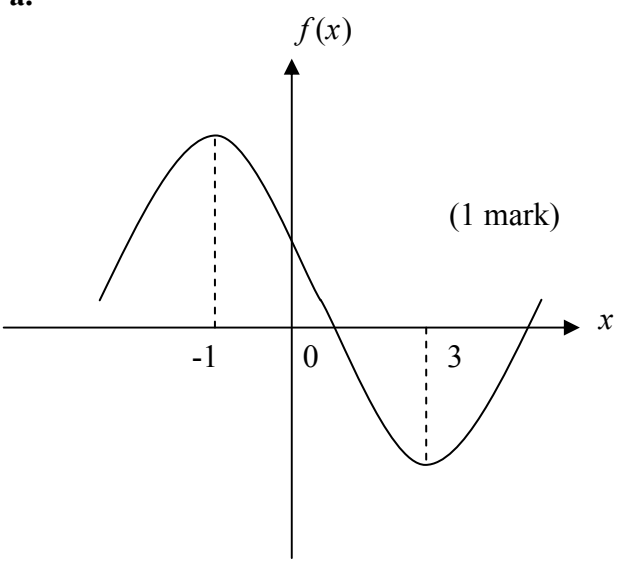
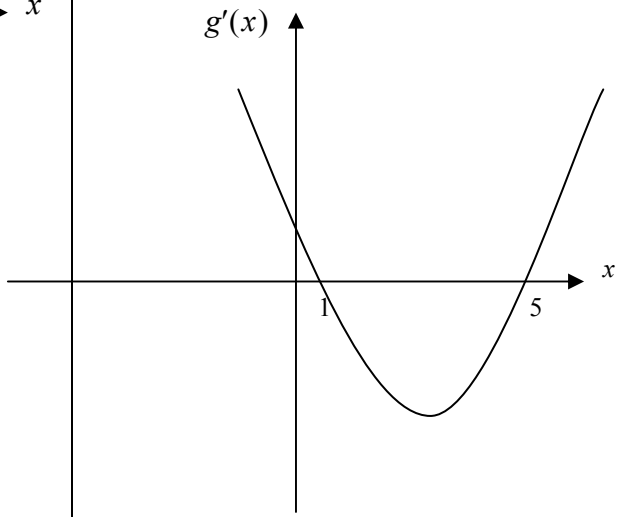
Question 6

<p>a.</p> $\int_0^a 4x^3 dx = x^4 \Big _0^a = \frac{1}{4} \quad (1 \text{ mark})$ $a^4 = \frac{1}{4}$ $\Rightarrow a^2 = \pm \frac{1}{2}$ $\Rightarrow a = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$ <p>But $a > 0$</p> $\Rightarrow a = \frac{\sqrt{2}}{2} \quad (1 \text{ mark})$	<p>b.</p> $\int_{1/2}^{3/4} 4x^3 dx = x^4 \Big _{1/2}^{3/4} \quad (1 \text{ mark})$ $= \frac{81}{256} - \frac{1}{16}$ $= \frac{81}{256} - \frac{16}{256}$ $= \frac{65}{256} \quad (1 \text{ mark})$
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Question 7

<p>a.</p> $\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ $\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$ $= 0.3 + 0.6 - 0.8$ $= 0.1 \quad (1 \text{ mark})$ $\Pr(A B) = \frac{0.1}{0.6} = \frac{1}{6} \quad (1 \text{ mark})$	<p>b.</p> $\Pr(X < 12 X > 8) = \frac{\Pr(X < 12 \cap X > 8)}{\Pr(X > 8)}$ $= \frac{\Pr(8 < X < 12)}{\Pr(X > 8)}$ $Z = \frac{12 - 8}{2} = 2$ <p>95% lie within 2 standard deviations of mean 5% lie outside 2 standard deviations of mean (1 mark) 2.5% are greater than $Z = 2$</p> $\therefore = \frac{\Pr(8 < X < 12)}{\Pr(X > 8)} = \frac{0.5 - 0.025}{0.5} \quad (1 \text{ mark})$ $= \frac{0.475}{0.5}$ $= 0.475 \times 2$ $= 0.95 \quad (1 \text{ mark})$
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Question 8

<p>a.</p>  <p style="text-align: right;">(1 mark)</p>	<p>b.</p> <p>Increasing when $g'(x) > 0$</p> $g'(x) = 3x^2 - 18x + 15$ $g'(x) = 3(x^2 - 6x + 5) = 0$ $g'(x) = 3(x - 5)(x - 1) = 0 \quad (1 \text{ mark})$ $x = 5 \text{ or } x = 1$ $g'(x) > 0 \text{ when } x < 1 \text{ or } x > 5 \quad (1 \text{ mark})$ 
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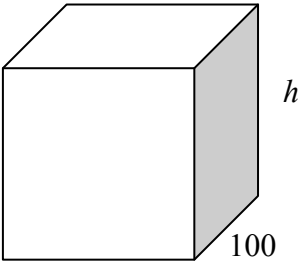
Question 9

<p>a.</p> $\sigma^2 = np(1 - p)$ $2.25 = 9p(1 - p)$ $\Rightarrow \frac{2.25}{9} = p(1 - p)$ $\Rightarrow 0.25 = p - p^2 \quad (1 \text{ mark})$ $\Rightarrow p^2 - p + 0.25 = 0$ $\Rightarrow 4p^2 - 4p + 1 = 0$ $\Rightarrow (2p - 1)(2p - 1) = 0$ $\Rightarrow 2p - 1 = 0$ $\Rightarrow p = \frac{1}{2} \quad (1 \text{ mark})$	<p>b.</p> $\Pr(X \geq 1) = 1 - \Pr(X = 0)$ $= 1 - \binom{9}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^9$ $= 1 - \left(\frac{1}{2}\right)^9$ $= 1 - \frac{1}{512} = \frac{511}{512} \quad (1 \text{ mark})$
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Question 10

<p>a. $\frac{dy}{dx} = 3e^{3x} \cos 3x + 3e^{3x} \sin 3x$ (1 mark)</p>	<p>b. $\int (3 \sin 3x(e^{3x} - 1) + 3e^{3x} \cos 3x) dx$ $= \int (3 \sin 3x e^{3x} - 3 \sin 3x + 3e^{3x} \cos 3x) dx$ $= \int (3e^{3x} \cos 3x + 3e^{3x} \sin 3x) dx - \int 3 \sin 3x dx$ (1 mark) $= e^{3x} \sin 3x + \cos 3x + c$ where c is a constant (1 mark)</p>
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Question 11

<p>a.</p>  <p>$\frac{dV}{dt} = -50 \text{ ml sec}^{-1} = -50 \text{ cm}^3 \text{ sec}^{-1}$ (1 mark)</p> <p>$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt}$ where h = depth of water. $V = 100 \times 100 \times h = 10000h$</p> <p>$\frac{dV}{dh} = 10000$ (1 mark)</p> <p>$\frac{dh}{dt} = \frac{1}{10000} \times \frac{-50}{1} = -0.005$</p> <p>Height of water is falling at $0.005 \text{ cm sec}^{-1}$ (1 mark)</p>	<p>b.</p> <p>$\frac{60}{0.005} = \frac{60000}{5} = 12000$ seconds $12000 \text{ seconds} = \frac{12000}{60} \text{ minutes} = 200 \text{ minutes}$ $200 \text{ minutes} = \frac{200}{60} = 3\frac{2}{3} \text{ hours}$ (1 mark)</p>
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END OF SUGGESTED SOLUTIONS
2006 Mathematical Methods Trial Examination 1

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