# VCE 2006 Mathematical Methods Trial Examination 2

# **Suggested Solutions**

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# PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by multiple-choice questions and extended answer questions involving multi-stage solutions of increasing complexity.

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Question 1 D Sketch the graphs on the calculator. There are three solutions at $x = -0.77$ , 2 and 4.	Question 2 C $g\{f(x)\} = \log_e(x^2 - 9)$ $x^2 - 9 > 0$ $\Rightarrow x^2 > 9$
Ougstion 3 E	$\Rightarrow \pm x > 3$ $\Rightarrow x < -3 \text{ or } x > 3$ Domain $(-\infty, -3) \cup (3, \infty)$ Question 4 A
Question 3 E The inverse only exists for a one-to-one function. The graph is one-to-one for $-\frac{\pi}{2} \le 3x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{6} \le x \le \frac{\pi}{6}$	$x^{2} = 1 + 4 + 9 + 16 + 25$ $Pr(X = x) = 1$ $\Rightarrow 55k^{2} = 1$ $\Rightarrow k^{2} = \frac{1}{55}$ $\Rightarrow k = \pm \frac{1}{\sqrt{55}} \times \frac{\sqrt{55}}{\sqrt{55}} = \pm \frac{\sqrt{55}}{55}$
Question 5 E	Question 6 A
$2(1 - \cos^{2} \theta) - 7\cos \theta + 2 = 0$ $\Rightarrow 2 - 2\cos^{2} \theta - 7\cos \theta + 2 = 0$ $\Rightarrow -2\cos^{2} \theta - 7\cos \theta + 4 = 0$ $\Rightarrow 2\cos^{2} \theta + 7\cos \theta - 4 = 0$ $\Rightarrow (2\cos \theta - 1)(\cos \theta + 4) = 0$ $\Rightarrow 2\cos \theta - 1 = 0 \text{ or } \cos \theta + 4 = 0$ $\cos \theta = \frac{1}{2} \text{ or } \cos \theta = -4$ But $-1 \le \cos \theta \le 1$ $\Rightarrow \cos \theta = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$ Sum $= \frac{\pi}{3} + \frac{5\pi}{3} = 2\pi$	$f(x) = a(x+1)^{2} (x-3)$ When $x = 0$ , $f(x) = 6$ $\Rightarrow 6 = a \times 1 \times -3$ $\Rightarrow a = -2$ $f(x) = -2(x+1)^{2}(x-3)$ $\Rightarrow f(x) = 2(x+1)^{2}(3-x)$

Question 7 B	Question 8 C
$y = 4 - \frac{1}{x - 3}$	$V = \frac{4}{3}\pi r^3$
Interchange x and y	$\frac{dV}{dr} = 4\pi r^2$
$\Rightarrow x = 4 - \frac{1}{y - 3}$	<i>ur</i>
	$\frac{dr}{dt} = 0.1$
$\Rightarrow \frac{1}{y-3} = 4 - x$	$\frac{dt}{dt} = \frac{dV}{dt} \times \frac{dr}{dt} = 4\pi r^2 \times 0.1 = 0.4\pi r^2$
$\Rightarrow y - 3 = \frac{1}{4 - x}$	When $r = 7$ ,
$\Rightarrow y = \frac{1}{4-x} + 3 = 3 + \frac{-1}{x-4}$	$\frac{dV}{dt} = 0.4 \pi \times 49 = 61.6 \mathrm{cm}^3 \mathrm{sec}^{-1}$
$\Rightarrow y = 3 - \frac{1}{x - 4}$ Question 9 B	
	Question 10 C
Use a graphics calculator to sketch the graphs and find that the points of intersection are	$\int \frac{1}{(3x+2)^{1/2}} dx$
x = 1.57 and $x = 2.2Use the calculator to find the area between the$	$= \int (3x+2)^{-1/2} dx$
curve and the <i>X</i> axis for $y =  6x - 11 $ from x = 1.57 to $x = 2.2$ Area = 1.188	$=\frac{2}{3}(3x+2)^{1/2}+c$
Use the calculator to find the area between the	5
curve and the X axis for $y =  x $ from	$=\frac{2\sqrt{3x+2}}{3}+c$
x = 1.57 to $x = 2.2$ Area = 0.576	$=\frac{2\sqrt{3x+2}}{2\sqrt{3x+2}}$
Area between these graphs = $1.188 - 0.576 = 0.612$	$=\frac{1}{3}$

Question 11 E	Question 12 E
When $t = 0, A = A_0 = 200$	The area in answer B can be checked as
$A = 200e^{-kt}$	incorrect by using a graphics calculator.
When $t = 20, A = 100$	dy = x + x + 1
$0.5 = e^{-20k}$	gradient = $\frac{dy}{dx} = xe^x + e^x \times 1$
$\log_e 0.5 = -20k$	$= e^{x}(x+1) = 0$ when $x = -1$
k = 0.035	When $x < -1, \frac{dy}{dr} < 0$
$A = 200e^{-0.035t}$	ил
$\frac{dA}{dt} = 200 \times (-0.035)e^{-0.035t}$	Therefore, negative gradient when $x < -1$
When $t = 30$ , $\frac{dA}{dt} = -2.45$	
The isotope is decaying at 2.45 g/day.	
Question 13 A	Question 14 B
$k \int_{0}^{9} x^{2} dx = 1$ $\Rightarrow \frac{kx^{3}}{3} \Big]_{0}^{9} = 1$	$\sqrt{f(x)}$ only exists when $f(x) \ge 0$ . That is, when y on the original graph is above or on the X axis.
$\Rightarrow \frac{729k}{3} - 0 = 1$	
$\Rightarrow 243k = 1$	
$\Rightarrow k = \frac{1}{243} = 0.004$	

Question 15 P	Question 16 D
Question 15 B	Question 16 D
	dy = 2 + 4 - 2 + 8x - 4 - 8x - 2
The gradient is positive everywhere	$\frac{dy}{dx} = \frac{2}{2x-1} + 4 = \frac{2+8x-4}{2x-1} = \frac{8x-2}{2x-1}$
except when $x \leq -1$ and when $x = 0$ .	dv
x = -1 is a minimum as the graph	When $x = 1, \frac{dy}{dx} = 6$
goes from a negative to a positive	
gradient. $x = 0$ is a stationary point	So, gradient of normal = $-\frac{1}{6}$
of inflexion as the graph goes from a	$\frac{50}{6}$
positive to a positive gradient.	1
r	Equation of normal is $y = -\frac{1}{6}x + c$
	6
	When $x = 1$ on curve $y = \log_e 1 + 4 = 4$
	So point (1, 4) lies on the normal.
	1 25
	Therefore, $4 = -\frac{1}{6} + c \Rightarrow \frac{25}{6} = c$
	6 6
	1 25
	$\Rightarrow y = -\frac{1}{6}x + \frac{25}{6} \Rightarrow 6y + x - 25 = 0$
	0 0

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Question 17 E	Question 18 B
$A = \int_{1}^{2} \frac{12}{12 - 5x} dx$	B 0.7
$2^{2} -5$	$B \xrightarrow{0.7} D \xrightarrow{0.3}$
$=\frac{2}{-5}\int_{1}^{2}\frac{-5}{12-5x}dx$	
2, $(12)$ , $(12)$	
$= -\frac{2}{5} \log_e (12 - 5x) \Big]_1^2$	B <sup>/</sup> 0.9
$= -\frac{2}{5}\log_e(2) + \frac{2}{5}\log_e(7)$	B' 0.1 $0.7$
$=-\frac{1}{5}\log_{e}(2)+\frac{1}{5}\log_{e}(7)$	
$=\frac{2}{5}\left[\log_{e}(7) - \log_{e}(2)\right]$	$B \sim 0.3$
<b>č</b>	$B' \sim B = 0.1$
$= 0.4 \log_e \left(\frac{7}{2}\right)$	B <sup>/</sup> 0.9
$= 0.4 \log_e 3.5$	B B B = $0.5 \times 0.7 \times 0.7 = 0.245$
<i>Detailed</i>	$B B' B = 0.5 \times 0.3 \times 0.1 = 0.015$
	$B' B B = 0.5 \times 0.1 \times 0.7 = 0.035$
	$B'B'B = 0.5 \times 0.9 \times 0.1 = 0.045$
Question 19 C	Total = 0.34 Question 20 E
$E(\cos x)$	1
$\frac{\pi \rho^2}{5} 2$	$\Pr(A) = \frac{1}{6}$
$=\int_{0}^{\pi/2} \frac{2}{\pi} \cos x dx$	$\Pr(B) = \frac{1}{2}$
$\lceil 2 \rceil^{\pi/2}$	2
$= \left[\frac{2}{\pi}\sin x\right]_{0}^{\pi/2}$	$\Pr(A \cap B) = \frac{3}{36} = \frac{1}{12}$
	30 12
$=\frac{2}{\pi}\left[\sin\frac{\pi}{2} - \sin 0\right]$	$\Pr(A \cap B) = \Pr(A) \times \Pr(B) = \frac{1}{12}$
$=\frac{2}{\pi}[1-0]=\frac{2}{\pi}=0.637$	Therefore, independent events.
$\begin{bmatrix} -\frac{\pi}{\pi} L^{1-0} \end{bmatrix} - \frac{\pi}{\pi} - 0.057$	$\Pr(A \cup B)$
	$= \Pr(A) + \Pr(B) - \Pr(A \cap B)$
	$=\frac{1}{6}+\frac{1}{2}-\frac{1}{12}=\frac{7}{12}$
	-
	Therefore, not mutually exclusive.

<b>Question 21</b> A Use a graphics calculator. Enter data in the <b>list</b> menu.	Question 22 D Pr(B   GG)
Use <b>binompdf</b> to get probabilities. Then use <b>graph</b> to	$\Pr(B \cap GG)$
get A.	$= \frac{1}{\Pr(B \cap GG) + \Pr(Y \cap GG)}$
	$\frac{3}{2} \times \frac{2}{2}$
	$= \frac{\overline{6}^{5}}{\frac{3}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5}}$
	= 0.75

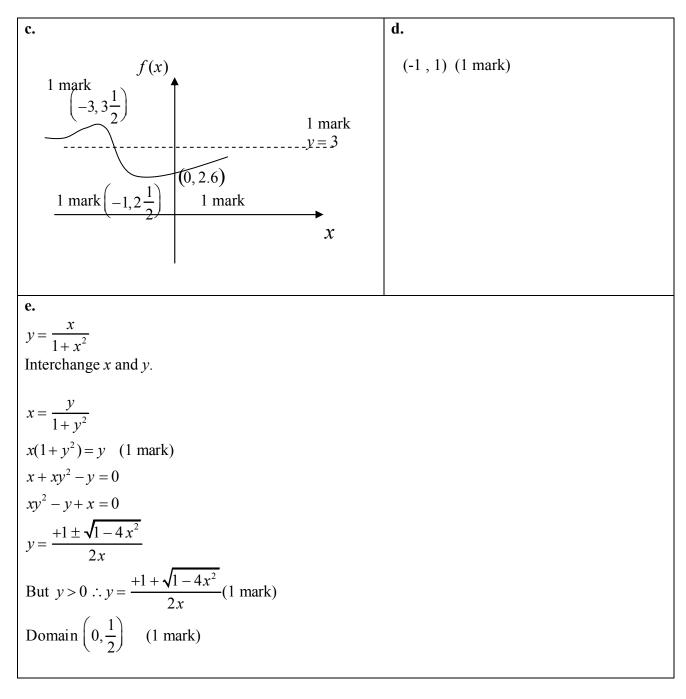
# Suggested Solutions Part II

#### **Question 1**

Question 1	1
a.	b.
$f(x) = \frac{x^2}{1+x^2}$	<ul> <li>Graph is reflected in the X axis.</li> <li>Graph is translated 2 units to the left parallel</li> </ul>
$f'(x) = \frac{(1+x^2)1 - x \times 2x}{(1+x^2)^2}$	<ul> <li>to the X axis.</li> <li>Graph is translated 3 units up parallel to the Y axis.</li> </ul>
	1 wild.
$=\frac{1+x^2-2x^2}{(1+x^2)^2}$	(1 mark for each)
$=\frac{1-x^2}{(1+x^2)^2}=0$ for turning point (1 mark)	
$\Rightarrow 1 - x^2 = 0$	
$\Rightarrow x = \pm 1 (1 \text{ mark})$	
When $x = 1$ , $f(x) = \frac{1}{2}$	
When $x = -1$ , $f(x) = \frac{1}{2}$	
When $x < -1$ , $f'(x) < 0$	
When $-1 < x < 1$ , $f'(x) > 0$	
When $x > 1$ , $f'(x) < 0$	
Maximum $\left(1,\frac{1}{2}\right)$ Minimum $\left(-1,\frac{1}{2}\right)$ (1 mark)	

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#### **Question 1 (continued)**



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# Question 2

a.	b.
Maximum value of $\sin\left(\frac{2\pi t}{3}\right) = 1$	Minimum value of $\sin\left(\frac{2\pi t}{3}\right) = -1$
Max length of spring = $(a + b)$ cm (1 mark)	Min length of spring = $(a - b)$ cm (1 mark)
c. $8 \sin\left(\frac{2\pi t}{3}\right) = 4$ $\Rightarrow \sin\left(\frac{2\pi t}{3}\right) = \frac{1}{2}  (1 \text{ mark})$ $\Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}  (1 \text{ mark})$ $\Rightarrow t = \frac{\pi}{6} \times \frac{3}{2\pi}, \frac{5\pi}{6} \times \frac{3}{2\pi}, \frac{13\pi}{6} \times \frac{3}{2\pi}, \frac{17\pi}{6} \times \frac{3}{2\pi}$ $\Rightarrow t = \frac{1}{4}, 1\frac{1}{4}, 3\frac{1}{4}, 4\frac{1}{4}  (1 \text{ mark})$	d. Period = $2\pi \div \frac{2\pi}{3} = 3$ seconds (1 mark)
e. $64 + 8\sin\left(\frac{2\pi t}{3}\right) > 60$ This is true when $8\sin\left(\frac{2\pi t}{3}\right) > -4 \Rightarrow \sin\left(\frac{2\pi t}{3}\right) > -\frac{1}{2}$ Let $\sin\left(\frac{2\pi t}{3}\right) = \frac{1}{2}$ $\Rightarrow \frac{2\pi t}{3} = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\Rightarrow t = \frac{7\pi}{6} \times \frac{3}{2\pi}, \frac{11\pi}{6} \times \frac{3}{2\pi}$ $\Rightarrow t = \frac{7}{4}, \frac{11}{4}  (1 \text{ mark})$ $64 + 8\sin\left(\frac{2\pi t}{3}\right) > 60 \text{ from 0 to } 1.75 \text{ seconds.}$ Between 1.75 seconds and 2.75 seconds, the function is less than 60.	f. When $t = 0$ , length $(x) = 64$ When $t = 0.25$ , length $(x) = 68$ (1 mark) Average rate of change $= \frac{\Delta x}{\Delta t} = \frac{4}{0.25} = 16 \text{ cm/sec} \qquad (1 \text{ mark})$
$\frac{\mathbf{g}}{\frac{dx}{dt}} = \frac{16\pi}{3} \cos\left(\frac{2\pi t}{3}\right)  (1 \text{ mark})$ When $t = 2.75$ , $\frac{dx}{dt} = \frac{16\pi}{3} \cos\left(\frac{2\pi \times 2.75}{3}\right) = \frac{8\pi\sqrt{3}}{3}$	$\frac{3}{2}$ cm/sec (1 mark)

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#### **Question 3**

a. b. Diameter = distance from -10 to +10 = 20 $y = a(x-b)^2 + c$  (b = 0)  $v = ax^2 + c$ (1 mark)When x = 0, y = 1 so  $y = ax^{2} + 1$  (1 mark) When x = -3, y = 3.25 so 3.25 = 9a + 1 $\Rightarrow 2.25 = 9a$  $\Rightarrow \frac{2.25}{9} = a$  $\Rightarrow a = 0.25$  (1 mark) c. d. Express x in terms of y.  $\int 4\pi (y-1)dy \quad (1 \text{ mark})$  $y = \frac{x^2}{4} + 1$ a and b are the y values.  $\Rightarrow y-1=\frac{x^2}{4}$ a = 1 (1 mark) b is the value of v when x = 10 (1 mark)  $\Rightarrow x^2 = 4(y-1)$ When  $x = 10, y = \frac{1}{4} \times 100 + 1 = 26$  $\Rightarrow x = \pm \sqrt{4(y-1)}$  (1 mark)  $\Rightarrow b = 26$  $V = 4\pi \left[\frac{y^2}{2} - y\right]_{1}^{26} = 3927$  cubic units (1 mark) f. e.  $r = 3 \Longrightarrow x = 3$  $\frac{dy}{dr} = \frac{1}{9} \left[ e^{3x} \times 3 + (3x+2) \times 3e^{3x} \right] \quad (1 \text{ mark})$ When x = 3,  $y = \frac{1}{4} \times 9 + 1 = \frac{13}{4}$  $=\frac{1}{3}e^{3x}\times\left[1+3x+2\right]$  $V = 4\pi \int_{1}^{13/4} (y-1)dy$  (1 mark)  $=\frac{1}{2}e^{3x}\times[3x+3]$  $V = 4\pi \left[\frac{y^2}{2} - y\right]^{13/4}$  $=e^{3x}(x+1)$  (1 mark)  $V = 4\pi \left[ \frac{169}{32} - \frac{13}{4} - \frac{1}{2} + 1 \right]$  $V = 4\pi \times \frac{81}{32} = \frac{81\pi}{8}$  cubic units (1 mark)

#### 2006 Mathematical Methods Trial Examination 2 Suggested Solutions Part II Question 3 (continued)

g. i.	g. ii.
dh $dh$ $dh$ $dh$ $dh$ $dh$ $dh$ $dh$	$\frac{dV}{dV} = \frac{dV}{dh} \frac{dh}{dh}$
$\frac{dh}{dt} = 4te^{3t} \Longrightarrow h = \int_{a}^{1} 4te^{3t} dt$	$\frac{dt}{dt} = \frac{dh}{dt} \frac{dt}{dt}$
0	h = y
Now $\int te^{3t} + \int e^{3t} = \frac{1}{9}e^{3t}(3t+2) + c$ (1 mark)	$\frac{dV}{dt} = \frac{dV}{dy}\frac{dy}{dt}$
	dt dy dt
$\int te^{3t} = \frac{1}{9}e^{3t}(3t+2) - \frac{1}{3}e^{3t} + c$	$\frac{dV}{dy} = 4\pi(y-1)  (1 \text{ mark})$
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	
$h = 4 \int_{0}^{1} t e^{3t} dt = 4 \left  \frac{1}{9} e^{3t} (3t+2) - \frac{1}{3} e^{3t} \right _{0}^{1}  (1 \text{ mark})$	$\frac{dh}{dt} = \frac{dy}{dt} = 4te^{3t} = 4e^3 \text{ when } t = 1$
	dt dt
$h = 4 \left[ \frac{5}{9}e^3 - \frac{1}{3}e^3 - \frac{2}{9} + \frac{1}{3} \right]$	$\frac{dV}{dt} = 4\pi (18.2983 - 1) \times 4e^3$
h = 18.2983 units (1 mark)	= 17464.52 cubic units/time unit (1 mark)
	-1/404.52 cubic units/time unit (1 mark)

# **Question 4**

a. i.	a. ii.
$E(X) = \sum x \Pr(X = x)$	$\sigma^2 = E(X^2) - \mu^2$
= 0 + 0.01 + 0.1 + 0.6 + 1.6 + 1.5	$E(X^2) = 0.01 + 0.2 + 1.8 + 6.4 + 7.5$
= 3.81 (1  mark)	= 15.91 (1  mark)
	$\sigma^2 = 15.91 - (3.81)^2$
	$\sigma = \sqrt{15.91 - (3.81)^2} = 1.18$ (1 mark)
b. i.	<b>b. ii.</b> Probability(will <b>not</b> flower in 2008) 2006 2007 2008
1 - 0.85 = 0.15 (1 mark)	F F $F' = 0.8 \times 0.8 \times 0.2$
	F $F' = 0.8 \times 0.2 \times 0.85$
	$\begin{array}{ccccc} F' & F' & F' & = & 0.2 \times 0.15 \times 0.2 \\ F' & F & F' & = & 0.2 \times 0.85 \times 0.85 \end{array}$
	$F'$ F $F'$ = $0.2 \times 0.85 \times 0.85$ (1 mark)
	= 0.128 + 0.136 + 0.006 + 0.1445
	= 0.4145 (1 mark)
c. i.	c. ii.
$a\int_{0}^{1} (t^{2} - t^{3})dt = 1$	$12\int_{0}^{0.23} (t^2 - t^3) dt$
$\Rightarrow a \left[ \frac{t^3}{3} - \frac{t^4}{4} \right]_0^1 = 1$	$12\int_{0}^{0.25} (t^2 - t^3) dt$ $= 12\left[\frac{t^3}{3} - \frac{t^4}{4}\right]_{0}^{0.25}$
$\Rightarrow a\left(\frac{1}{3} - \frac{1}{4}\right) = 1$	= 0.05 (1 mark)
$\Rightarrow \frac{a}{12} = 1 \Rightarrow a = 12  (1 \text{ mark})$	

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# **Question 4 (continued)**

c. iii.  

$$Pr(X \le 3) | Pr(X \le 6) = \frac{Pr(X \le 3) \cap Pr(X \le 6)}{Pr(X \le 6)}$$

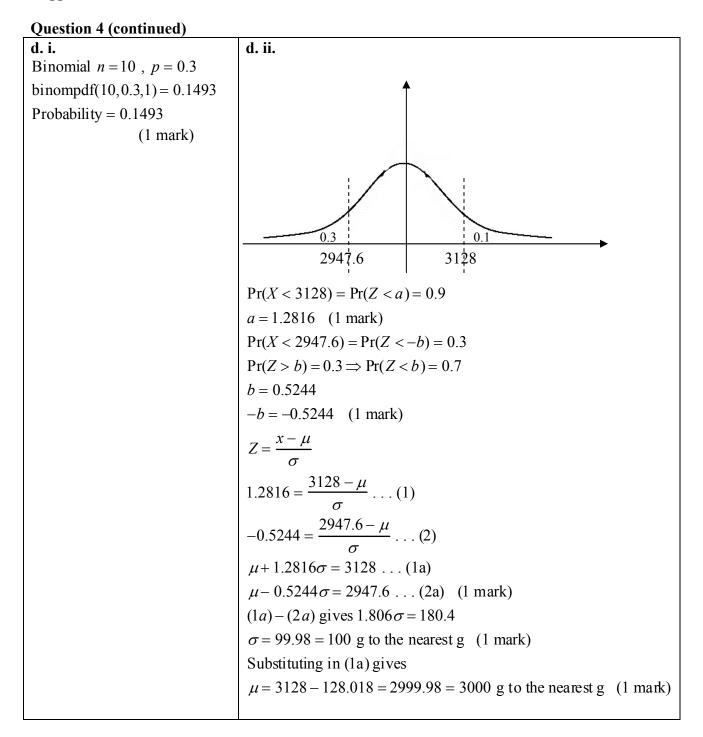
$$= \frac{Pr(X \le 3)}{Pr(X \le 6)}$$

$$Pr(X \le 6) = 12 \left[ \frac{t^3}{3} - \frac{t^4}{4} \right]_0^{0.5} = 0.3125 \quad (1 \text{ mark})$$

$$Pr(X \le 3) | Pr(X \le 6) = \frac{0.05078125}{0.3125}$$

$$= 0.1625$$

$$= 0.16 \text{ to 2 decimal places} \quad (1 \text{ mark})$$



#### END OF SUGGESTED SOLUTIONS 2006 Mathematical Methods Trial Examination 2

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