Year 2006

VCE

Mathematical Methods

Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING **PO BOX 2227** **KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com

ABN 47 065 111 373

PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by multiple-choice questions and extended answer questions involving multi-stage solutions of increasing complexity.

IMPORTANT COPYRIGHT NOTICE

VICTORIAN CERTIFICATE OF EDUCATION
2006

	STUDENT NUMBER								Letter
Figures									
Words									

MATHEMATICAL METHODS

Trial Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.

Materials supplied
- Question and answer book of 22 pages, with a detachable sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct **and** sign your name in the space provided to verify this.
- All written responses must be in English

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E((X-\mu)^2) = E(x^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum xp(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET

VCE MATHEMATICAL METHODS 2006
Trial Written Examination 2
ANSWER SHEET

NAME: _____

STUDENT NUMBER _____

SIGNATURE_____

Instructions

- Write your name in the space provided above.
- Write your student number in the space provided above. Sign your name.
- Use a **PENCIL** for **ALL** entries.
  If you make a mistake, **ERASE** it - **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example.

A	**B**	C	D	E

1	A	B	C	D	E	**12**	A	B	C	D	E
2	A	B	C	D	E	**13**	A	B	C	D	E
3	A	B	C	D	E	**14**	A	B	C	D	E
4	A	B	C	D	E	**15**	A	B	C	D	E
5	A	B	C	D	E	**16**	A	B	C	D	E
6	A	B	C	D	E	**17**	A	B	C	D	E
7	A	B	C	D	E	**18**	A	B	C	D	E
8	A	B	C	D	E	**19**	A	B	C	D	E
9	A	B	C	D	E	**20**	A	B	C	D	E
10	A	B	C	D	E	**21**	A	B	C	D	E
11	A	B	C	D	E	**22**	A	B	C	D	E

Please DO NOT fold, bend or staple this form

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The number of solutions of the equation $2^x = x^2$ is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 2

If $f(x) = x^2 - 9$ and $g(x) = \log_e x$, then $g\{f(x)\}$ equals

A. $\log_e(x^2 - 9), x \in [-3, 3]$

B. $\log_e(x^2 - 9), x \in (-3, 3)$

C. $\log_e(x^2 - 9), x \in (-\infty, -3) \cup (3, \infty)$

D. $[\log_e x]^2 - 9, x \in (0, \infty)$

E. $2\log_e x - 9, x \in (0, \infty)$

Question 3

If $f(x) = \sin(3x)$, then a restricted domain for which $f(x)$ has an inverse could be

A. $\left[0, \frac{2\pi}{3}\right]$

B. $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$

C. $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

D. $\left[0, \frac{\pi}{3}\right]$

E. $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

Question 4

A discrete random variable X is defined only for values of x = 1, 2, 3, 4, and 5, with probabilities given by $\Pr(X = x) = (kx)^2$ where k is a constant. The value of k is

A. $\pm\frac{\sqrt{55}}{55}$

B. $\frac{\sqrt{55}}{55}$

C. $\pm\frac{\sqrt{15}}{15}$

D. $\frac{\sqrt{15}}{15}$

E. $\frac{1}{15}$

Question 5

The sum of the solutions of $2\sin^2\theta = 7\cos\theta - 2,\ 0 \le \theta \le 2\pi$, is

A. $\frac{\pi}{3}$

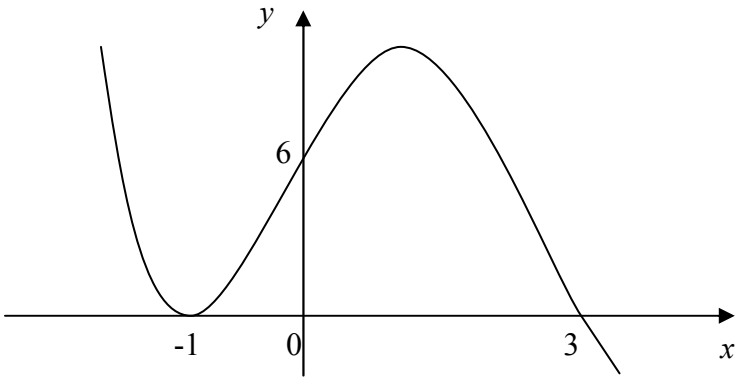
B. $\frac{5\pi}{3}$

C. π

D. $\frac{11\pi}{6}$

E. 2π

Question 6



The graph of the function shown above could be

A. $y = 2(x+1)^2(3-x)$

B. $y = -2(x+1)^2(3-x)$

C. $y = -2(x-1)^2(x+3)$

D. $y = -2(x+1)(x-3)^2$

E. $y = 2(x+1)(x-3)^2$

Question 7

Given that $f(x) = 4 - \frac{1}{x-3}$, then $f^{-1}(x)$ equals

A. $3 + \frac{1}{x-4}$

B. $3 - \frac{1}{x-4}$

C. $3 + \frac{1}{x+4}$

D. $3 - \frac{1}{x+4}$

E. $4 + \frac{1}{x-3}$

Question 8

The volume of a spherical balloon of radius r cm is $\frac{4}{3}\pi r^3$. The balloon is being inflated at a rate of 0.1 cm sec^{-1}. The rate at which the volume is increasing when the radius is 7 cm, is closest to

A. 6.2 cm^3 sec^{-1}

B. 14.4 cm^3 sec^{-1}

C. 61.6 cm^3 sec^{-1}

D. 143.7 cm^3 sec^{-1}

E. 615 cm^3 sec^{-1}

Question 9

The area of the region bounded by the graphs of $y = |6x - 11|$ and $y = |x|$ is closest to

A. 0.576

B. 0.612

C. 1.188

D. 6.601

E. 11.11

Question 10

An antiderviative of $\frac{1}{\sqrt{3x+2}}$ could be

A. $\frac{\log_e\left(\sqrt{3x+2}\right)}{3}$

B. $\frac{2\log_e\left(\sqrt{3x+2}\right)}{3}$

C. $\frac{2\sqrt{3x+2}}{3}$

D. $\frac{\sqrt{3x+2}}{2}$

E. $\frac{\sqrt{3x+2}}{6}$

Question 11
A radioactive isotope decays according to the equation $A = A_0 e^{-kt}$ where A is the amount of the isotope remaining after t days. Initially, there is 200 g of the isotope which has a half life of 20 days. The rate at which the isotope is decaying when t = 30 days, is

A. 0.099 g/day.

B. -0.996 g/day.

C. -0.885 g/day.

D. 1.856 g/day.

E. 2.45 g/day.

Question 12
Given the function $f : R \to R$ where $f(x) = xe^x$, which one of the following statements is true?

A. The gradient is positive for all values of x.

B. The area between the X axis and this graph and the ordinates $x = 0$ and $x = 2$ is $2e^x$.

C. The graph has a stationary point of inflexion when $x = 1$.

D. The gradient is positive when $x \geq -1$.

E. The gradient is negative when $x < -1$.

Question 13
A continuous random variable X has a probability density function

$$f(x) = \begin{cases} kx^2 & 0 \leq x < 9 \\ 0 & \text{elsewhere} \end{cases}$$

The value of k is closest to

A. 0.004

B. 0.006

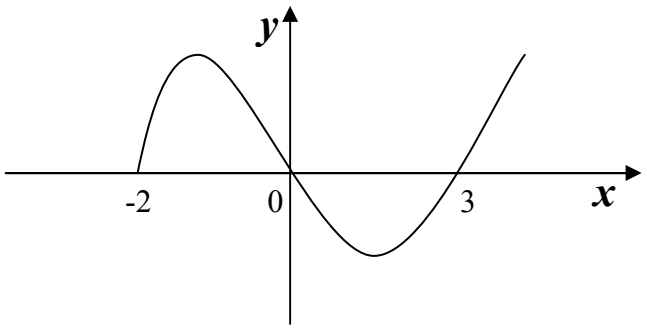
C. 0.02

D. 0.04

E. 0.06

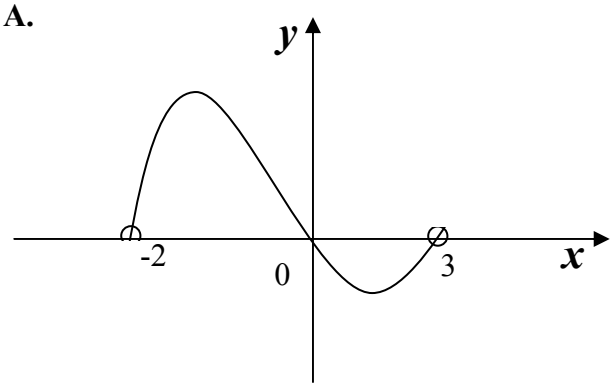
Question 14

The graph of $y = f(x)$ is shown below.

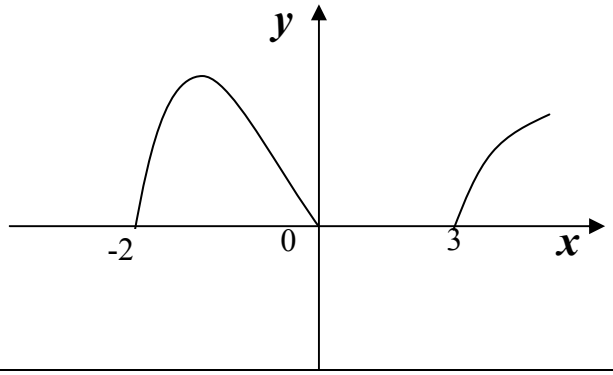


Which one of the following could be the graph of $y = \sqrt{f(x)}$?

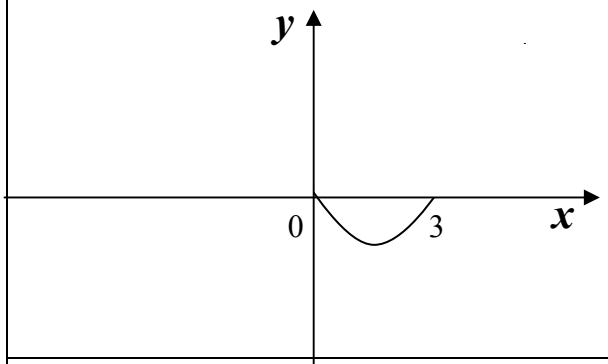
A.



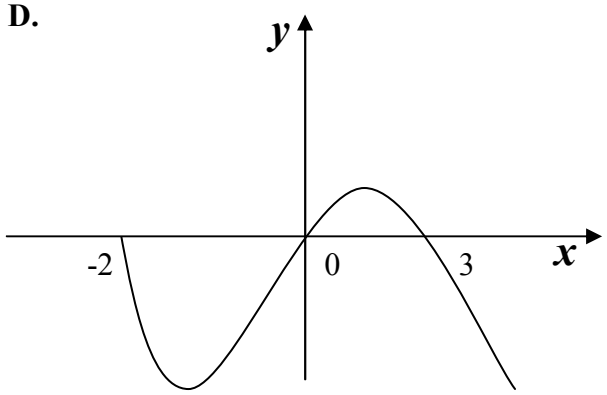
B.



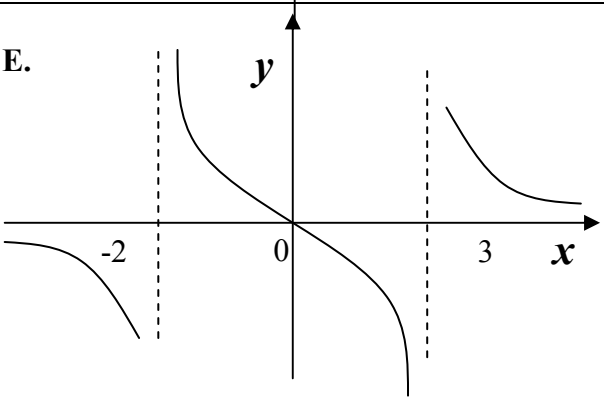
C.



D.

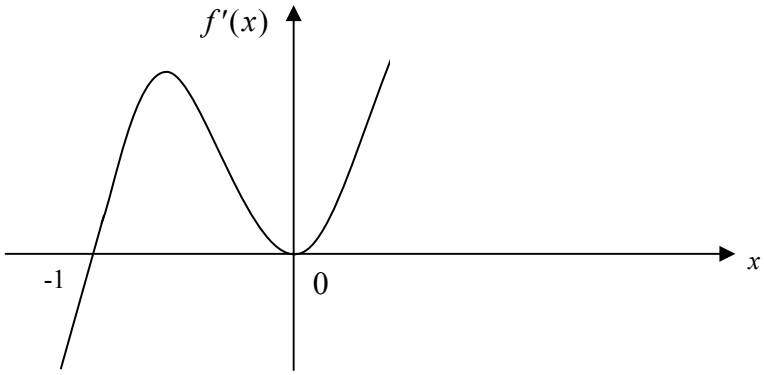


E.



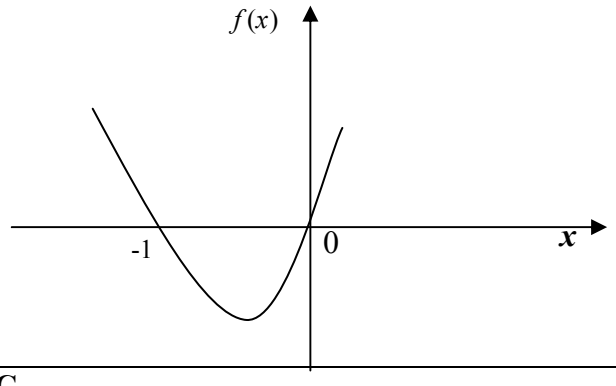
Question 15

The graph of $y = f'(x)$ is shown below

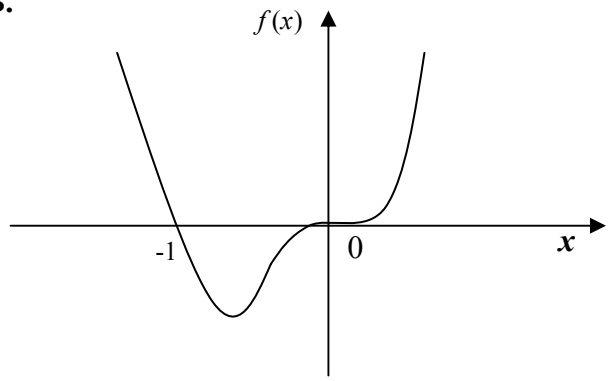


Which one of the following could be the graph of $y = f(x)$?

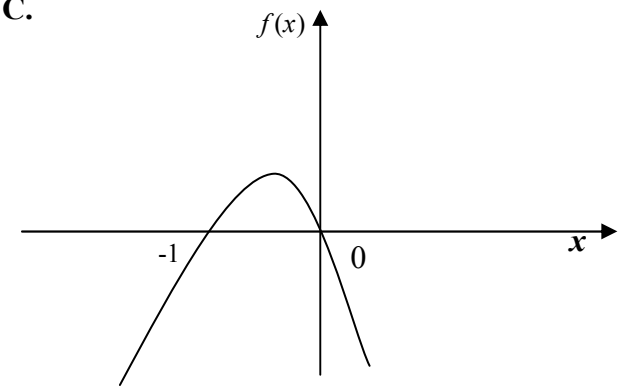
A.



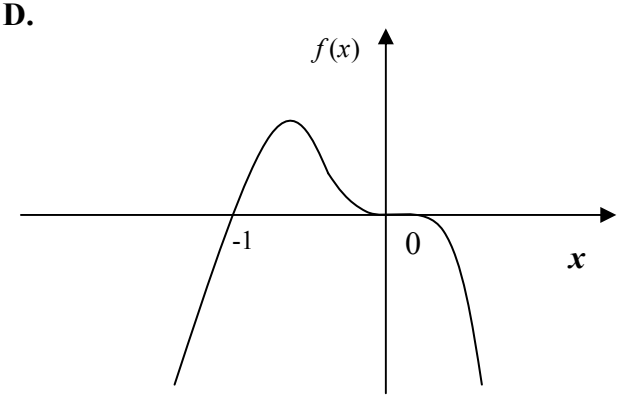
B.



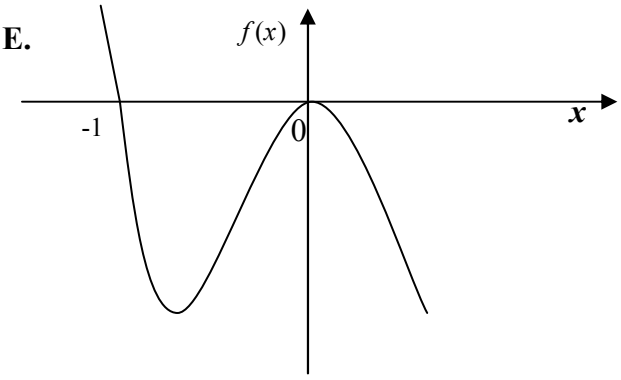
C.



D.



E.



Question 16

The equation of the normal to the curve $f(x) = \log_e(2x-1) + 4x$ at the point where $x = 1$, is

A. $y = 6x - 2$

B. $y = \frac{1}{6}x - 2$

C. $6y + x - 2 = 0$

D. $6y + x - 25 = 0$

E. $6y - x - 23 = 0$

Question 17

The area between the X axis, the curve $f(x) = \frac{2}{12-5x}$ and the ordinates $x = 1$ and $x = 2$ can be given by

A. $2[\log_e 7 - \log_e 2]$

B. $\frac{1}{2}[\log_e 7 - \log_e 2]$

C. $0.4 \log_e 7$

D. $2.5 \log_e(3.5)$

E. $0.4 \log_e(3.5)$

Question 18
When John gets a birdie on a hole in golf, the probability he will get a birdie on the next hole is 0.7. When he does not get a birdie on a particular hole, the probability that he will not get a birdie on the next hole is 0.9. If the probability that John will get a birdie on the first hole is 0.5, then the probability that he will get a birdie on the third hole is

A. 0.12

B. 0.34

C. 0.63

D. 0.67

E. 0.68

Question 19
X is a continuous random variable such that the probability density function is given by

$$f(x) = \begin{cases} \frac{2}{\pi} & 0 \le x \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

The expected value of $\cos(x)$ is closest to

A. 0.32

B. 0.5

C. 0.64

D. 1

E. 1.28

Question 20
Tran rolls a six sided die twice.

A is the event "getting a six on the first roll".
B is the event "getting an even number on the second roll".

Which one of the following statements is true?

A. A and B are mutually exclusive events.

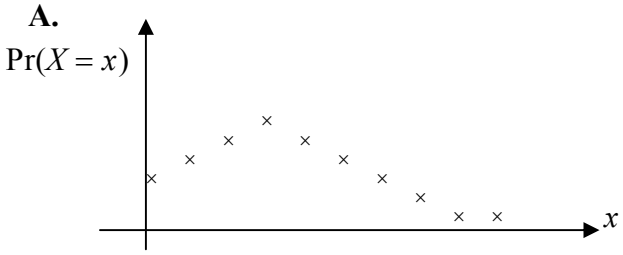
B. $\Pr(A \cup B) = \frac{5}{12}$

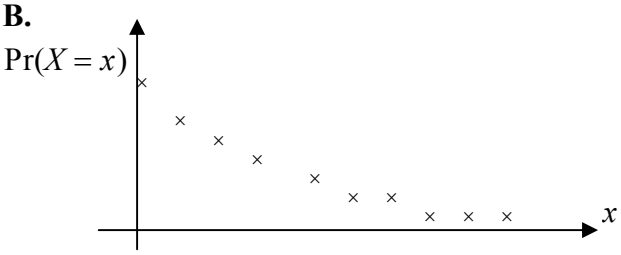
C. $\Pr(A \cap B) = 0$

D. $\Pr(A) \times \Pr(B) = \frac{1}{36}$

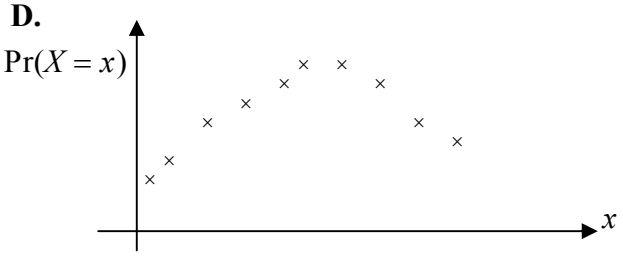
E. A and B are independent events.

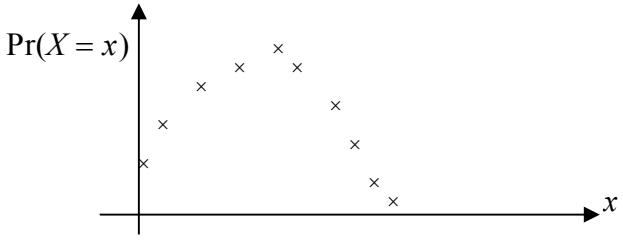
Question 21
Which one of the following graphs best represents the shape of a distribution where there are 10 independent trials, each with a probability of 0.3 of a success.

A.


B.


C.


D.


E.


Question 22
Thee are 2 boxes each containing red and green balls of the same size. The yellow box has 4 red and 2 green balls. The black box has 3 red and 3 green balls. A box is chosen at random and 2 balls are chosen from this box, without replacement, and found to be both green. The probability that it was the black box that was chosen is closest to

A. 0.42

B. 0.53

C. 0.67

D. 0.75

E. 0.80

END OF PART I
MULTIPLE CHOICE QUESTIONS

Question 1

All parts of this question refer to the function $f(x) = \dfrac{x}{1+x^2}$.

a. Use calculus to find the turning points of the graph $f(x) = \dfrac{x}{1+x^2}$ and state the types of turning points.

3 marks

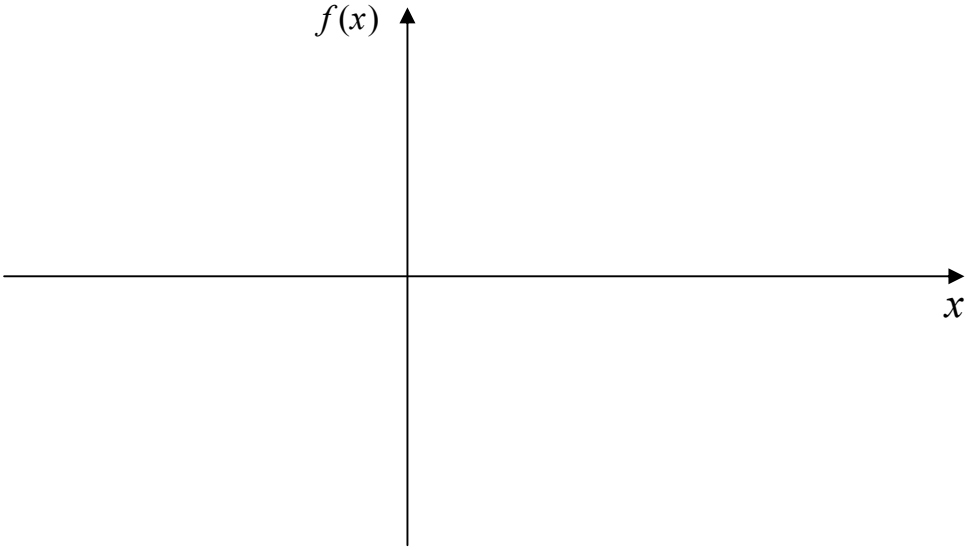
b. State the transformations that have occurred in changing $f(x)$ to $-f(x+2)+3$

3 marks

Question 1 (continued)

All parts of this question refer to the function $f(x) = \dfrac{x}{1+x^2}$.

c. On the axes below, sketch the graph of $-f(x+2)+3$, showing all turning points, intercepts with the axes and the equations of any asymptotes.



4 marks

d. State the maximal domain of $f(x)$ such that $f(x)$ has a positive gradient.

1 mark

e. Find $f^{-1}(x)$ if $f(x)$ has a domain $(0,\infty)$ and give the domain of $f^{-1}(x)$.

2 marks

Total = 13 marks

Question 2
A mass is suspended from the ceiling on the end of a spring. It is pulled down and then released. The mass oscillates up and down. The length of the spring x cm at any time t seconds, is given by the equation:

$$x = a + b\sin\left(\frac{2\pi t}{3}\right) \text{ where } a > b$$

a. What is the maximum length of the spring?

_____

1 mark

b. What is the minimum length of the spring?

_____

1 mark

c. If $a = 64$ and $b = 8$, find the first 4 times when the spring has a length of 68 cm.
Give **exact** values for these answers.

_____

_____

_____

_____

_____

_____

_____

3 marks

d. How long does one complete oscillation take?

_____

_____

1 mark

Question 2 (continued)

e. For how long is the length of the spring greater than 60 cm in the first 2 seconds?

2 marks

f. What is the average rate of change of the length of the spring from $t = 0$ to $t = 0.25$ seconds?

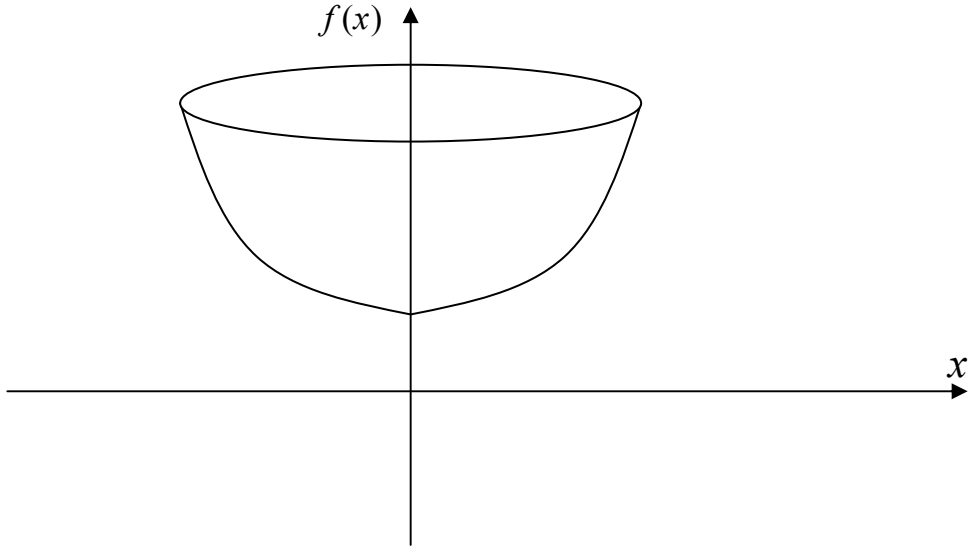
2 marks

g. What is the **exact** value of the rate of change of the length of the spring when $t = 2.75$ seconds?

2 marks

Total = 12 marks

Question 3



The curved part of a bowl shown on the axes above can be modelled by the equation of a parabola that contains the points (0 , 1) and (-3 , 3.25) , $-10 \leq x \leq 10$

a. Find the equation of the parabola.

2 marks

b. Find the diameter of the circular top of the bowl.

1 mark

c. Express x in terms of y.

1 mark

Question 3 (continued)

d. The volume of the bowl can be found using the formula $V = \int_a^b \pi x^2 dy$.

Find the volume of the bowl and give your answer to the nearest cubic unit.

4 marks

e. Water is poured into the bowl so that the radius of the top of the water level is 3 units.
Give the **exact** value of the volume of water in the container.

2 marks

f. Differentiate $\frac{1}{9}e^{3x}(3x + 2)$.

2 marks

Question 3 (continued)

g. The bowl is being filled with water so that the rate of change of height of water at any time t is $4te^{3t}$.

i. Find the depth of water in the bowl when $t = 1$. Give your answer to 4 decimal places.

3 marks

ii. Find the rate of change of volume when $t = 1$. Give your answer to 2 decimal places.

2 marks

Total = 17 marks

Question 4

a. A retailer sells tulip bulbs in packets of 5. The probability that the bulbs in the packet will flower has the probability distribution given in the table below.

x	0	1	2	3	4	5
$\Pr(X = x)$	0.04	0.01	0.05	0.2	0.4	0.3

i. How many bulbs would you expect to flower from each packet?
Give you answer to 2 decimal places

_____

_____

1 mark

ii. What is the standard deviation of this distribution. Give your answer to 2 decimal places.

_____

_____

_____

_____

2 marks

b. From past experience, the retailer knows that if a tulip bulb flowers one year, it has an 80% chance of flowering the next year, and if a tulip bulb does **not** flower one year, it has an 85% chance of **not** flowering the next year.

i. What is the probability that a tulip bulb that did **not** flower in 2005 will flower in 2006?

_____

1 mark

ii. What is the probability that a tulip bulb that flowered in 2005 will **not** flower in 2008?
Give your answer to 4 decimal places.

_____

_____

_____

_____

_____

2 marks

Question 4 (continued)

c. The time, t , in years, taken for a bulb to flower after it is planted is described by the probability density function:

$$f(t) = \begin{cases} at^2(1-t) & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

i. Find the value of a.

1 mark

ii. What is the probability that a bulb will flower within 3 months of planting? Give your answer to 2 decimal places.

1 mark

iii. Given that a particular tulip flowered within 6 months of planting, what is the probability that it flowered within 3 months? Give your answer to 4 decimal places.

2 marks

Question 4 (continued)

d. A nursery wholesaler sells tulip bulbs in boxes of 1000. It is known that the weight of the boxes is normally distributed and that 10% of the boxes weigh more than 3128 g while 30% of the boxes weigh less than 2947.6 g.

i. A nursery retailer buys 10 boxes of tulip bulbs from the wholesaler. What is the probability that not more than one of the boxes will have a weight less than 2947.6 g?
Give your answer to 4 decimal places.

1 mark

ii. What is the mean weight and the standard deviation of the wholesaler's boxes.
Give your answers to the nearest g.

5 marks

Total = 16 marks

END OF QUESTION AND ANSWER BOOK

KILBAHA MULTIMEDIA PUBLISHING **PO BOX 2227** **KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com