

Year 2006

Mathematical Methods

Examination 2

Suggested Solutions



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SECTION 1

Question 1 **Answer C**

$y = x^2$ translate 3 units down $y = x^2 - 3$
 translate 2 units to the right $y = (x - 2)^2 - 3$

Question 2 **Answer B**

$\tan(2x) = 1$
 $2x = \tan^{-1}(1) = \frac{\pi}{4}$
 $x = \frac{\pi}{8}$ the smallest positive value

Question 3 **Answer A**

$f(x) = 5 - x$ $f(-2) = 7$ $f(7) = -2$
 domain $[-2, 7)$ range $(-2, 7]$

Question 4 **Answer B**

$f(x) = 3\sin\left(\frac{\pi}{2} - \pi\right) + 4,$
 the range is 4 ± 3 $[1, 7]$

Question 5 **Answer D**

3 white, 7 yellow, total 10 balls,
 without replacement

$$\Pr(YYY) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{7}{24}$$

Question 6 **Answer D**

$f : [a, \infty) \rightarrow R$ $f(x) = \log_e(x^4)$
 is not a one-one function, to have
 an inverse function, it must be one-one
 so $a > 0$

Question 7 **Answer B**

$g(x) = \log_e|x - b|$ $b \in R,$
 the maximal domain is
 $x - b \neq 0$ $R \setminus \{b\}$

Question 8 **Answer E**

$y = \log_3(x)$
 reflect in the x -axis $y = -\log_3(x)$
 translate 5 units up $y = -\log_3(x) + 5$
 translate 2 units right $y = -\log_3(x - 2) + 5$

Question 9 **Answer B**

$$y = 3a^{2x} + b \qquad y - b = 3a^{2x}$$

$$a^{2x} = \frac{y - b}{3} \qquad 2x = \log_a\left(\frac{y - b}{3}\right)$$

$$x = \frac{1}{2} \log_a\left(\frac{y - b}{3}\right)$$

Question 10 **Answer A**

Given $\frac{dr}{dt} = 3 \text{ cm/min}$

$$\frac{dV}{dt} = ? \text{ when } r = 6$$

$$V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 12\pi r^2$$

$$\left. \frac{dV}{dt} \right|_{r=6} = 432\pi$$

Question 11 **Answer D**

$|2k + 1| = k + 1$ then

$$2k + 1 = k + 1 \quad \text{or} \quad -2k - 1 = k + 1$$

$$k = 0 \qquad \qquad \qquad -3k = 2$$

$$k = 0 \quad \text{or} \quad k = -\frac{2}{3}$$

Question 12 **Answer C**

$$X \sim Bi\left(n = 10, p = \frac{1}{2}\right)$$

$$\Pr(X \geq 8) = 0.0547$$

Question 13 Answer E

$$f \quad y = \frac{2}{3x+6} - 1 \quad f^{-1} \quad x = \frac{2}{3y+6} - 1$$

transposing $x+1 = \frac{2}{3y+6}$

$$3y+6 = \frac{2}{x+1} \quad 3y = \frac{2}{x+1} - 6$$

$$y = f^{-1}(x) = \frac{2}{3x+3} - 2$$

Question 14 Answer A

$f'(x)$ or the gradient is negative
for $-3 < x < 3$

Question 15 Answer D

Let $A_1 = \int_{-1}^1 f(x) dx$ but $A_1 < 0$

$$A_2 = \int_1^4 f(x) dx \quad \text{and} \quad A_3 = \int_4^6 f(x) dx$$

but $A_3 < 0$ the area is $A_2 - A_1 - A_3$
 $= \int_1^4 f(x) dx - \int_{-1}^1 f(x) dx - \int_4^6 f(x) dx$

Question 16 Answer A

$$f'(x) = g'(x) + 3$$

$$f(x) = g(x) + 3x + c \quad \text{at} \quad x = 0$$

$$f(0) = g(0) + 0 + c$$

$$2 = 1 + c \quad c = 1$$

so $f(x) = g(x) + 3x + 1$

Question 17 Answer A

let $y = \frac{\sin(3x)}{2e^x - x}$

Quotient rule

$$u = \sin(3x) \quad v = 2e^x - x$$

$$\frac{du}{dx} = 3\cos(3x) \quad \frac{dv}{dx} = 2e^x - 1$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(2e^x - x)3\cos(3x) - \sin(3x)(2e^x - 1)}{(2e^x - x)^2}$$

$$\frac{dy}{dx} = \frac{3(2e^x - x)\cos(3x) - (2e^x - 1)\sin(3x)}{4e^{2x} - 4xe^x + x^2}$$

Question 18 Answer D

Since $\sum \Pr(X = x) = 1$

$$a + b + 0.4 = 1$$

and $E(X) = \sum x \Pr(X = x)$

$$E(X) = -a + 0.4 = 0.3$$

so $a = 0.1 \quad b = 1 - 0.4 - a = 0.5$

Question 19 Answer A

$$\int_0^k (1 + e^{\frac{x}{k}}) dx = \left[x + ke^{\frac{x}{k}} \right]_0^k$$

$$= (k + ke^1) - (0 + k) = 1$$

$$ke = 1$$

$$k = \frac{1}{e} = e^{-1}$$

Question 20 Answer E

Let $y = f(\sin(4x)) = f(u)$

$$u = \sin(4x) \quad \frac{dy}{du} = f'(u)$$

$$\frac{du}{dx} = 4\cos(4x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 4\cos(4x) f'(\sin(4x))$$

Question 21 Answer D

$$X \sim N(\mu = 200, \sigma^2 = 10^2)$$

$$\Pr(X < 208) = 0.788$$

Question 22 Answer C

$y = a \log_e(x - b)$ where $a < 0 \quad b > 0$

reflect $y = \log_e(x)$ in the x -axis,

translate b units to the right.

SECTION 2

Question 1

$$f : [0, 2\pi] \rightarrow \mathbb{R} \quad f(x) = 2 \sin(x)$$

a. i. $f'(x) = 2 \cos(x)$

ii. $|f'(x)|_{\max} = 2 \quad |f'(x)|_{\min} = 0$

b. i. $f'(x) = 2 \cos(x) = 1 \quad \cos(x) = \frac{1}{2}$

$$x = \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

ii. $f'\left(\frac{\pi}{3}\right) = 2 \cos\left(\frac{\pi}{3}\right) = 1 = m_T$

$$f\left(\frac{\pi}{3}\right) = 2 \sin\left(\frac{\pi}{3}\right) = \sqrt{3} \quad \text{at } A\left(\frac{\pi}{3}, \sqrt{3}\right) \quad \text{tangent } y - \sqrt{3} = 1\left(x - \frac{\pi}{3}\right)$$

$$y = x + \sqrt{3} - \frac{\pi}{3}$$

iii. tangent crosses x -axis when $y = 0 \quad x = \frac{\pi}{3} - \sqrt{3} \quad \left(\frac{\pi}{3} - \sqrt{3}, 0\right)$

tangent crosses y -axis when $x = 0 \quad y = \sqrt{3} - \frac{\pi}{3} \quad \left(0, \sqrt{3} - \frac{\pi}{3}\right)$

c. At $B \quad x = \frac{5\pi}{3} \quad f\left(\frac{5\pi}{3}\right) = -\sqrt{3} \quad B\left(\frac{5\pi}{3}, -\sqrt{3}\right)$

tangent at $B \quad y + \sqrt{3} = 1\left(x - \frac{5\pi}{3}\right) \quad y = x - \sqrt{3} - \frac{5\pi}{3}$

so $x_A = \sqrt{3} - \frac{\pi}{3} \quad x_B = -\sqrt{3} - \frac{5\pi}{3}$

$$m = x_A - x_B = 2\sqrt{3} + \frac{4\pi}{3}$$

Question 2

a. i. $\Pr(PPP) = 0.4 \times 0.3^2 = 0.036 = \frac{9}{250}$

ii. $\Pr(GPP) + \Pr(PGP) + \Pr(PPG)$

$= 0.6 \times 0.4 \times 0.3 + 0.4 \times 0.7 \times 0.4 + 0.4 \times 0.3 \times 0.7 = 0.268 = \frac{67}{250}$

b. $X \sim N(\mu = 60, \sigma^2 = ?)$

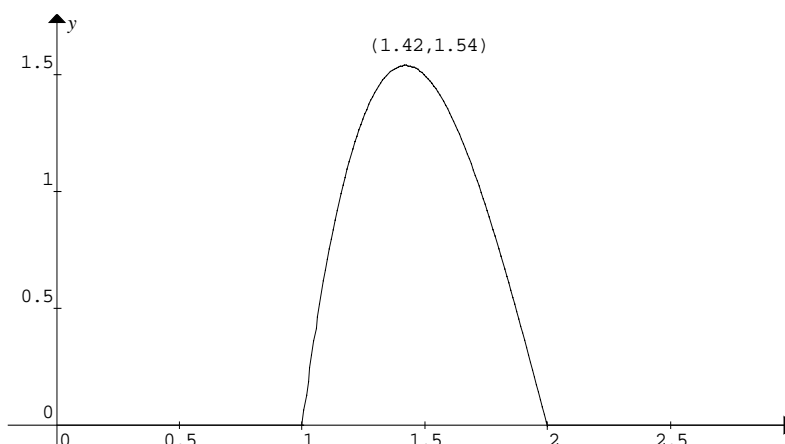
$\Pr(X < 50) = 0.2$

$Z = \frac{X - \mu}{\sigma} \quad \frac{50 - 60}{\sigma} = \frac{-10}{\sigma} = -0.8416$

$\sigma = \frac{10}{0.6416} = 11.9 \text{ minutes}$

c.

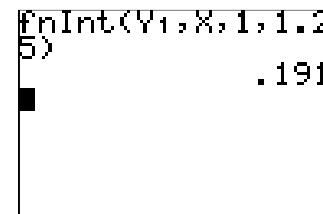
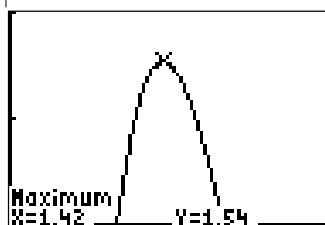
$$f(t) = \begin{cases} 4t^3 - 24t^2 + 44t - 24 & \text{for } 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$



d. 75 minutes = 1.25 hours

$\Pr(T < 1.25) = \int_1^{1.25} (4t^3 - 24t^2 + 44t - 24) dt$

$= 0.191$



e. $Y \sim \text{Bi}(n = 5, p = 0.8086)$

$\Pr(Y = 4) = \binom{5}{4} 0.8086^4 \times 0.1914$

$= 0.41$

f. $\int_1^m (4t^3 - 24t^2 + 44t - 24) dt = \frac{1}{2}$

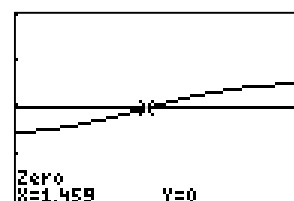
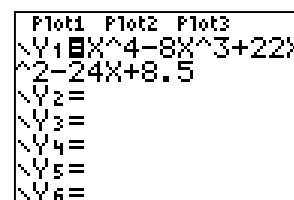
$[t^4 - 8t^3 + 22t^2 - 24t]_1^m = \frac{1}{2}$

$(m^4 - 8m^3 + 22m^2 - 24m) - (1 - 8 + 22 - 24) = \frac{1}{2}$

$m^4 - 8m^3 + 22m^2 - 24m + 8.5 = 0$

$m = 1.4588 \text{ hours}$

$m = 88 \text{ minutes}$



Question 3

a. $f(x) = 3 - e^x - e^{-x}$

$$f(0) = 3 - 1 - 1 = 1 = b$$

b. When $y = 0$ $3 - e^x - e^{-x} = 0$ let $u = e^x$

$$3 - u - \frac{1}{u} = 0 \text{ multiply by } u$$

$$3u - u^2 - 1 = 0$$

$$u^2 - 3u + 1 = 0$$

$$\Delta = 9 - 4 = 5$$

$$u = e^x = \frac{3 \pm \sqrt{5}}{2} \text{ but } u > 0 \text{ take positive}$$

$$a = \log_e \left(\frac{3 + \sqrt{5}}{2} \right)$$

c. i.

x	-0.5	0	0.5
y	0.74	1.00	0.74

ii. Area = $0.5 \times 0.74 + 0.5 \times 1 + 0.5 \times 0.74$
 $= 1.2 \text{ km}^2$

iii. $V = 1.2 \text{ mV}$

d. i. $(0,1)(1,0)(-1,0) y = ax^2 + c$

$$c = 1 \quad a = -1 \quad y = 1 - x^2$$

ii. $A = \int_{-1}^1 (1 - x^2) dx = 2 \int_0^1 (1 - x^2) dx$

$$= 2 \left[x - \frac{1}{3} x^3 \right]_0^1 = 2 \left[\left(1 - \frac{1}{3} \right) - (0) \right]$$

$$= \frac{4}{3} = 1.33 \text{ km}^2$$

Question 4

a. i. $f(x) = px^3 + qx^2 + rx + s$ at $B(0,7)$ $f(0) = 7 = s = 7$

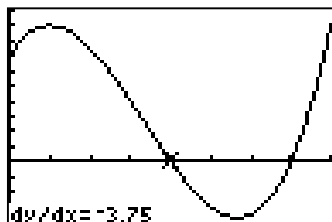
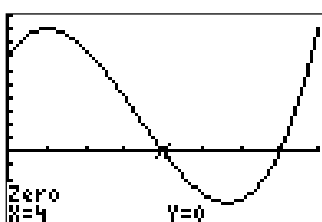
ii. $f'(x) = 3px^2 + 2qx + r$ $f'(0) = 4.25 = r$

b. at $C(1,9)$ $f(1) = 9 \Rightarrow 9 = p + q + 4.25 + 7 \Rightarrow p + q = -2.25$ (1)

$f'(1) = 0 \Rightarrow 3p + 2q + 4.25 = 0 \Rightarrow 3p + 2q = -4.25$ (2)

so $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$

c. $D(4,0)$ $F(7,0)$ from CALC



d. $f'(4) = -3.75$

e. $E(5.66, -3.704)$ so the distance is 3.70

f. inflexion point is $(3.33, 2.648)$ max value $f'(3.33) = -4.08$

absolute value of $f'(3.33) = 4.1$ from CALC

g. $g(x) = \frac{a}{1-bx}$ at $B(0,7)$ $g(0) = 7 = a$ $a = 7$

$g'(x) = ba(1-bx)^{-2} = \frac{ba}{(1-bx)^2}$ $g'(0) = ba = 4.25$ $b = \frac{4.25}{7} = \frac{17}{28}$

h. $A_1 = \int_{-2}^0 \frac{a}{1-bx} dx = -\frac{a}{b} \log_e((1-bx)) \Big|_{-2}^0 = \frac{-196}{17} [\log_e(1) - \log_e(1+2b)]$
 $= \frac{196}{17} \log_e\left(\frac{31}{14}\right) \approx 9.1651$

$A_2 = \int_0^4 (0.25x^3 - 2.5x^2 + 4.25x + 7) dx = \left[\frac{1}{16}x^4 - \frac{5}{6}x^3 + \frac{17}{8}x^2 + 7x \right]_0^4$
 $= \left[\frac{1}{16} \times 256 - \frac{5}{6} \times 64 + \frac{17}{8} \times 16 + 7 \times 4 - 0 \right] = 24 \frac{2}{3}$

total area $A_1 + A_2 = 9.1651 + 24.667 = 33.83$ units²



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