Year 2006

Mathematical Methods

Examination 2

Suggested Solutions

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SECTION 1

Question 1 **Answer C**

$y = x^2$ translate 3 units down $y = x^2 - 3$

translate 2 units to the right $y = (x-2)^2 - 3$

Question 2 **Answer B**

$\tan(2x) = 1$

$2x = \tan^{-1}(1) = \frac{\pi}{4}$

$x = \frac{\pi}{8}$ the smallest positive value

Question 3 **Answer A**

$f(x) = 5 - x \quad f(-2) = 7 \quad f(7) = -2$

domain $[-2, 7)$ range $(-2, 7]$

Question 4 **Answer B**

$f(x) = 3\sin\left(\frac{\pi}{2} - \pi\right) + 4,$

the range is $4 \pm 3 \quad [1, 7]$

Question 5 **Answer D**

3 white, 7 yellow, total 10 balls,
without replacement

$\Pr(YYY) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{7}{24}$

Question 6 **Answer D**

$f : [a, \infty) \to R \quad f(x) = \log_e(x^4)$

is not a one-one function, to have
an inverse function, it must be one-one
so $a > 0$

Question 7 **Answer B**

$g(x) = \log_e|x - b| \quad b \in R,$

the maximal domain is

$x - b \neq 0 \quad R \setminus \{b\}$

Question 8 **Answer E**

$y = \log_3(x)$

reflect in the *x*-axis $y = -\log_3(x)$

translate 5 units up $y = -\log_3(x) + 5$

translate 2 units right $y = -\log_3(x-2) + 5$

Question 9 **Answer B**

$y = 3a^{2x} + b \qquad y - b = 3a^{2x}$

$a^{2x} = \frac{y-b}{3} \qquad 2x = \log_a\left(\frac{y-b}{3}\right)$

$x = \frac{1}{2}\log_a\left(\frac{y-b}{3}\right)$

Question 10 **Answer A**

Given $\frac{dr}{dt} = 3\text{cm/min}$

$\frac{dV}{dt} = ?$ when $r = 6$

$V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 12\pi r^2$

$\left.\frac{dV}{dt}\right|_{r=6} = 432\pi$

Question 11 **Answer D**

$|2k+1| = k+1$ then

$2k+1 = k+1$ or $-2k-1 = k+1$

$k = 0 \qquad -3k = 2$

$k = 0$ or $k = -\frac{2}{3}$

Question 12 **Answer C**

$X \sim Bi\left(n = 10,\ p = \frac{1}{2}\right)$

$\Pr(X \geq 8) = 0.0547$

Question 13 Answer E

$f \quad y = \dfrac{2}{3x+6} - 1 \qquad f^{-1} \quad x = \dfrac{2}{3y+6} - 1$

transposing $\quad x+1 = \dfrac{2}{3y+6}$

$3y+6 = \dfrac{2}{x+1} \qquad 3y = \dfrac{2}{x+1} - 6$

$y = f^{-1}(x) = \dfrac{2}{3x+3} - 2$

Question 14 Answer A

$f'(x)$ or the gradient is negative

for $-3 < x < 3$

Question 15 Answer D

Let $A_1 = \int_{-1}^{1} f(x)\,dx$ but $A_1 < 0$

$A_2 = \int_{1}^{4} f(x)\,dx$ and $A_3 = \int_{4}^{6} f(x)\,dx$

but $A_3 < 0$ the area is $A_2 - A_1 - A_3$

$= \int_{1}^{4} f(x)\,dx - \int_{-1}^{1} f(x)\,dx - \int_{1}^{6} f(x)\,dx$

Question 16 Answer A

$f'(x) = g'(x) + 3$

$f(x) = g(x) + 3x + c$ at $x = 0$

$f(0) = g(0) + 0 + c$

$2 = 1 + c \qquad c = 1$

so $f(x) = g(x) + 3x + 1$

Question 17 Answer A

let $y = \dfrac{\sin(3x)}{2e^x - x}$

Quotient rule

$u = \sin(3x) \qquad v = 2e^x - x$

$\dfrac{du}{dx} = 3\cos(3x) \qquad \dfrac{dv}{dx} = 2e^x - 1$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{(2e^x - x)3\cos(3x) - \sin(3x)(2e^x - 1)}{(2e^x - x)^2}$

$\dfrac{dy}{dx} = \dfrac{3(2e^x - x)\cos(3x) - (2e^x - 1)\sin(3x)}{4e^{2x} - 4xe^x + x^2}$

Question 18 Answer D

Since $\quad \sum \Pr(X = x) = 1$

$a + b + 0.4 = 1$

and $\quad E(X) = \sum x \Pr(X = x)$

$E(X) = -a + 0.4 = 0.3$

so $\quad a = 0.1 \quad b = 1 - 0.4 - a = 0.5$

Question 19 Answer A

$\int_0^k \left(1 + e^{\frac{x}{k}}\right) dx = \left[x + ke^{\frac{x}{k}}\right]_0^k$

$= \left(k + ke^1\right) - (0 + k) = 1$

$ke = 1$

$k = \dfrac{1}{e} = e^{-1}$

Question 20 Answer E

Let $\quad y = f(\sin(4x)) = f(u)$

$u = \sin(4x) \qquad \dfrac{dy}{du} = f'(u)$

$\dfrac{du}{dx} = 4\cos(4x)$

$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = 4\cos(4x)\, f'(\sin(4x))$

Question 21 Answer D

$X \sim N(\mu = 200,\ \sigma^2 = 10^2)$

$\Pr(X < 208) = 0.788$

Question 22 Answer C

$y = a\log_e(x - b)$ where $a < 0 \quad b > 0$

reflect $y = \log_e(x)$ in the x-axis,

translate b units to the right.

SECTION 2

Question 1

$f:[0,2\pi] \to R \quad f(x) = 2\sin(x)$

a. **i.** $f'(x) = 2\cos(x)$

ii. $|f'(x)|_{\max} = 2 \qquad |f'(x)|_{\min} = 0$

b. **i.** $f'(x) = 2\cos(x) = 1 \qquad \cos(x) = \frac{1}{2}$

$x = \frac{\pi}{3} \;,\; 2\pi - \frac{\pi}{3} = \frac{\pi}{3} \;,\; \frac{5\pi}{3}$

ii. $f'\left(\frac{\pi}{3}\right) = 2\cos\left(\frac{\pi}{3}\right) = 1 = m_T$

$f\left(\frac{\pi}{3}\right) = 2\sin\left(\frac{\pi}{3}\right) = \sqrt{3}$ at $A\left(\frac{\pi}{3}, \sqrt{3}\right)$ tangent $y - \sqrt{3} = 1\left(x - \frac{\pi}{3}\right)$

$y = x + \sqrt{3} - \frac{\pi}{3}$

iii. tangent crosses x-axis when $y = 0 \qquad x = \frac{\pi}{3} - \sqrt{3} \qquad \left(\frac{\pi}{3} - \sqrt{3}, 0\right)$

tangent crosses y-axis when $x = 0 \qquad y = \sqrt{3} - \frac{\pi}{3} \quad \left(0, \sqrt{3} - \frac{\pi}{3}\right)$

c. At B $x = \frac{5\pi}{3} \quad f\left(\frac{5\pi}{3}\right) = -\sqrt{3} \quad B\left(\frac{5\pi}{3}, -\sqrt{3}\right)$

tangent at $B \quad y + \sqrt{3} = 1\left(x - \frac{5\pi}{3}\right) \qquad y = x - \sqrt{3} - \frac{5\pi}{3}$

so $x_A = \sqrt{3} - \frac{\pi}{3} \qquad x_B = -\sqrt{3} - \frac{5\pi}{3}$

$m = x_A - x_B = 2\sqrt{3} + \frac{4\pi}{3}$

Question 2

a. **i.** $\Pr(PPP) = 0.4 \times 0.3^2 = 0.036 = \dfrac{9}{250}$

ii. $\Pr(GPP) + \Pr(PGP) + \Pr(PPG)$

$= 0.6 \times 0.4 \times 0.3 + 0.4 \times 0.7 \times 0.4 + 0.4 \times 0.3 \times 0.7 = 0.268 = \dfrac{67}{250}$

b. $X \sim N(\mu = 60, \sigma^2 = ?)$

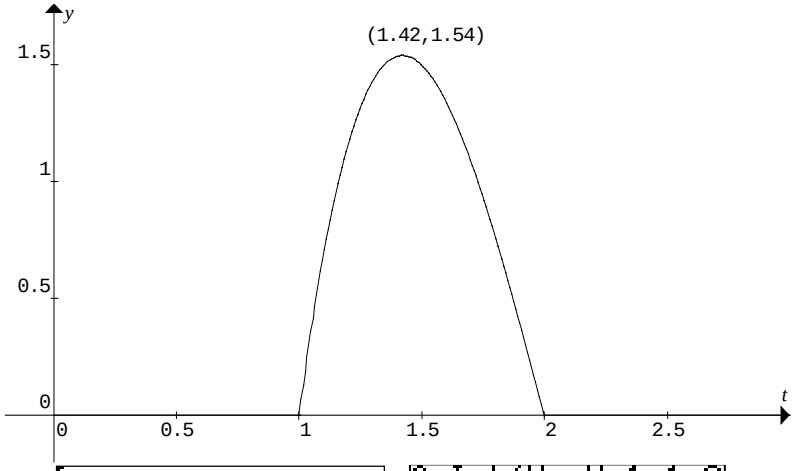
$\Pr(X < 50) = 0.2$

$Z = \dfrac{X - \mu}{\sigma} \qquad \dfrac{50 - 60}{\sigma} = \dfrac{-10}{\sigma} = -0.8416$

$\sigma = \dfrac{10}{0.6416} = 11.9 \text{ minutes}$

c.

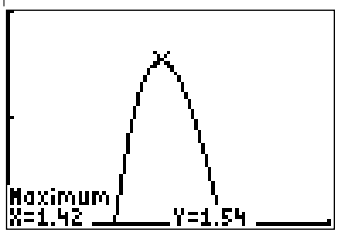
$$f(t) = \begin{cases} 4t^3 - 24t^2 + 44t - 24 & \text{for } 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

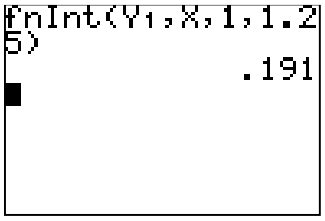


d. $75 \text{ minutes} = 1.25 \text{ hours}$

$\Pr(T < 1.25) = \int_1^{1.25} (4t^3 - 24t^2 + 44t - 24)\,dt$

$= 0.191$





e. $Y \sim \text{Bi}(n = 5, p = 0.8086)$

$\Pr(Y = 4) = \binom{5}{4} 0.8086^4 \times 0.1914$

$= 0.41$

f. $\int_1^m (4t^3 - 24t^2 + 44t - 24)\,dt = \dfrac{1}{2}$

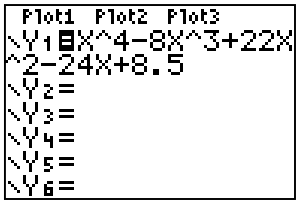
$\left[t^4 - 8t^3 + 22t^2 - 24t\right]_1^m = \dfrac{1}{2}$

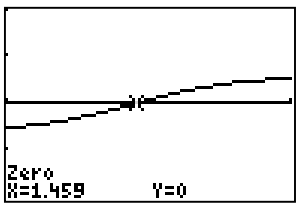
$(m^4 - 8m^3 + 22m^2 - 24m) - (1 - 8 + 22 - 24) = \dfrac{1}{2}$

$m^4 - 8m^3 + 22m^2 - 24m + 8.5 = 0$

$m = 1.4588 \text{ hours}$

$m = 88 \text{ minutes}$





Question 3

a. $f(x) = 3 - e^x - e^{-x}$

$f(0) = 3 - 1 - 1 = 1 = b$

b. When $y = 0$ $3 - e^x - e^{-x} = 0$ let $u = e^x$

$$3 - u - \frac{1}{u} = 0 \text{ multiply by } u$$

$$3u - u^2 - 1 = 0$$

$$u^2 - 3u + 1 = 0$$

$$\Delta = 9 - 4 = 5$$

$$u = e^x = \frac{3 \pm \sqrt{5}}{2} \quad \text{but} \quad u > 0 \quad \text{take positive}$$

$$a = \log_e\left(\frac{3+\sqrt{5}}{2}\right)$$

c. **i.**

x	-0.5	0	0.5
y	0.74	1.00	0.74

ii. Area = $0.5 \times 0.74 + 0.5 \times 1 + 0.5 \times 0.74$

$= 1.2\,\text{km}^2$

iii. $V = 1.2mV$

d. **i.** $(0,1)(1,0)(-1,0)\ y = ax^2 + c$

$c = 1 \quad a = -1 \quad y = 1 - x^2$

ii. $A = \int_{-1}^{1}(1 - x^2)\,dx = 2\int_{0}^{1}(1 - x^2)\,dx$

$$= 2\left[x - \frac{1}{3}x^3\right]_0^1 = 2\left[\left(1 - \frac{1}{3}\right) - (0)\right]$$

$$= \frac{4}{3} = 1.33\,\text{km}^2$$

Question 4

a. **i.** $f(x) = px^3 + qx^2 + rx + s$ at $B(0,7)$ $f(0) = 7 = s = 7$

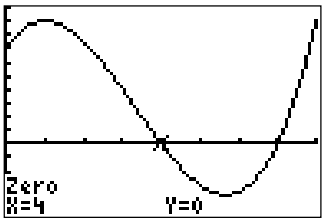
ii. $f'(x) = 3px^2 + 2qx + r$ $f'(0) = 4.25 = r$

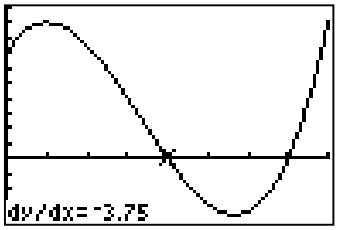
b. at $C(1,9)$ $f(1) = 9 \Rightarrow 9 = p + q + 4.25 + 7$ $\Rightarrow p + q = -2.25 \quad (1)$

$f'(1) = 0 \Rightarrow 3p + 2q + 4.25 = 0$ $\Rightarrow \quad 3p + 2q = -4.25 \quad (2)$

so $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$

c. $D(4,0)$ $F(7,0)$ from CALC





d. $f'(4) = -3.75$

e. $E(5.66, -3.704)$ so the distance is 3.70

f. inflexion point is $(3.33, 2.648)$ max value $f'(3.33) = -4.08$

absolute value of $f'(3.33) = 4.1$ from CALC

g. $g(x) = \dfrac{a}{1-bx}$ at $B(0,7)$ $g(0) = 7 = a$ $a = 7$

$g'(x) = ba(1-bx)^{-2} = \dfrac{ba}{(1-bx)^2}$ $g'(0) = ba = 4.25$ $b = \dfrac{4.25}{7} = \dfrac{17}{28}$

h. $A_1 = \displaystyle\int_{-2}^{0} \frac{a}{1-bx}\,dx = -\frac{a}{b}\log_e\left((1-bx)\right]_{-2}^{0} = \frac{-196}{17}\left[\log_e(1) - \log_e(1+2b)\right]$

$= \dfrac{196}{17}\log_e\left(\dfrac{31}{14}\right) \simeq 9.1651$

$A_2 = \displaystyle\int_0^4 \left(0.25x^3 - 2.5x^2 + 4.25x + 7\right)dx = \left[\frac{1}{16}x^4 - \frac{5}{6}x^3 + \frac{17}{8}x^2 + 7x\right]_0^4$

$= \left[\dfrac{1}{16}\times 2.56 - \dfrac{5}{6}\times 64 + \dfrac{17}{8}\times 16 + 7\times 4 - 0\right] = 24\dfrac{2}{3}$

total area $A_1 + A_2 = 9.1651 + 24.667 = 33.83$ units^2

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Mathematics 2007

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