The Mathematical Association of Victoria

2006 MATHEMATICAL METHODS (CAS)

VCAA Sample Examination 1 and 2

Suggested Answers and Solutions

Examination 1

The examination will consist of short answer questions which are to be answered without the use of technology.

Examination 2

The examination will consist of two parts. Part 1 will be a multiple-choice section containing 22 questions and Part II will consist of extended answer questions, involving multi-stage solutions of increasing complexity.

The sample exams are found at:
www.vcaa.vic.edu.au/vce/studies/mathematics/cas/pastexams /2006cas_sample.pdf

These answers and solutions have been written and published to assist students in their preparations for the 2006 Mathematical Methods (CAS) Examinations. The answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority.

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"Cliveden", 61 Blyth Street, Brunswick, 3056
Phone: (03) 9380 2399 Fax: (03) 9389 0399
E-mail: office@mav.vic.edu.au website: http://www.mav.vic.edu.au

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2006 Mathematical Methods 3&4 VCAA Sample Examination 1 Suggested answers and solutions

Question 1

a Let $y = 2\log_e(x+1)$

For the inverse $x = 2\log_e(y+1)$

$$\frac{x}{2} = \log_e(y+1)$$

$$y + 1 = e^{\frac{x}{2}}$$

$$y = e^{\frac{x}{2}} - 1$$

$$f^{-1}(x) = e^{\frac{x}{2}} - 1$$

b Dom of $f^{-1}(x)$ = range of $f(x)$

$= R$

Note that the graph of f is a transformation of the graph of $y = \log_e(x)$, with a translation of 1 unit to the left and a dilation by a scale factor of two from the X axis. f therefore has the same range as $y = \log_e(x)$, i.e. R.

Question 2

a Let $y = uv$

where $u = 3x^4$ and $v = \tan x$

$$\frac{du}{dx} = 12x^3 \text{ and } \frac{dv}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 3x^4\sec^2 x + 12x^3\tan x$$

Alternatively,

$$\frac{dy}{dx} = \frac{12x^3\sin(x)}{\cos(x)} + \frac{3x^4}{\cos^2(x)}$$

$$= \frac{12x^3\sin(x)\cos(x) + 3x^4}{\cos^2(x)}$$

b

$$f(x) = \int f'(x)dx$$

$$\therefore \quad f(x) = \int \frac{1}{x-2}dx$$

$$f(x) = \log_e|x-2| + c$$

$$6 = \log_e|1-2| + c$$

$$= \log_e 1 + c$$

$$c = 6$$

$$f(x) = \log_e|x-2| + 6$$

The rule of f is $f(x) = \log_e(-x+2) + 6$.

Alternatively, $f(x) = \log_e(2-x) + 6$

Question 3

$$\tan(x) = \sqrt{3}$$

$$x = \frac{-2\pi}{3}, \frac{\pi}{3}$$

Question 4

a Amplitude $= 3$

Period $= \frac{2\pi}{2} = \pi$

b Endpoints: When $x = \pi$,

$$f(x) = 3\sin\left(2\left(\pi + \frac{\pi}{3}\right)\right)$$

$$= 3\sin\frac{8\pi}{3}$$

$$= 3\sin\frac{\pi}{3}$$

$$= \frac{3\sqrt{3}}{2}$$

Since the period is π; $\left(-\pi, \frac{3\sqrt{3}}{2}\right)$, $\left(0, \frac{3\sqrt{3}}{2}\right)$ and $\left(\pi, \frac{3\sqrt{3}}{2}\right)$ are points on the graph.

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x axis intercepts: When $y = 0$,

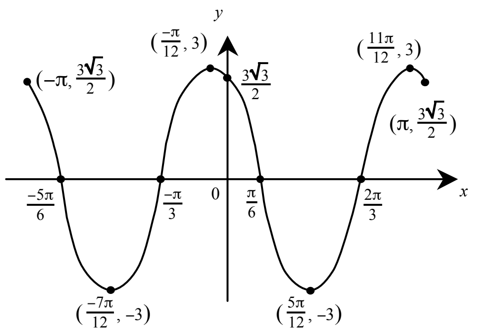
$$3\sin\left(2\left(x+\tfrac{\pi}{3}\right)\right) = 0$$

$$\sin\left(2\left(x+\tfrac{\pi}{3}\right)\right) = 0$$

$$2\left(x+\tfrac{\pi}{3}\right) = -\pi,\ 0,\ \pi,\ 2\pi$$

$$x+\frac{\pi}{3} = \frac{-\pi}{2},\ 0,\ \frac{\pi}{2},\ \pi$$

$$x = \frac{-5\pi}{6},\ \frac{-\pi}{3},\ \frac{\pi}{6},\ \frac{2\pi}{3}$$



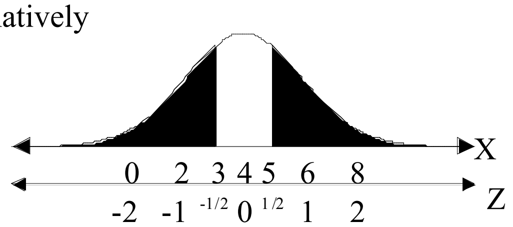
Question 5

a $\Pr(X > 4) = 0.5$

b
$$\begin{aligned}\Pr(X>5) &= \Pr(X<3)\\ &= \Pr\left(Z<\frac{3-4}{2}\right)\\ &= \Pr\left(Z<-\frac{1}{2}\right)\end{aligned}$$

$$b = -\frac{1}{2}$$

Alternatively



$$z = \frac{x-\mu}{\sigma}$$

$$\therefore z = \frac{5-4}{2} = \frac{1}{2}$$

$$\therefore \Pr(X>5) = \Pr\left(Z>\frac{1}{2}\right) = \Pr\left(Z<-\frac{1}{2}\right)\ (\textit{by symmetry})$$

$$\therefore b = -\frac{1}{2}$$

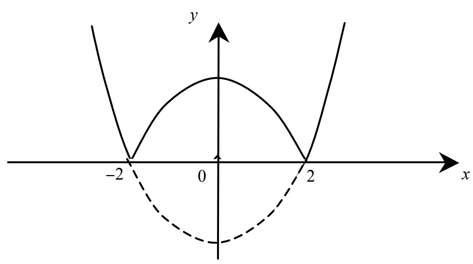
Question 6

a
$$\begin{aligned}\int_0^2 f(x)\,dx &= 1\\ \int_0^2 ax(2-x)\,dx &= 1\\ a\int_0^2 2x - x^2\,dx &= 1\\ a\left[x^2-\tfrac{1}{3}x^3\right]_0^2 &= 1\\ a\left[2^2-\tfrac{1}{3}\times 2^3\right] &= 1\\ \tfrac{4}{3}a &= 1\\ a &= \tfrac{3}{4}\end{aligned}$$

b
$$\begin{aligned}\Pr\left(X<\tfrac{1}{2}\right) &= \tfrac{3}{4}\int_0^{\frac{1}{2}} x(2-x)\,dx\\ &= \tfrac{3}{4}\left[x^2-\tfrac{1}{3}x^3\right]_0^{\frac{1}{2}}\\ &= \tfrac{3}{4}\left[\left(\tfrac{1}{2}\right)^2-\tfrac{1}{3}\times\left(\tfrac{1}{2}\right)^3\right]\\ &= \tfrac{3}{4}\times\tfrac{5}{24}\\ &= \frac{5}{32}\end{aligned}$$

Question 7

a



b
$$\begin{aligned}\text{Area} &= \int_{-2}^{2} |x^2-4| - (x^2-4)\,dx\\ &= -2\int_{-2}^{2} x^2-4\,dx\\ &= -2\left[\tfrac{1}{3}x^3-4x\right]_{-2}^{2}\\ &= -2\left[\left(\tfrac{8}{3}-8\right)-\left(\tfrac{-8}{3}+8\right)\right]\\ &= \tfrac{64}{3}\end{aligned}$$

Question 8

a $g(f(x)) = \log_e(x^2+1)$

b $g'(f(x)) = \frac{1}{x^2+1} \times 2x$

$= \frac{2x}{x^2+1}$

c $\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$

$= \frac{1}{2}\int g'(f(x))\,dx$

$= \frac{1}{2} g(f(x)) + c$

$= \frac{1}{2}\log_e(x^2+1)$, if $c = 0$

Alternatively, the antiderivative may be expressed as $\log_e(\sqrt{x^2+1}) + c$. Any real value of c may be used as the question asks for *an* antiderivative.

Question 9

For the curve, $\frac{dy}{dx} = 4x^3$

At the point of intersection of the curve and the tangent, $\frac{dy}{dx} = 4$

$4x^3 = 4$

$x^3 = 1$

$x = 1$

For the tangent, when $x = 1$, $y = 3$.

At the point (1, 3) on the curve,

$y = x^4 + c$

becomes $3 = 1^4 + c$

$c = 2$

Question 10

Required to find $\frac{dh}{dt}$ when $h = 3$.

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ [1]

Given $\frac{dV}{dt} = 3$ [2]

For a cone, $V = \frac{1}{3}\pi r^2 h$

and $\frac{r}{h} = \frac{2}{4}$ so $r = \frac{1}{2}h$

Area between the curves is $\frac{64}{3}$ units^2

therefore $V = \frac{1}{3}\pi \times \left(\frac{1}{2}h\right)^2 \times h$

$= \frac{\pi h^3}{12}$

$\frac{dV}{dh} = \frac{\pi h^2}{4}$

$\frac{dh}{dV} = \frac{4}{\pi h^2}$

Using [1] and [2]

$\frac{dh}{dt} = \frac{4}{\pi h^2} \times 3$

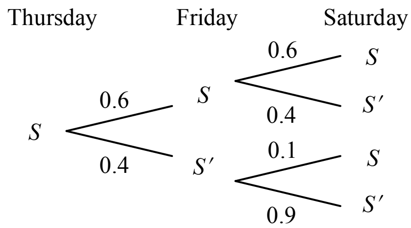
$= \frac{12}{\pi h^2}$

When $h = 3$, $\frac{dh}{dt} = \frac{12}{\pi \times 3^2}$

$= \frac{4}{3\pi}$

The water is rising at $\frac{4}{3\pi}$ m/min when the depth is 3 m.

Question 11



Pr('no snow' on Saturday)

= Pr('snow' on Friday and 'no snow' on Saturday) + Pr('no snow' on Friday and 'no snow' on Saturday)

= 0.6 × 0.4 + 0.4 × 0.9

= 0.24 + 0.36

= 0.6

END OF PAPER

2006 Mathematical Methods 3&4 CAS VCAA Sample Examination 2 – Section 1 Suggested answers and solutions for Multiple-choice questions

1	A	**2**	C	**3**	A	**4**	A	**5**	D
6	A	**7**	B	**8**	A	**9**	D	**10**	A
11	A	**12**	D	**13**	E	**14**	A	**15**	C
16	C	**17**	A	**18**	B	**19**	B	**20**	A
21	C	**22**	E						

1

$$\text{Average value} = \frac{1}{\pi - 0}\int_0^{\pi} \sin(x)\,dx$$
$$= \frac{-1}{\pi}[\cos(x)]_0^{\pi}$$
$$= \frac{-1}{\pi}(\cos(\pi) - \cos(0))$$
$$= \frac{2}{\pi}$$

A

2

$$(m-2)x + 3y = 6 \quad \boxed{1}$$
$$2x + (m+2)y = m \quad \boxed{2}$$

$2 \times \boxed{1} - (m-2) \times \boxed{2}$

$$6 - (m-2)(m+2)y = 12 - m(m-2)$$
$$y = \frac{12 - m(m-2)}{6 - (m-2)(m+2)}$$
$$= \frac{12 - m^2 + 2m}{10 - m^2}$$

There is a unique solution if $10 - m^2 \neq 0$, i.e., $m \in R \setminus \{-\sqrt{10}, \sqrt{10}\}$.

C

3 If $|p + 3| > 3$, then

$$p + 3 > 3 \text{ when } p + 3 \text{ is non-negative} \quad \boxed{1}$$

and

$$-(p + 3) > 3 \text{ when } p + 3 \text{ is negative} \quad \boxed{2}$$

From $\boxed{1}$, $p > 0$, and from $\boxed{2}$,

$$p + 3 < -3$$
$$p < -6$$

Hence for $|p + 3| > 3$, $\quad p > 0$ or $p < -6$

A

4 From the matrix equation, $\quad x' = x + 3$ and $y' = 2y + 2$

Therefore $\quad x = x' - 3$ and $y = \dfrac{y' - 2}{2}$

So $y = x^2$ becomes

$$\frac{y-2}{2} = (x-3)^2$$
$$y = 2(x-3)^2 + 2$$

A

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5 This question is intended for graphics calculator use, but the analytical solution is provided here for information.

$$h = 0.5\left(1 - e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right)\right)$$

$$\frac{dh}{dt} = 0.5\left(0 + 0.05e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right) + \frac{3\pi}{2}e^{-0.05t}\sin\left(\frac{3\pi t}{2}\right)\right)$$

$$= 0.025e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right) + \frac{3\pi}{4}e^{-0.05t}\sin\left(\frac{3\pi t}{2}\right)$$

When $t = 2.5$,

$$\frac{dh}{dt} = 0.025e^{-0.125}\cos\left(\frac{15\pi}{4}\right) + \frac{3\pi}{4}e^{-0.125}\sin\left(\frac{15\pi}{4}\right)$$

$$= 0.025e^{-0.125} \times \frac{1}{\sqrt{2}} + \frac{3\pi}{4}e^{-0.125} \times \frac{-1}{\sqrt{2}}$$

$= -1.45$, correct to two decimal places **D**

6

$$\text{Average rate of change} = \frac{N(10) - N(0)}{10}$$

$$= \frac{1000e - 1000}{10}$$

$= 172$, to the nearest integer **A**

7

$$y = ax^3 + bx^2 + cx + d$$

The graph intersects the y axis at 24, so $d = 24$.

From the shape of the graph, a is positive, so **B** is the answer. **B**

8

$$f(x) = e^{2x}$$
$$[f(x)]^2 = [e^{2x}]^2$$
$$= e^{4x}$$
$$= e^{2(2x)}$$
$$= f(2x)$$

$= f(y)$ where $y = 2x$ **A**

9 The graph shown could have rule $y = x^{\frac{1}{3}}$. **D**

10 For $f(x) = \log_e(x^2) + 1$, x^2 must be greater than 0, so $x \in R\backslash\{0\}$. **A**

11

$$f(x) = 2x^3 - 3x^2 + 6$$
$$f'(x) = 6x^2 - 6x$$

The graph has turning points where $f'(x) = 0$,

$$6x^2 - 6x = 0$$
$$6x(x - 1) = 0$$
$$x = 0 \text{ or } 1$$

For an inverse to exist, the graph must be one-to-one, and since the domain of f is given as $[a, \infty)$ the domain must be $[1, \infty)$, or a subset of this. Hence $a \geq 1$ is the answer. **A**

12 For $\int_0^t f(x)\,dx > 0$, $t > 0$, so $t \in (0, b]$ only. **D**

13

$$\lim_{\delta x \to 0} \sum_{i=1}^{n} (x_i\,\delta x) = \lim_{n \to 0} \frac{b - a}{n} \sum_{i=1}^{n} (x_i)$$

$$= \int_a^b x\,dx$$

$= \int_0^4 x\,dx$

$= \frac{1}{2}[x^2]_0^4$

$= \frac{1}{2}(16 - 0)$

$= 8$ **E**

14 $y = |\sin(x)|, \pi < x < 2\pi$

is equivalent to $y = -\sin(x), \pi < x < 2\pi$

$\frac{dy}{dx} = -\cos(x)$

When $x = k$, $\frac{dy}{dx} = -\cos(k)$ **A**

15 Since the gradient is zero at $x = 2$, and negative both to the immediate left and right of $x = 2$, there is a stationary point of inflection at $x = 2$. **C**

16 The graph of the function with equation $y = f(x)$ is transformed into the graph of the function $y = g(x)$ by a dilation of scale factor 2 from the x axis and a reflection in the x axis. **C**

17 For the curve with equation $y = 2x^{\frac{3}{2}}$, when $x = 4$, $y = 16$.

$\frac{dy}{dx} = 3x^{\frac{1}{2}}$

The gradient of the curve when $x = 4$ is 6, so the gradient of the normal to the curve is $\frac{-1}{6}$.

The equation of the normal to the curve at the point (4, 16) is

$y - 16 = \frac{-1}{6}(x - 4)$

$y = \frac{-1}{6}x + \frac{50}{3}$ **A**

18 The function $y = f(x)$ has positive gradient, increasing from left to right. **B**

19 $\Pr(X > 15) = \Pr\left(\frac{X - \mu}{\sigma} > \frac{15 - \mu}{\sigma}\right)$

$= \Pr\left(Z > \frac{15 - 12.2}{1.4}\right)$

$= \Pr(Z > 2)$ **B**

20 The distribution associated with this is Binomial with $p = 0.15$.

$\Pr(X \geq 1) > 0.95$

$1 - \Pr(X = 0) > 0.95$

$\Pr(X = 0) < 0.05$

$\binom{n}{0} p^0 (1 - p)^n < 0.05$

$(1 - 0.15)^n < 0.05$

$0.85^n < 0.05$

$\log_e 0.85^n < \log_e 0.05$

$n \log_e 0.85 < \log_e 0.05$

$n > \frac{\log_e 0.05}{\log_e 0.85}$

The smallest value n can take is 19. **A**

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21 Pr(2 families have the same number of children)
= Pr(both have 0 children) + Pr(both have 1 child) + Pr(both have 2 children) + Pr(both have 3 children)
= $\Pr(X = 0) \times \Pr(X = 0) + \Pr(X = 1) \times \Pr(X = 1) + \Pr(X = 2) \times \Pr(X = 2) + \Pr(X = 3) \times \Pr(X = 3)$
= $0.4 \times 0.4 + 0.3 \times 0.3 + 0.2 \times 0.2 + 0.1 \times 0.1$
= $0.16 + 0.09 + 0.04 + 0.01$
= 0.30 **C**

22

$$\begin{aligned}\Pr(X > a) &= \int_a^{\pi} \frac{1}{2}\sin(x)\,dx \\ &= \frac{-1}{2}[\cos(x)]_a^{\pi} \\ &= \frac{-1}{2}(\cos(\pi) - \cos(a)) \\ &= \frac{-1}{2}(-1 - \cos(a)) \\ &= \frac{1}{2}(1 + \cos(a))\end{aligned}$$

When $\Pr(X > a) = 0.25$,

$$\begin{aligned}\frac{1}{2}(1 + \cos(a)) &= 0.25 \\ 1 + \cos(a) &= 0.5 \\ \cos(a) &= -0.5 \\ a &= \frac{2\pi}{3} \approx 2.09\end{aligned}$$

E

2006 Mathematical Methods 3&4 CAS
VCAA Sample Examination 2 – Section 2
Suggested answers and solutions for Extended-response questions

CAS may be used but is not essential to complete most questions.

Question 1

a i $f'(x) = 3ax^2 + 2bx + c$

When $x = 1$, $f(x) = 1$ and $f'(x) = 0$,

$\therefore \quad 1 = a + b + c + 2 \quad \boxed{1}$

and $\quad 0 = 3a + 2b + c \quad \boxed{2}$

$\boxed{2} - \boxed{1}$ gives $\quad b = 1 - 2a \quad \boxed{3}$

Sub $\boxed{3}$ in $\boxed{1}$: $\quad a = c + 2 \quad \boxed{4}$

Sub $\boxed{4}$ in $\boxed{1}$: $\quad b = -2c - 3 \quad \boxed{5}$

ii If $f(2) = 0$, $\quad 0 = 8a + 4b + 2c + 2$

Substituting $\boxed{4}$ and $\boxed{5}$ into the above equation yields $c = -3$.

b i $f'(x) = 3x^2 - 8x + 5$

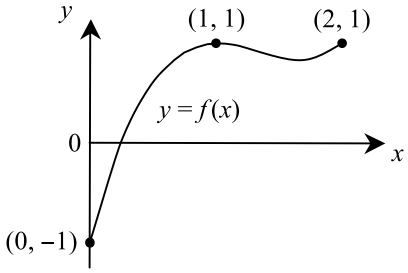
ii Let $f'(x) = 0$, $\quad 0 = 3x^2 - 8x + 5$

$\therefore \quad 0 = (x - 1)(3x - 5)$

$\therefore \quad x = 1$ or $\frac{5}{3}$

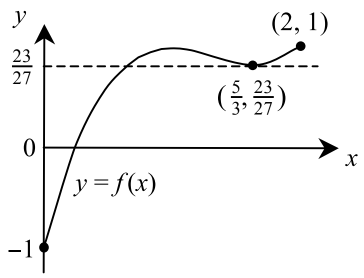
$f(1) = 1, f\left(\frac{5}{3}\right) = \frac{23}{27}, \therefore m = 1, n = \frac{5}{3}$

iii



Absolute maximum value of function is 1.
Absolute minimum value of function is -1.

iv



From the graph, $f(x) = p$ has one solution where $p \in [-1, \frac{23}{27})$.

c i The graph of $y = f(x)$ is transformed into the graph of $y = f\left(\frac{x}{k}\right) - 1$ by a dilation of factor k from the y axis, and a translation of 1 unit in the negative direction of the y axis. Order is not important.

ii For $f(x) - 1 = (x - 1)^2(x - 2)$, the x-axis intercepts are at $x = 1$ and $x = 2$.

The graph of $y = f\left(\frac{x}{k}\right) - 1$ is the graph of $y = f(x) - 1$ dilated by a factor of k from the y axis, therefore the x-axis intercepts are $x = k$ and $x = 2k$.

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Question 1, continued...

c iii For $f(x) = 1$,

$$f(x) - 1 = 0$$

$$\therefore (x-1)^2(x-2) = 0$$

$$\therefore x = 1 \text{ or } 2$$

For $f(x+h) = 1$, it follows that $x = 1 - h$ or $x = 2 - h$, since the graph of $y = f(x+h)$ is a translation of h units in the negative direction of the x axis of the graph of $y = f(x)$. For only one solution to be positive, $1 - h \le 0$

$\therefore h \ge 1$

and $2 - h > 0$

$\therefore h < 2$

Therefore $1 \le h < 2$.

Question 2

a At (2, 3), $3 = (2 \times 2^2 - 3 \times 2)e^{2a}$

$\therefore 3 = 2e^{2a}$

$\therefore e^{2a} = \frac{3}{2}$

$\therefore 2a = \log_e\left(\frac{3}{2}\right)$

$\therefore a = \frac{1}{2}\log_e\left(\frac{3}{2}\right)$

b i At the point A, $y = 0$,

$\therefore (2x^2 - 3x)e^{ax} = 0$

$\therefore x(2x-3) = 0$

$\therefore x = 0 \text{ or } \frac{3}{2}$

At the point A, $x = \frac{3}{2}$.

ii Area of lake $= -\int_0^{\frac{3}{2}} (2x^2 - 3x)e^{ax}\,dx$

$\therefore -\int_0^{\frac{3}{2}} (2x^2 - 3x)e^{ax}\,dx = 10$

Using CAS, $a = 2.474$, correct to 3 d.p.

c When $a = 1$, $y = (2x^2 - 3x)e^x$

$\therefore \frac{dy}{dx} = (2x^2 + x - 3)e^x$

When $\frac{dy}{dx} = 0$, $0 = 2x^2 + x - 3$

$\therefore 0 = (x-1)(2x+3)$

$\therefore x = 1 \text{ or } -\frac{3}{2}$

When $x = 1$, $y = -e$

$\therefore B = (1, -e)$.

d i When $x = 0$, $y = 0$ and $\frac{dy}{dx} = -3$.

$\therefore$ equation of tangent is $y = -3x$.

ii Find the point of intersection of the two graphs,

i.e. let $(2x^2 - 3x)e^x = -3x$

Using CAS, $x = 0$ or 0.874 217...

When $x = 0.874\,217...$, $y = -2.622\,65...$

$\therefore D = (0.87, -2.62)$, correct to 2 d.p.

Question 3

a From the transition matrix,

Pr(hunts on north side next night) $= \frac{4}{5}$.

b $$\begin{bmatrix} \frac{2}{5} & \frac{4}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{68}{125} \\ \frac{57}{125} \end{bmatrix}$$

Pr(north side on Thursday) $= \frac{68}{125}$ or 0.544.

c Use CAS to consider large powers.

$$\begin{bmatrix} \frac{2}{5} & \frac{4}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix}^{30} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0.571\,429 \\ 0.428\,571 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{5} & \frac{4}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix}^{40} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0.571\,429 \\ 0.428\,571 \end{bmatrix}$$

The fox will hunt on the north side of the creek on 57% of the nights.

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Question 3, continued...

d Pr(fox spends longer than 3 hours hunting)

$$= \frac{3}{32}\int_3^4 t(4-t)\,dt$$

$$= \frac{5}{32} \text{ (using CAS)}$$

e Binomial, $n = 3$, $p = \frac{5}{32}$

$$\Pr(X \geq 2) = \binom{3}{2}\left(\frac{5}{32}\right)^2\left(\frac{27}{32}\right)^1 + \binom{3}{3}\left(\frac{5}{32}\right)^3\left(\frac{27}{32}\right)^0$$

$= 0.066$, correct to 3 d.p.

f $\frac{3}{32}\int_3^T t(4-t)\,dt = 0.104$

Using CAS, $T = 0.8$ hours

$= 0.8 \times 60$ minutes

$= 48$ minutes

$\therefore n = 48$.

Question 4

a Range of $h(t) = [-60 + 62, 60 + 62]$

$= [2, 122]$

$\therefore$ Maximum height = 122

b From above

Minimum height = 2

c Period $= 2\pi \div \frac{5\pi}{2}$

$= 2\pi \times \frac{2}{5\pi}$

$= 0.8$ hours

$= 0.8 \times 60$ minutes

$= 48$ minutes

P returns to its lowest point at 1.48 pm.

d i $92 = 62 + 60\sin\left(\frac{(5t-1)\pi}{2}\right)$

$\therefore \sin\left(\frac{(5t-1)\pi}{2}\right) = \frac{1}{2}$ [1]

$\therefore \frac{(5t-1)\pi}{2} = \frac{\pi}{6}$ (1st positive solution)

$\therefore 5t - 1 = \frac{\pi}{6} \times \frac{2}{\pi} = \frac{1}{3}$

$\therefore t = \frac{4}{15}$ hour

$= \frac{4}{15} \times 60$ minutes

$= 16$ minutes

P reaches a height of 92 m at 1.16 pm.

ii From [1] above, the 2nd positive solution is

$\frac{(5t-1)\pi}{2} = \frac{5\pi}{6}$

$\therefore 5t - 1 = \frac{5\pi}{6} \times \frac{2}{\pi} = \frac{5}{3}$

$\therefore t = \frac{8}{15}$ hour

$= \frac{8}{15} \times 60$ minutes

$= 32$ minutes

P is above 92 m from 1.16 pm to 1.32 pm, i.e. for 16 minutes during one rotation.

e i $h'(t) = 60 \times \frac{5\pi}{2}\cos\left(\frac{(5t-1)\pi}{2}\right)$

$\therefore h'(t) = 150\pi\cos\left(\frac{(5t-1)\pi}{2}\right)$

or $h'(t) = 150\pi\sin\left(\frac{5\pi t}{2}\right)$ [2]

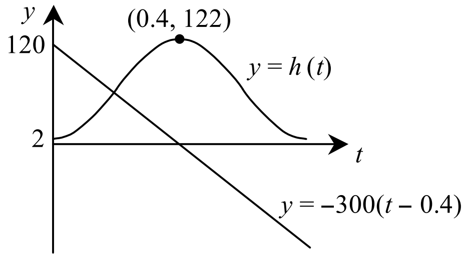
ii When $t = 1$, from [2] above,

$h'(t) = 150\pi\sin\left(\frac{5\pi}{2}\right)$

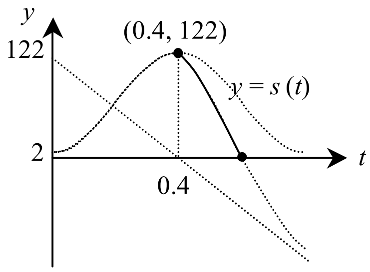
$= 150\pi$ m/h

Question 4, continued...

f i



ii Use addition of ordinates to draw the graph of $y = s(t)$ for $t > 0.4$.



iii When the spider reaches the ground, $s(t) = 0$

$\therefore\ 62 + 60\sin\left(\frac{(5t-1)\pi}{2}\right) - 300(t - 0.4) = 0$

Using CAS, $t = 0.603$, correct to 3 d.p.

The spider leaves car C when $t = 0.4$.

The spider reaches the ground after

$(0.603 - 0.4)$ hours $= 0.203$ hours

$= 0.203 \times 60$ minutes

$= 12.18$ minutes

It takes the spider 12 minutes to reach the ground.

END OF PAPER