The Mathematical Association of Victoria

2006 MATHEMATICAL METHODS

VCAA Sample Examination 1 and 2

Suggested Answers and Solutions

Examination 1

The examination will consist of short answer questions which are to be answered without the use of technology.

Examination 2

The examination will consist of two parts. Part 1 will be a multiple-choice section containing 22 questions and Part II will consist of extended answer questions, involving multi-stage solutions of increasing complexity.

The sample exams are found at:
www.vcaa.vic.edu.au/vce/studies/mathematics/methods/pastexams/2006mmsample.pdf

These answers and solutions have been written and published to assist students in their preparations for the 2006 Mathematical Methods Examinations. The answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority.

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2006 Mathematical Methods 3&4
VCAA Sample Examination 1
Suggested answers and solutions

Question 1

a Let $y = 2\log_e(x+1)$

For the inverse $x = 2\log_e(y+1)$

$$\frac{x}{2} = \log_e(y+1)$$

$$y + 1 = e^{\frac{x}{2}}$$

$$y = e^{\frac{x}{2}} - 1$$

$$f^{-1}(x) = e^{\frac{x}{2}} - 1$$

b Dom of $f^{-1}(x)$ = range of $f(x)$

$= R$

Note that the graph of f is a transformation of the graph of $y = \log_e(x)$, with a translation of 1 unit to the left and a dilation by a scale factor of two from the X axis. f therefore has the same range as $y = \log_e(x)$, i.e. R.

Question 2

a Let $y = uv$

where $u = 3x^4$ and $v = \tan x$

$$\frac{du}{dx} = 12x^3 \text{ and } \frac{dv}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 3x^4\sec^2 x + 12x^3\tan x$$

Alternatively,

$$\frac{dy}{dx} = \frac{12x^3\sin(x)}{\cos(x)} + \frac{3x^4}{\cos^2(x)}$$

$$= \frac{12x^3\sin(x)\cos(x) + 3x^4}{\cos^2(x)}$$

b

$$f(x) = \int f'(x)dx$$

$$\therefore \quad f(x) = \int \frac{1}{x-2}dx$$

$$f(x) = \log_e|x-2| + c$$

$$6 = \log_e|1-2| + c$$

$$= \log_e 1 + c$$

$$c = 6$$

$$f(x) = \log_e|x-2| + 6$$

The rule of f is $f(x) = \log_e(-x+2) + 6$.

Alternatively, $f(x) = \log_e(2-x) + 6$

Question 3

$$\tan(x) = \sqrt{3}$$

$$x = \frac{-2\pi}{3}, \frac{\pi}{3}$$

Question 4

a Amplitude $= 3$

Period $= \frac{2\pi}{2} = \pi$

b Endpoints: When $x = \pi$,

$$f(x) = 3\sin\left(2\left(\pi + \frac{\pi}{3}\right)\right)$$

$$= 3\sin\frac{8\pi}{3}$$

$$= 3\sin\frac{\pi}{3}$$

$$= \frac{3\sqrt{3}}{2}$$

Since the period is π; $\left(-\pi, \frac{3\sqrt{3}}{2}\right)$, $\left(0, \frac{3\sqrt{3}}{2}\right)$ and $\left(\pi, \frac{3\sqrt{3}}{2}\right)$ are points on the graph.

x axis intercepts: When $y = 0$,

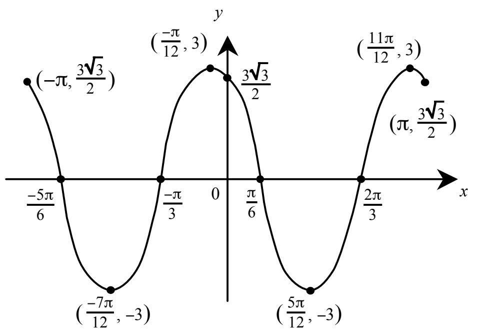
$$3\sin\left(2\left(x+\frac{\pi}{3}\right)\right)=0$$

$$\sin\left(2\left(x+\frac{\pi}{3}\right)\right)=0$$

$$2\left(x+\frac{\pi}{3}\right)=-\pi,\ 0,\ \pi,\ 2\pi$$

$$x+\frac{\pi}{3}=\frac{-\pi}{2},\ 0,\ \frac{\pi}{2},\ \pi$$

$$x=\frac{-5\pi}{6},\ \frac{-\pi}{3},\ \frac{\pi}{6},\ \frac{2\pi}{3}$$

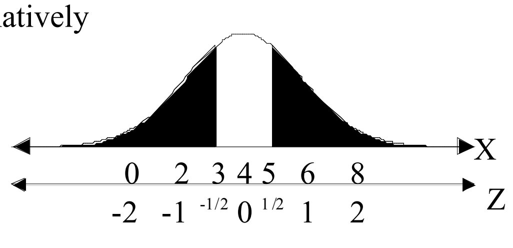


Question 5

a $\Pr(X > 4) = 0.5$

b
$$\Pr(X>5)=\Pr(X<3)$$
$$=\Pr\left(Z<\frac{3-4}{2}\right)$$
$$=\Pr\left(Z<-\frac{1}{2}\right)$$
$$b=-\frac{1}{2}$$

Alternatively



$$z=\frac{x-\mu}{\sigma}$$

$$\therefore z=\frac{5-4}{2}=\frac{1}{2}$$

$$\therefore \Pr(X>5)=\Pr\left(Z>\frac{1}{2}\right)=\Pr\left(Z<-\frac{1}{2}\right)\ (by\ symmetry)$$

$$\therefore b=-\frac{1}{2}$$

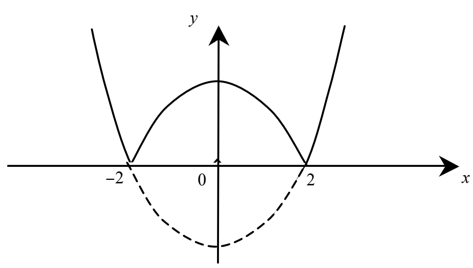
Question 6

a
$$\int_0^2 f(x)\,dx=1$$
$$\int_0^2 ax(2-x)\,dx=1$$
$$a\int_0^2 2x-x^2\,dx=1$$
$$a\left[x^2-\tfrac{1}{3}x^3\right]_0^2=1$$
$$a\left[2^2-\tfrac{1}{3}\times 2^3\right]=1$$
$$\tfrac{4}{3}a=1$$
$$a=\tfrac{3}{4}$$

b
$$\Pr\left(X<\tfrac{1}{2}\right)=\tfrac{3}{4}\int_0^{\frac{1}{2}} x(2-x)\,dx$$
$$=\tfrac{3}{4}\left[x^2-\tfrac{1}{3}x^3\right]_0^{\frac{1}{2}}$$
$$=\tfrac{3}{4}\left[\left(\tfrac{1}{2}\right)^2-\tfrac{1}{3}\times\left(\tfrac{1}{2}\right)^3\right]$$
$$=\tfrac{3}{4}\times\tfrac{5}{24}$$
$$=\frac{5}{32}$$

Question 7

a



b
$$\text{Area}=\int_{-2}^{2}|x^2-4|-(x^2-4)\,dx$$
$$=-2\int_{-2}^{2}x^2-4\,dx$$
$$=-2\left[\tfrac{1}{3}x^3-4x\right]_{-2}^{2}$$
$$=-2\left[\left(\tfrac{8}{3}-8\right)-\left(\tfrac{-8}{3}+8\right)\right]$$
$$=\tfrac{64}{3}$$

Question 8

a $g(f(x)) = \log_e(x^2 + 1)$

b $g'(f(x)) = \frac{1}{x^2+1} \times 2x$

$= \frac{2x}{x^2+1}$

c $\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$

$= \frac{1}{2}\int g'(f(x))\,dx$

$= \frac{1}{2} g(f(x)) + c$

$= \frac{1}{2}\log_e(x^2 + 1)$, if $c = 0$

Alternatively, the antiderivative may be expressed as $\log_e(\sqrt{x^2+1}) + c$. Any real value of c may be used as the question asks for *an* antiderivative.

Question 9

For the curve, $\frac{dy}{dx} = 4x^3$

At the point of intersection of the curve and the tangent, $\frac{dy}{dx} = 4$

$4x^3 = 4$

$x^3 = 1$

$x = 1$

For the tangent, when $x = 1$, $y = 3$.

At the point (1, 3) on the curve,

$y = x^4 + c$

becomes $3 = 1^4 + c$

$c = 2$

Question 10

Required to find $\frac{dh}{dt}$ when $h = 3$.

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ [1]

Given $\frac{dV}{dt} = 3$ [2]

For a cone, $V = \frac{1}{3}\pi r^2 h$

and $\frac{r}{h} = \frac{2}{4}$ so $r = \frac{1}{2}h$

Area between the curves is $\frac{64}{3}$ units^2

therefore $V = \frac{1}{3}\pi \times \left(\frac{1}{2}h\right)^2 \times h$

$= \frac{\pi h^3}{12}$

$\frac{dV}{dh} = \frac{\pi h^2}{4}$

$\frac{dh}{dV} = \frac{4}{\pi h^2}$

Using [1] and [2]

$\frac{dh}{dt} = \frac{4}{\pi h^2} \times 3$

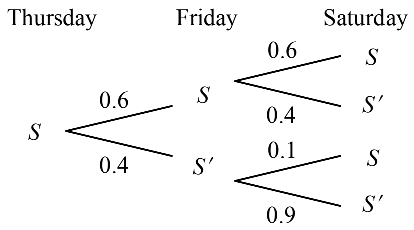
$= \frac{12}{\pi h^2}$

When $h = 3$, $\frac{dh}{dt} = \frac{12}{\pi \times 3^2}$

$= \frac{4}{3\pi}$

The water is rising at $\frac{4}{3\pi}$ m/min when the depth is 3 m.

Question 11



Pr('no snow' on Saturday)

= Pr('snow' on Friday and 'no snow' on Saturday) + Pr('no snow' on Friday and 'no snow' on Saturday)

$= 0.6 \times 0.4 + 0.4 \times 0.9$

$= 0.24 + 0.36$

$= 0.6$

END OF PAPER

2006 Mathematical Methods 3&4
VCAA Sample Examination 2 - Section 1
Suggested answers and solutions for Multiple-choice questions

1	C	**2**	B	**3**	A	**4**	D	**5**	D
6	A	**7**	B	**8**	D	**9**	D	**10**	A
11	A	**12**	A	**13**	C	**14**	A	**15**	C
16	D	**17**	A	**18**	B	**19**	B	**20**	A
21	C	**22**	E						

1

$$\begin{aligned} 2\sin(3x) - 1 &= 0 \\ \sin(3x) &= \frac{1}{2} \\ 3x &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \\ x &= \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18} \end{aligned}$$

$$\frac{\pi}{18} + \frac{5\pi}{18} + \frac{13\pi}{18} + \frac{17\pi}{18} = 2\pi$$ **C**

2

$$\begin{aligned} e^{2x} - 3e^x + 2 &= 0 \\ (e^x)^2 - 3e^x + 2 &= 0 \\ (e^x - 1)(e^x - 2) &= 0 \\ e^x &= 1 \text{ or } 2 \\ x &= 0 \text{ or } \log_e 2 \end{aligned}$$ **B**

3 If $|p + 3| > 3$, then

$p + 3 > 3$ when $p + 3$ is non-negative [1]

and $-(p + 3) > 3$ when $p + 3$ is negative [2]

From [1], $p > 0$, and from [2],

$$\begin{aligned} p + 3 &< -3 \\ p &< -6 \end{aligned}$$

Hence for $|p + 3| > 3$, $p > 0$ or $p < -6$ **A**

4 Total area $= \int_a^b f(x)\,dx - \int_b^c f(x)\,dx = \int_a^b f(x)\,dx + \int_c^b f(x)\,dx$ **D**

5 This question is intended for graphics calculator use, but the analytical solution is provided here for information.

$$\begin{aligned} h &= 0.5\left(1 - e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right)\right) \\ \frac{dh}{dt} &= 0.5\left(0 + 0.05e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right) + \frac{3\pi}{2}e^{-0.05t}\sin\left(\frac{3\pi t}{2}\right)\right) \\ &= 0.025e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right) + \frac{3\pi}{4}e^{-0.05t}\sin\left(\frac{3\pi t}{2}\right) \end{aligned}$$

When $t = 2.5$,

$$\begin{aligned} \frac{dh}{dt} &= 0.025e^{-0.125}\cos\left(\frac{15\pi}{4}\right) + \frac{3\pi}{4}e^{-0.125}\sin\left(\frac{15\pi}{4}\right) \\ &= 0.025e^{-0.125} \times \frac{1}{\sqrt{2}} + \frac{3\pi}{4}e^{-0.125} \times \frac{-1}{\sqrt{2}} \\ &= -1.45, \text{ correct to two decimal places} \end{aligned}$$ **D**

6 Average rate of change $= \dfrac{N(10) - N(0)}{10}$

$= \dfrac{1000e - 1000}{10}$

$= 172$, to the nearest integer **A**

7 $y = ax^3 + bx^2 + cx + d$

The graph intersects the y axis at 24, so $d = 24$.

From the shape of the graph, a is positive, so **B** is the answer. **B**

8 One method of solution is to rearrange $\log_5 6 = x$ to the form $5^x = 6$. The graphics calculator can be used to show that $5^{1.113}$ is the closest approximation to 6.

Alternative solution: $x = \dfrac{\log 6}{\log 5} = 1.11328...$ **D**

9 The graph shown could have rule $y = x^{\frac{1}{3}}$. **D**

10 For $f(x) = \log_e(x^2) + 1$, x^2 must be greater than 0, so $x \in R\backslash\{0\}$. **A**

11 $f(x) = 2x^3 - 3x^2 + 6$

$f'(x) = 6x^2 - 6x$

The graph has turning points where $f'(x) = 0$,

$6x^2 - 6x = 0$

$6x(x - 1) = 0$

$x = 0$ or 1

For an inverse to exist, the graph must be one-to-one, and since the domain of f is given as $[a, \infty)$ the domain must be $[1, \infty)$, or a subset of this. Hence $a \geq 1$ is the answer. **A**

12 $\int_a^b (4 - 2f(x))\,dx = \int_a^b 4\,dx - 2\int_a^b f(x)\,dx$

$= 4[x]_a^b - 2 \times 3$ since $\int_a^b f(x)\,dx = 3$

$= 4(b - a) - 6$ **A**

13 The graph of the function with equation $y = f(x)$ is transformed into the graph of the function $y = g(x)$ by a dilation of scale factor 2 from the x axis and a reflection in the x axis. **C**

14 $y = |\sin(x)|$, $\pi < x < 2\pi$

is equivalent to $y = -\sin(x)$, $\pi < x < 2\pi$

$\dfrac{dy}{dx} = -\cos(x)$

When $x = k$, $\dfrac{dy}{dx} = -\cos(k)$ **A**

15 Since the gradient is zero at $x = 2$, and negative both to the immediate left and right of $x = 2$, there is a stationary point of inflection at $x = 2$. **C**

16 For $\int_0^t f(x)\,dx > 0$, $t > 0$, so $t \in (0, b]$ only. **D**

17 For the curve with equation $y = 2x^{\frac{3}{2}}$, when $x = 4$, $y = 16$.

$\dfrac{dy}{dx} = 3x^{\frac{1}{2}}$

The gradient of the curve when $x = 4$ is 6, so the gradient of the normal to the curve is $\dfrac{-1}{6}$.

The equation of the normal to the curve at the point (4, 16) is

$$y - 16 = \frac{-1}{6}(x - 4)$$

$$y = \frac{-1}{6}x + \frac{50}{3}$$ **A**

18 The function $y = f(x)$ has positive gradient, increasing from left to right. **B**

19

$$\Pr(X > 15) = \Pr\left(\frac{X - \mu}{\sigma} > \frac{15 - \mu}{\sigma}\right)$$
$$= \Pr\left(Z > \frac{15 - 12.2}{1.4}\right)$$
$$= \Pr(Z > 2)$$ **B**

20 The distribution associated with this is Binomial with $p = 0.15$.

$$\Pr(X \geq 1) > 0.95$$
$$1 - \Pr(X = 0) > 0.95$$
$$\Pr(X = 0) < 0.05$$
$$\binom{n}{0}p^0(1 - p)^n < 0.05$$
$$(1 - 0.15)^n < 0.05$$
$$0.85^n < 0.05$$
$$\log_e 0.85^n < \log_e 0.05$$
$$n \log_e 0.85 < \log_e 0.05$$
$$n > \frac{\log_e 0.05}{\log_e 0.85}$$

The smallest value n can take is 19. **A**

21 Pr(2 families have the same number of children)
= Pr(both have 0 children) + Pr(both have 1 child) + Pr(both have 2 children) + Pr(both have 3 children)
$= \Pr(X = 0) \times \Pr(X = 0) + \Pr(X = 1) \times \Pr(X = 1) + \Pr(X = 2) \times \Pr(X = 2) + \Pr(X = 3) \times \Pr(X = 3)$
$= 0.4 \times 0.4 + 0.3 \times 0.3 + 0.2 \times 0.2 + 0.1 \times 0.1$
$= 0.16 + 0.09 + 0.04 + 0.01$
$= 0.30$ **C**

22

$$\Pr(X > a) = \int_a^{\pi} \frac{1}{2}\sin(x)\, dx$$
$$= \frac{-1}{2}[\cos(x)]_a^{\pi}$$
$$= \frac{-1}{2}(\cos(\pi) - \cos(a))$$
$$= \frac{-1}{2}(-1 - \cos(a))$$
$$= \frac{1}{2}(1 + \cos(a))$$

When $\Pr(X > a) = 0.25$,

$$\frac{1}{2}(1 + \cos(a)) = 0.25$$
$$1 + \cos(a) = 0.5$$
$$\cos(a) = -0.5$$

$$a = \frac{2\pi}{3} \approx 2.09$$ **E**

2006 Mathematical Methods 3&4
VCAA Sample Examination 2 – Section 2
Suggested answers and solutions for Extended-response questions

Question 1

a $f'(x) = (x-1)^2 + 2(x-1)(x-2)$

$= (x-1)(x-1+2(x-2))$

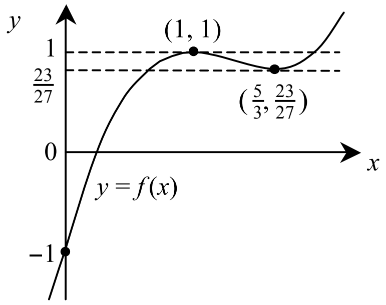
$= (x-1)(3x-5)$

$\therefore\ u = 3$ and $v = -5$

b When $f'(x) = 0$, $x = 1$ or $\frac{5}{3}$

$f(1) = 1, f\left(\frac{5}{3}\right) = \frac{23}{27}, \therefore\ a = 1, b = \frac{5}{3}$

c



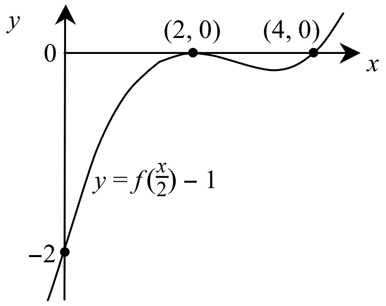
From the graph, $f(x) = p$ has one solution where $p < \frac{23}{27}$ or $p > 1$.

d i The graph of $y = f(x)$ is transformed into the graph of $y = f\left(\frac{x}{2}\right) - 1$ by a dilation of factor 2 from the y axis, and a translation of 1 unit in the negative direction of the y axis. Order is not important.

ii For $f(x) - 1 = (x-1)^2(x-2)$, the x-axis intercepts are at $x = 1$ and $x = 2$.

The graph of $y = f\left(\frac{x}{2}\right) - 1$ is the graph of $y = f(x) - 1$ dilated by a factor of 2 from the y axis, therefore the x-axis intercepts are $x = 2$ and $x = 4$.

iii



$$\text{Area} = -\int_2^4 \left(\frac{x}{2}-1\right)^2\left(\frac{x}{2}-1\right)dx$$

$= 0.17$, correct to 2 d.p. (using a graphics calculator)

e For $f(x) = 1$,

$f(x) - 1 = 0$

$\therefore\ (x-1)^2(x-2) = 0$

$\therefore\ x = 1$ or 2

For $f(x+h) = 1$, it follows that $x = 1 - h$ or $x = 2 - h$, since the graph of $y = f(x+h)$ is a translation of h units in the negative direction of the x axis of the graph of $y = f(x)$.

For only one solution to be positive,

$1 - h \le 0$

$\therefore\ h \ge 1$

and $2 - h > 0$

$\therefore\ h < 2$

Therefore $1 \le h < 2$.

Question 2

a At (2, 3), $3 = (2 \times 2^2 - 3 \times 2)e^{2a}$

$\therefore \quad 3 = 2e^{2a}$

$\therefore \quad e^{2a} = \frac{3}{2}$

$\therefore \quad 2a = \log_e\left(\frac{3}{2}\right)$

$\therefore \quad a = \frac{1}{2}\log_e\left(\frac{3}{2}\right)$

$= 0.203$, correct to 3 d.p.

b At the point A, $y = 0$,

$\therefore \quad (2x^2 - 3x)e^x = 0$

$\therefore \quad x(2x - 3) = 0$

$\therefore \quad x = 0$ or $\frac{3}{2}$

At the point A, $x = \frac{3}{2}$.

c i $\frac{dy}{dx} = (2x^2 - 3x)e^x + (4x - 3)e^x$

$= (2x^2 + x - 3)e^x$

$\therefore p = 2$, $q = 1$ and $r = -3$.

ii When $\frac{dy}{dx} = 0$, $0 = 2x^2 + x - 3$

$\therefore \quad 0 = (x - 1)(2x + 3)$

$\therefore \quad x = 1$ or $-\frac{3}{2}$

When $x = 1$, $y = -e$

$\therefore B = (1, -e)$.

d LHS $= (2x^2 + mx + n)e^x + (4x + m)e^x$

$= (2x^2 + (m + 4)x + m + n)e^x$

$\therefore (2x^2 + (m + 4)x + m + n)e^x = (2x^2 - 3x)e^x$

Equating coefficients, $m + 4 = -3$

$\therefore \quad m = -7$

and $\quad m + n = 0$

$\therefore \quad n = 7$

$\therefore \frac{d}{dx}\{(2x^2 - 7x + 7)e^x\} = (2x^2 - 3x)e^x$

$$[2x^2 - 7x + 7)e^x]_0^{\frac{3}{2}} = \int_0^{\frac{3}{2}} (2x^2 - 3x)e^x\,dx$$

Now $\quad$ Area of lake $= -\int_0^{\frac{3}{2}} (2x^2 - 3x)e^x\,dx$

$= -[2x^2 - 7x + 7)e^x]_0^{\frac{3}{2}}$

$= 7 - e^{\frac{3}{2}}$ km squared.

Question 3

a i Pr(hunts on north side next 3 nights)

$= \left(\frac{2}{5}\right)^3 = \frac{8}{125}$

ii Pr(on north side on 2 of next 3 nights)

= Pr(NNS) + Pr(NSN) + Pr(SNN)

$= \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} + \frac{2}{5} \times \frac{3}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{4}{5} \times \frac{2}{5}$

$= \frac{12}{25}$ or 0.48

iii Pr(south side on Thursday or Friday)

= Pr(NNS) + Pr(NNNS)

$= \left(\frac{2}{5}\right)^2 \times \frac{3}{5} + \left(\frac{2}{5}\right)^3 \times \frac{3}{5}$

$= \frac{84}{625}$ or 0.1344

b Pr(fox spends longer than 3 hours hunting)

$= \frac{3}{32}\int_3^4 t(4 - t)\,dt$

$= \frac{5}{32}$ (using graphics calculator)

c Binomial, $n = 3$, $p = \frac{5}{32}$

$$\Pr(X \geq 2) = \binom{3}{2}\left(\frac{5}{32}\right)^2\left(\frac{27}{32}\right)^1 + \binom{3}{3}\left(\frac{5}{32}\right)^3\left(\frac{27}{32}\right)^0$$

$= 0.066$, correct to 3 d.p.

Question 3, continued...

d $\frac{3}{32}\int_0^T t(4-t)\,dt = 0.104$

$\int_0^T 4t - t^2\,dt = 0.104 \times \frac{32}{3}$

$[2t^2 - \frac{1}{3}t^3]_0^T = \frac{416}{375}$

$2T^2 - \frac{1}{3}T^3 = \frac{416}{375}$

Finding the 1st positive solution using a graphics calculator,

$T = 0.8$ hours

$= 0.8 \times 60$ minutes

$= 48$ minutes

$\therefore \quad n = 48.$

Question 4

a Range of $h(t) = [-60 + 62, 60 + 62]$

$= [2, 122]$

$\therefore$ Maximum height = 122

b From above

Minimum height = 2

c Period $= 2\pi \div \frac{5\pi}{2}$

$= 2\pi \times \frac{2}{5\pi}$

$= 0.8$ hours

$= 0.8 \times 60$ minutes

$= 48$ minutes

P returns to its lowest point at 1.48 pm.

d i $92 = 62 + 60\sin\left(\frac{(5t-1)\pi}{2}\right)$

$\therefore \sin\left(\frac{(5t-1)\pi}{2}\right) = \frac{1}{2}$ [1]

$\therefore \frac{(5t-1)\pi}{2} = \frac{\pi}{6}$ (1st positive solution)

$\therefore 5t - 1 = \frac{\pi}{6} \times \frac{2}{\pi} = \frac{1}{3}$

$\therefore t = \frac{4}{15}$ hour

$= \frac{4}{15} \times 60$ minutes

$= 16$ minutes

P reaches a height of 92 m at 1.16 pm.

ii From [1] above, the 2nd positive solution is

$\frac{(5t-1)\pi}{2} = \frac{5\pi}{6}$

$\therefore 5t - 1 = \frac{5\pi}{6} \times \frac{2}{\pi} = \frac{5}{3}$

$\therefore t = \frac{8}{15}$ hour

$= \frac{8}{15} \times 60$ minutes

$= 32$ minutes

P is above 92 m from 1.16 pm to 1.32 pm, i.e. for 16 minutes during one rotation.

e i $h'(t) = 60 \times \frac{5\pi}{2}\cos\left(\frac{(5t-1)\pi}{2}\right)$

$\therefore h'(t) = 150\pi\cos\left(\frac{(5t-1)\pi}{2}\right)$

or $h'(t) = 150\pi\sin\left(\frac{5\pi t}{2}\right)$ [2]

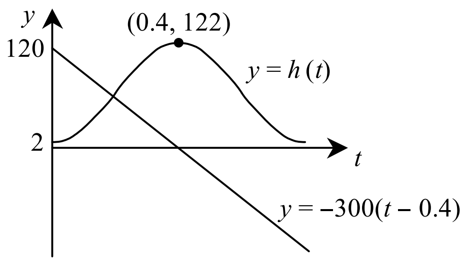
ii When $t = 1$, from [2] above,

$h'(t) = 150\pi\sin\left(\frac{5\pi}{2}\right)$

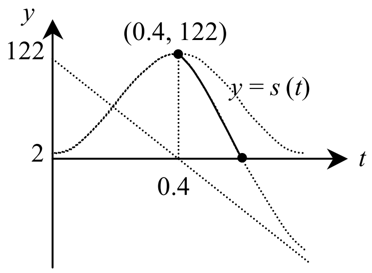
$= 150\pi$ m/h

Question 4, continued...

f i



ii Use addition of ordinates to draw the graph of $y = s(t)$ for $t > 0.4$.



iii When the spider reaches the ground, $s(t) = 0$

$$\therefore\ 62 + 60\sin\left(\frac{(5t-1)\pi}{2}\right) - 300(t - 0.4) = 0$$

Using a graphics calculator, $t = 0.603$, correct to 3 d.p.

The spider leaves car C when $t = 0.4$.

The spider reaches the ground after

$(0.603 - 0.4)$ hours $= 0.203$ hours

$= 0.203 \times 60$ minutes

$= 12.18$ minutes

It takes the spider 12 minutes to reach the ground.

END OF PAPER