Mathematical Methods

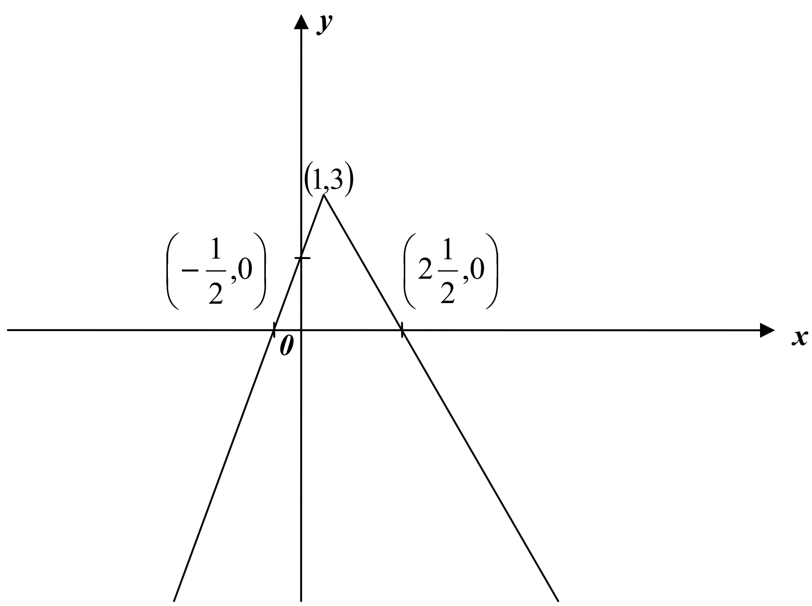
Written examination 1

2006 Trial Examination

SOLUTIONS

Question 1

Vertex (1, 3) and correct shape for graph	A1
(0, 1) y-intercept marked in or stated	A1
x-intercepts $\left(-\frac{1}{2},0\right) and \left(2\frac{1}{2},0\right)$ (<u>both</u> marked in or stated)	A1



Question 2

a. Inverse has equation $x+1=\dfrac{3}{y+2}$ M1

So $y = \dfrac{3}{x+1} - 2$ A1

b. Domain of this is $x<-1$. A1

Question 3

a. $f'(x)=-8e^{-2x}\cos 5x-20e^{-2x}\sin 5x$ M1 product rule and some correct derivatives, A1

b. $f(x)=\dfrac{2}{3}(x-3)^{\frac{3}{2}}+c$ A1

$f(4)=1 \Rightarrow 1=\dfrac{2}{3}+c \Rightarrow c=\dfrac{1}{3}$ A1

Question 4

$\cos x=-\dfrac{1}{2}\sqrt{3}$. Second or third quadrant, related angle $\dfrac{\pi}{6}$. M1

$x=\pm\dfrac{5\pi}{6}$. A1 + A1

Question 5

a. $f'(x) = \dfrac{x \times \dfrac{3}{3x} - \ln(3x)}{x^2} = \dfrac{1-\ln(3x)}{x^2}$ M1 (quotient rule) + A1

b. Turning point at $\ln(3x)=1$ so $3x=e, x=\dfrac{1}{3}e$ M1 (must involve a log equation)

$f\left(\dfrac{1}{3}e\right)=\dfrac{3}{e}$, so co ordinates are $\left(\dfrac{1}{3}e, \dfrac{3}{e}\right)$. A1

Question 6

a. $x=200-\dfrac{1}{2} of\ 30=185$. A1

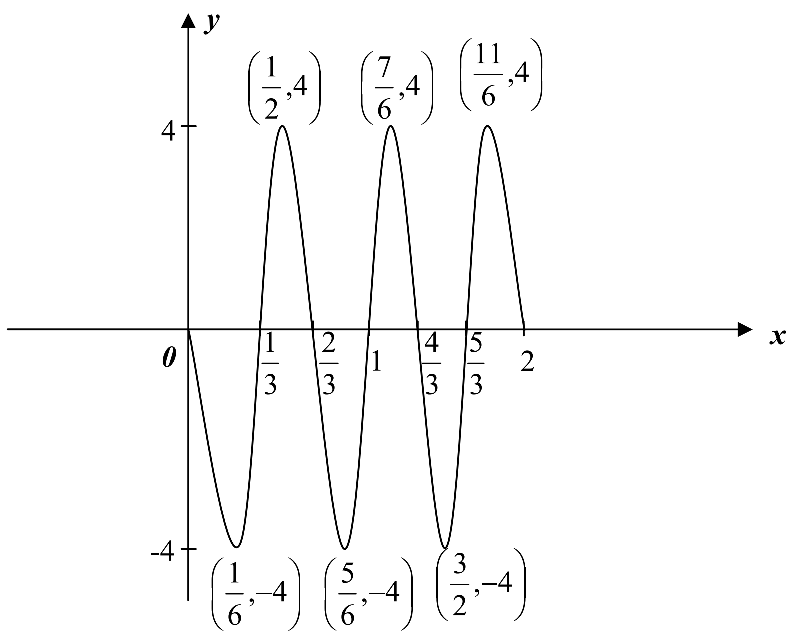
b. By symmetry, k is the Z value corresponding to the X value 240 = 200 + 40, which is $1\frac{1}{3}$ standard deviation's above the mean, so Z is $1\frac{1}{3}$. A1

Question 7

a. Period $\frac{2}{3}$, amplitude 4 . A1

b. Correct axial scales and placement of graph on their scale A1

Correct turning point coordinates and shape of graph A1



c. Area $= -\int_0^{\frac{1}{3}} 4\sin 3\pi x \, dx$ (or equivalent)

$= \frac{4}{3\pi}\left[\cos 3\pi x\right]_0^{\frac{1}{3}}$ The indefinite integral must be shown! A1

$= \frac{4}{3\pi}$ A1

Question 8

a. $\int_{-\infty}^{\infty} f(x)\,dx = 1 = \int_0^4 k(4x - x^2)\,dx$ M1

$= k\left[2x^2 - \frac{x^3}{3}\right]_0^4$ H1

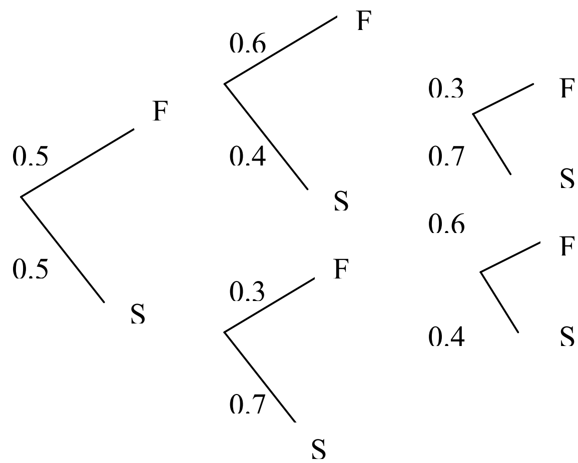
$= \frac{32k}{3}$ thus $k = \frac{3}{32}$ A1

b. $\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx = \frac{3}{32}\int_0^4 (4x^2 - x^3)\,dx$ H1

$= \frac{3}{32}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4 = 2$ A1

Question 9

a.



M1 correct format of tree and attempt at probabilities

b. Pr(S wins) $= 0.5 \times 0.7 + 0.5 \times 0.4 \times 0.7 + 0.5 \times 0.3 \times 0.4$ H1

$= 0.55$ A1

c. Pr(3 games | S wins) $= \dfrac{(0.5 \times 0.4 \times 0.7 + 0.5 \times 0.3 \times 0.4)}{0.55} = \dfrac{4}{11}$ A1

Question 10

a. $f(g(x)) = 5\sin^3 x$. A1

b. $\dfrac{d}{dx}[f(g(x))] = 15\sin^2 x\cos x$. A1

Question 11

Gradients of both curves must be equal. This is so if $-3x^2 + 4 = -2$ M1

Thus $x^2 = 2, x = \pm\sqrt{2}$. A1

If $x = \pm\sqrt{2}$ in the formula $y = -x^3 + 4x, y = \pm 2\sqrt{2}$. A1

Tangent $y = -2x + c$ goes through $(\sqrt{2}, 2\sqrt{2})$ if $c = 4\sqrt{2}$. M1

Other value of c (by symmetry or other) is $c = -4\sqrt{2}$. A1