Mathematical Methods

Written examination 2

2006 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

1.	D	12.	B
2.	B	13.	C
3.	E	14.	C
4.	E	15.	E
5.	C	16.	D
6.	A	17.	D
7.	A	18.	B
8.	E	19.	A
9.	E	20.	A
10.	D	21.	E
11.	D	22.	B

SECTION 2

Question 1

a.

$f'(x) = 2(1+x)(3-x) - 1(1+x)^2$

$= 2(1+x)(3-x) - (1+x)^2$

$= (1+x)\left[2(3-x) - (1+x)\right]$

$= (1+x)(6-2x-1-x)$

$= (1+x)(-3x+5)$

$f'(x) = -(1+x)(3x-5)$

$a = 3,\ b = -5$

3 marks

b.

$f'(x) = -(1+x)(3x-5)$

$x = -1,\ x = \frac{5}{3}$

$x = -1;\ f(x) = (1-1)^2(4) - 4 = -4$

$x = \frac{5}{3};\ \ f\left(\frac{5}{3}\right) = \left(1+\frac{5}{3}\right)^2\left(3-\frac{5}{3}\right) - 4$

$= \frac{64}{9}\left(\frac{4}{3}\right) - 2$

$= \frac{148}{27}$

$Min(-1,-4), Max\left(\frac{5}{3}, \frac{148}{27}\right)$

2 marks

c.

i.

$f'(-2) = -(1-2)(-6-5) = -11,\ f(-2) = 1$

$g(x) = -x^2 + bx + c$

$g'(x) = -2x + b$

Let ,

$g'(-2) = -11$

$-11 = -2 \times -2 + b$

$-15 = b$

$g(-2) = 1$

$1 = -4 - 2b + c$

$5 = 2 \times 15 + c$

$c = -25$

$\therefore g(x) = -x^2 - 15x - 25$

4 marks

ii. $g'(x) = -2x - 15 = 0$

$2x = -15$

$x = \dfrac{-15}{2}$

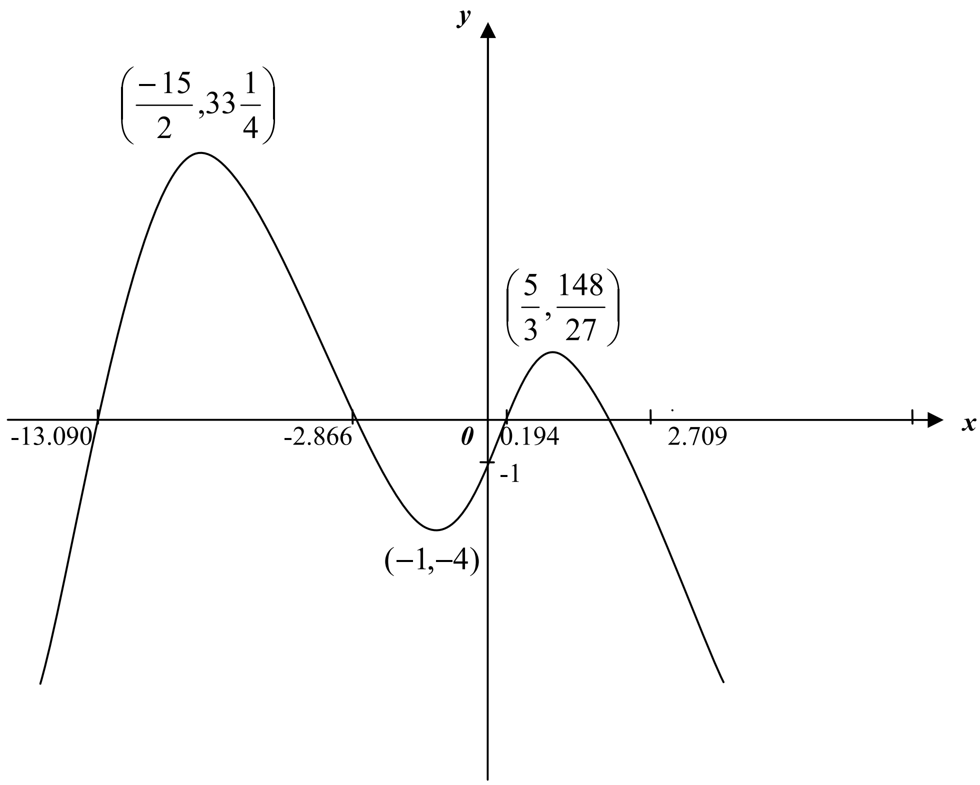
$g\left(\dfrac{-15}{2}\right) = -\left(-\dfrac{15}{2}\right)^2 - 15\left(-\dfrac{15}{2}\right) - 25$

$= 31\dfrac{1}{4}$

$\left(\dfrac{-15}{2}, 31\dfrac{1}{4}\right)$

2 marks

d.



3 marks
Total 14 marks

Question 2

a. (4,3); $3 = e^{4b}(4^2 - 12)$

$3 = e^{4b}(4)$

$3 = 4e^{4b}$

$\frac{3}{4} = e^{4b}$

$\log_e\left(\frac{3}{4}\right)^{\frac{1}{4}} = b$

$b = \frac{1}{4}\log_e\left(\frac{3}{4}\right)$

3 marks

b. $y = 0$

$0 = e^x(x^2 - 3x)$

$e^x = 0, x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, \ x = 3$

2 marks

c.
i.

$\frac{dy}{dx} = \left[e^x(x^2 - 3x) + (2x-3)e^x)\right]$

$= e^x(x^2 - x - 3)$

4 marks

ii.
Using graphics calculator,

B is at (-1.30, 1.52) and *C* is at (2.30, -16.06)

2 marks

d.

$$\left[e^x\left(x^2+px+q\right)+e^x\left(2x+p\right)\right]=e^x(x^2-3x)$$
$$e^x\left[x^2+px+q+2x+p\right]=e^x(x^2-3x)$$
$$x^2+px+q+2x+p=x^2-3x$$
$$x\left(p+2\right)+q+p=-3x$$
$$p+2=-3,\quad q+p=0$$
$$p=-5,\ q=5$$
$$\therefore Area=-\int_0^3 e^x(x^2-3x)dx=-\left[e^x(x^2-5x+5)\right]_0^3$$
$$=-e^3(9-15+5)--\left[e^0(5)\right]$$
$$=e^3+5\,units^2$$

7 marks
Total 18 marks

Question 3

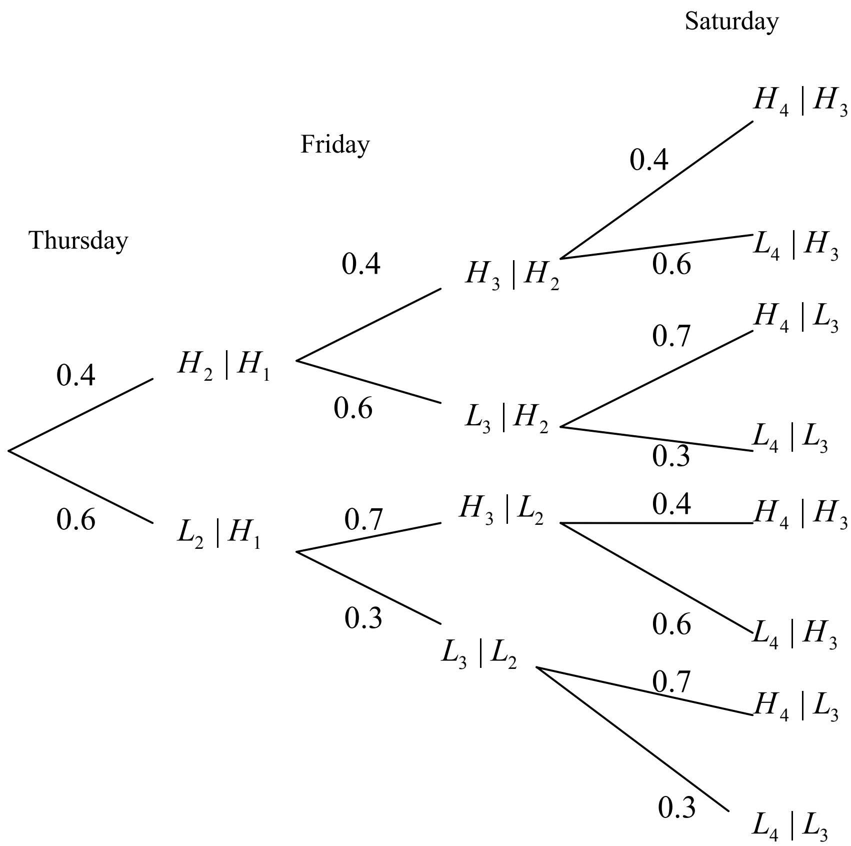
a. Let H_i be that John studies at home one night. Let L_i be that John studies at home one night.

i. $\Pr(H_{i+1} \mid H_i) = 0.4$

$\Pr(L_{i+1} \mid L_i) = 0.3$

$\Pr(H_{i+1} \mid L_i) = 0.7$

$\Pr(L_{i+1} \mid H_i) = 0.6$



Therefore

$\Pr(\text{Home on two of the next three nights}) = 0.4 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.7 + 0.6 \times 0.7 \times 0.4 = 0.432$

3 marks

ii. $\Pr(\text{Home on Friday}) = 0.4^2 + (0.6 \times 0.7) = 0.58$

1 mark

b.

$$\int_3^5 \frac{6}{125} t(5-t)\,dt = \frac{6}{125}\left[\frac{5}{2}t^2 - \frac{t^3}{3}\right]_3^5$$

$$= \frac{6}{125}\left(\frac{125}{2} - \frac{125}{3}\right) - \frac{6}{125}\left(\frac{45}{2} - 9\right) = \frac{44}{125}$$

2 marks

c. $\Pr(X \geq 2) = {}^3C_2\left(\frac{44}{125}\right)^2\left(\frac{81}{125}\right)^1 + {}^3C_3\left(\frac{44}{125}\right)^3 = 0.284$

or use $1 - bincdf\left(3, \frac{44}{125}, 1\right) = 0.284$

2 marks

d. $0.30 = \frac{6}{125}\int_0^n t(5-t)\,dt = \frac{6}{125}\left[\frac{5}{2}t^2 - \frac{t^3}{3}\right]_0^n$

$125 \times 0.3 = 15n^2 - 2n^3$.

Solve this cubic using CALC INTERSECT on the domain $0 < n < 5$ to get $n = 1.816$ (hours). The required time is 109 minutes.

4 marks
Total 12 marks

Question 4

a. 21m

1 mark

b. 9m

1 mark

c. Solve $17 = 6\sin\frac{1}{2}\pi(x-2)+15$ to obtain $x = 6.216$ using CALC INTERSECT

2 marks

d.

i. $f'(x) = 3ax^2 + 2bx + c$

$f'(6) = 108a + 12b + c$

2 marks

ii.

$h(x) = 6\cos\frac{\pi}{2}(x-2)+15$

$h'(x) = \frac{6}{2}\pi\cos\frac{\pi}{2}(x-2)$

$h'(6) = 3\pi\cos\frac{\pi}{2}(4)$

$= 3\pi\cos 2\pi = \frac{3}{2}\pi$

$f'(6) = h'(6)$

$108a + 12b + c = 3\pi$

2 marks

iii.

$f'(x) = 3ax^2 + 2bx + c$

But $f'(0) = 0,\ so \quad c = 0 \quad (1)$

Also $f(0) = 0$, so $d = 0 \quad (2)$

$h(6) = 3\sin\frac{\pi}{2}(4) + 15 = 15$

$\therefore\ f(6) = 15$

$15 = 216a + 36b + 6c + d$

$c = d = 0$

$15 = 216a + 36b \quad (3)$

$3\pi = 108a + 12b \quad (4)$

$(3) \div 3 \qquad 5 = 72a + 12b \quad (3a)$

$(4) - (3a)$ gives $\quad 3\pi - 5 = 36a$

$$a = \frac{3\pi - 5}{36}$$

Substitute for 72a in (3a)

$5 = 2(3\pi - 10) + 12b$

so $\quad b = \dfrac{25 - 6\pi}{12}$

$$f(x) = \left(\frac{3\pi - 5}{36}\right)x^3 + \left(\frac{25 - 6\pi}{12}\right)x^2$$

4 marks

e.

$$Area = \int_6^{16}\left(6\sin\frac{\pi}{2}(x-2) + 15\right)dx + \int_0^6\left(\left(\frac{3\pi - 5}{36}\right)x^3 + \left(\frac{25 - 6\pi}{12}\right)x^2\right)dx$$

$$= \left[-\frac{12}{\pi}\cos\left[\frac{\pi}{2}(x-2)\right] + 15x\right]_6^{16} + \left[\left(\frac{3\pi - 5}{144}\right)x^4 + \left(\frac{25 - 6\pi}{36}\right)x^3\right]_0^6$$

$$= \left[\left(\frac{-12}{\pi} \times -1\right) + 240\right] - \left[\frac{-12}{\pi} + 90\right] + \left[9(3\pi - 5) + 150 - 36\pi\right] - 0$$

$$= \frac{12}{\pi} + 150 + \frac{12}{\pi} + 27\pi - 45 + 150 + 36\pi$$

$$= \left(\frac{24}{\pi} - 9\pi + 300\right)\text{square units}$$

$= 279.365$ square units

4 marks

Total 16 marks