2006 Mathematical Methods & Mathematical Methods (CAS) GA 2: Exam 1

GENERAL COMMENTS

The number of students who sat for the 2006 examination was 16 604, comprising 16 065 Mathematical Methods students and 539 Mathematical Methods (CAS) students. Almost 16% scored 90% or more of the available marks and 3% received full marks. The mean and median scores were 22 out of a possible 40 marks. There was no significant difference between the responses of Mathematical Methods students and Mathematical Methods (CAS) students. The use of matrices by a relatively small number of CAS students was seen in Question 10.

The overall quality of responses was similar to that of recent years. There were many very good responses and it was rewarding to see the quite substantial number of students who worked through to obtain full marks. It was disappointing to see that a significant number of students did not evaluate simple expressions correctly.

Students need to be aware that the instruction to show working is applied rigorously when marking the papers. Failure to show appropriate working will result in marks not being awarded if only an answer is given in response to the question. Students must take care with their working. Graphs should be drawn showing correct features such as smoothness and endpoints. Incorrect notations cannot be given marks.

As noted in previous Assessment Reports, a number of students continue to have difficulty with algebraic skills, setting out, graphing skills and the correct use of mathematical notation. This was evident in some aspects of almost every question on the paper.

Question 1

Marks	0	1	Average
%	17	83	0.9

$$f\left(g\left(x\right)\right) = f\left(2x+1\right)$$
$$= \left(2x+1\right)^2 + 1$$

Most students obtained the correct answer. However, some students failed to gain the mark because they incorrectly expanded the correct expression. Students should be advised against proceeding beyond what is explicitly asked for in the question. Incorrect responses included finding the product of *f* and *g* or the composite function $g(f(x))$.

Question 2

2a.

Marks	0	1	2	Average
%	14	23	63	1.5

$$x = 3e^{2y} - 1$$
$$y = \frac{1}{2}\log_e\left(\frac{x+1}{3}\right)$$

This question was generally well answered, with almost all students knowing to interchange *x* and *y* in order to find the rule of the inverse function. Some students had difficulty transposing the equation, with a common error being to multiply by 2 rather than divide by 2. Other algebraic manipulation errors were also seen. It was pleasing that most students were able to handle the logarithm and exponential correctly, which was a marked improvement on recent years.

2b.

Marks	0	1	Average
%	55	45	0.5

Correct response:
dom f^{-1} = ran f = $(-1, \infty)$

Simply **stating** that the domain of f^{-1} is the same as the range of f was not sufficient to gain the mark. Students were expected to explicitly state the domain as a set. Common errors were R , $R\backslash\{0\}$, $R\backslash\{-1\}$ or $[-1, \infty)$.

Question 3

3a.

Marks	0	1	Average
%	48	52	**0.5**

$f'(x) = -\sin(x)e^{\cos(x)}$

Given that this was a standard application of chain rule for differentiation, the question was not well answered. Often the x was not included in responses, raising the question as to whether students understood circular functions. Common errors were $\sin(x)e^{\cos(x)}$, $\sin e^{\cos(x)}$, $\cos(x)e^{\cos(x)}$ and $\sin(x)\cos(x)e^{\cos(x)}$. Some students attempted to use the product rule.

3b.

Marks	0	1	2	3	Average
%	37	4	30	29	**1.6**

$f'(x) = \tan(x) + x\sec^2(x)$

$$f'\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} + \frac{2\pi}{9}$$

$$= \frac{3\sqrt{3} + 2\pi}{9}$$

Many students were able to obtain the correct derivative using the product rule but could not complete the corresponding evaluation successfully. This was usually the result of either not knowing the required exact circular function values or not being able to handle the subsequent working to find the answer. Occasionally students attempted to use the chain rule or incorrectly applied the product rule, simply multiplying the two derivatives of the individual functions together. Some students had difficulty in simplifying the expression as it contained a surd.

Question 4

4a.

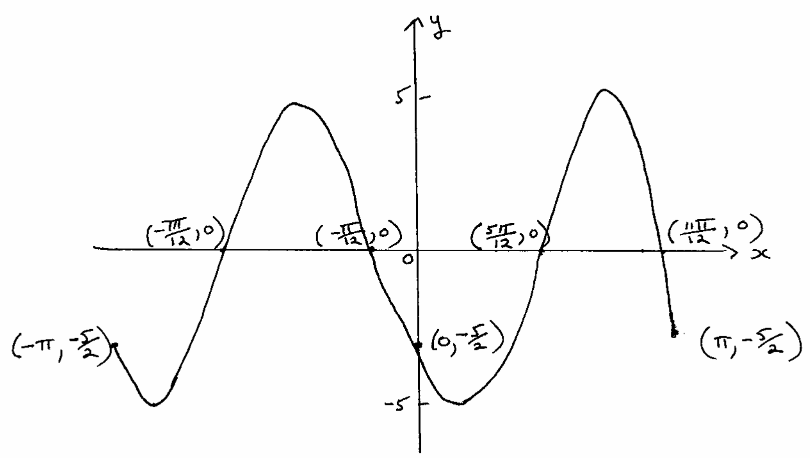
Marks	0	1	2	Average
%	6	15	79	**1.8**

amplitude = 5
period = π

This question was handled very well, as was expected of students at this level. The most common error was stating the period as 2π.

4b.

Marks	0	1	2	3	Average
%	32	43	12	14	**1.1**



Many students had difficulty sketching the graph and including all the required information in the correct form. It was disappointing to see the number of graphs that were either not smooth or not symmetric, or both. Many students did not realise that the amplitude and period found in part a. should be used for the graph. Although most graphs had an amplitude of 5, the period was rarely π. Endpoints were generally located correctly but rarely labelled, x-intercepts were often incorrect and not written as coordinates, the y-intercept was often ignored and graphs sometimes did not cover the entire domain. A few students also attempted to find turning points even though this was not asked for.

Question 5

Students tended to either obtain the correct answer or not attempt the question at all.

5a.

Marks	0	1	Average
%	39	61	**0.6**

$\Pr(X > 80) = \Pr(Z > 1) = 1 - \Pr(Z < 1) = 0.16$

Some students did not clearly understand the relationship between the random variables Z and X.

5b.

Marks	0	1	Average
%	55	45	**0.5**

$\Pr(64 < X < 72) = \Pr(-1 < Z < 0) = 0.34$

The most common incorrect responses were 0.32 and 0.68.

5c.

Marks	0	1	2	Average
%	47	26	27	**0.8**

$$\Pr(X < 64 \mid X<72) = \frac{\Pr(X < 64)}{\Pr(X < 72)} = \frac{\Pr(Z < -1)}{\Pr(Z < 0)} = \frac{0.16}{0.50} = 0.32 \left(\text{or } \frac{8}{25} \right)$$

Many students realised that conditional probability was required for this question but were not able to apply it correctly. Notation was poor and often made it difficult to ascertain if the student really understood what was required. The denominator was usually correct but the answer to part b. was often placed in the numerator. Some students did not further evaluate $\frac{0.16}{0.50}$ correctly.

Question 6

Students generally either did quite well on this question or did not attempt it at all. A few students did not realise the question involved a continuous random variable and required the use of integration.

6a.

Marks	0	1	2	Average
%	37	17	46	**1.1**

$$\int_1^3 \frac{x}{12} dx$$

$$= \frac{1}{3}$$

The main difficulty with this question was in students determining which terminals needed to be used. The majority of students knew the required anti-derivative, with the exception of a small number who obtained $\log_e(x)$. It was disappointing when students failed to include dx at the end of their integral expressions – this will be penalised in the future. A small number of students used 3 and 5 as their terminals and then subtracted their answer from 1. Even fewer determined the required value by using the graph and finding the area of the appropriate triangle.

6b.

Marks	0	1	2	Average
%	46	15	39	**1.0**

$\int_a^5 \frac{x}{12}dx = \frac{5}{8}$ or $\int_1^a \frac{x}{12}dx = \frac{3}{8}$

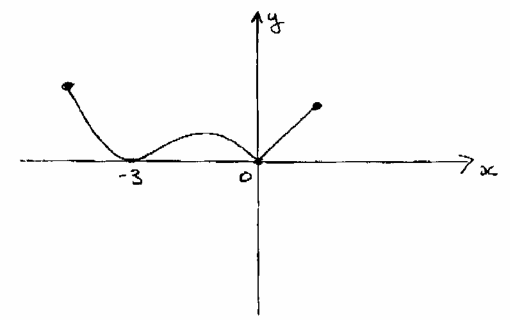
$a = \sqrt{10}$

For example, $\left[\frac{x^2}{24}\right]_1^a = \frac{3}{8}$ gives $\frac{a^2}{24} - \frac{1}{24} = \frac{3}{8}$, hence $a^2 = 10$

A substantial number of students were able to set up the correct integral equation but were unable to solve it to find *a*. Including $-\sqrt{10}$ (outside the domain) was a common error.

Question 7
7a.

Marks	0	1	2	Average
%	15	24	62	**1.5**



This question was generally well answered. Common errors included drawing the cusp point at $x = 0$ smoothly, not indicating endpoints or showing them as open circles, and not clearly indicating that the part of the graph to the right of *O* was also included.

7b.

Marks	0	1	Average
%	65	35	**0.4**

ran = [0, 20]

The responses to this question were disappointing. Common incorrect responses were *R*, (0, 20), [20, 0], R^+, and [0, 320]. The last response was obtained from incorrectly evaluating the function at $x = -5$.

Question 8

Marks	0	1	2	3	4	Average
%	42	18	9	3	29	**1.6**

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$-4 = -2\sqrt{x}$ or $\frac{1}{4} = \frac{1}{2\sqrt{x}}$

$x = 4,\ y = 2$

$y - 2 = -4(x - 4)$

$y = -4x + 18$

$a = 18$

Students found this question quite challenging, and a significant number seemed to have little or no idea of what was required. Many were able to find the correct derivative but did not know how to proceed, or equated the derivative to –4 instead of $-\frac{1}{4}$. Students who got to this point without error often did not solve correctly for x or y or both. It was not uncommon to see two values found for x and y and therefore two values for a, 18 and –14.

Question 9
This should have been a very straightforward question of a common application of a simple maximum/minimum type of problem. However this was not the case.

9a.

Marks	0	1	Average
%	63	37	**0.4**

$$A = 2a\left(9-3a^2\right)$$
$$= 18a - 6a^3$$

The answer was often not expressed just in terms of a but also in terms of b. Quite a few students had the correct response initially but then proceeded to expand the expression incorrectly.

9b.

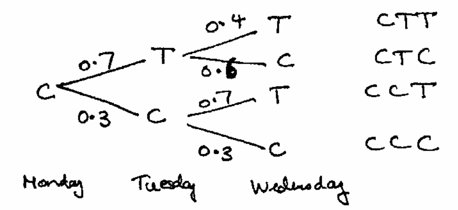
Marks	0	1	2	3	Average
%	64	4	10	21	**0.9**

$$\frac{dA}{da} = 18 - 18a^2$$
$$= 18(1-a)(1+a)$$
= 0, give maximum for the quadratic function
$a = 1$ since $a > 0$
Maximum value of A is 12

Many students had no idea what was required and attempted to try integrating $\int_{-3}^{3} 9 - 3x^2\, dx$ or solving $9 - 3x^2 = 0$ to obtain $\pm\sqrt{3}$. Those students who knew to differentiate and then equate to zero were usually successful. Some students forgot to obtain the area after correctly finding a.

Question 10

Marks	0	1	2	3	Average
%	14	16	9	61	**2.2**



Prob = $0.3 \times 0.7 + 0.7 \times 0.4$
$= 0.49$

This question was generally well answered with most students drawing a tree diagram and using it to find the required probability. It was disappointing to see that some students could not correctly multiply two simple decimals together, for example, writing 0.3 x 0.7 = 2.1. A few CAS students used matrices, but for a simple problem such as this, most still used tree diagrams. Able students were able to complete this question without using a tree diagram.

Question 11

Marks	0	1	2	3	4	5	Average
%	28	5	6	12	3	46	**3.1**

$$\int_0^3 \left(-x^2+ax+12\right)dx=45$$

$$\left[\frac{-x^3}{3}+\frac{ax^2}{2}+12x\right]_0^3=45$$

$$-9+\frac{9a}{2}+36=45$$

$$a=4$$

$$-x^2+4x+12=0$$

$$\left(6-x\right)\left(2+x\right)=0$$

$$m=6,\ n=-2$$

This question was well answered, with many students either obtaining three or five marks. Difficulty with algebraic skills was often the undoing of students. Occasionally students differentiated the expression even though they had written an integral, some used the quadratic equation to attempt to solve for m and n after finding a. Sometimes the values for m and n appeared from simply looking at the diagram and these values were then used to find a. This was not an appropriate solution as there could be an infinite set of solutions and students needed to show that their stated values satisfied all the conditions given.