



# Mathematical Methods

## Written examinations 1 and 2 – October/November

### Introduction

Mathematical Methods Examination 1 is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology. Mathematical Methods Examination 1 and Mathematical Methods Examination 1 CAS will be a common examination.

Mathematical Methods Examination 2 is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students are required to respond to multiple-choice questions in Part I of the paper and to extended answer questions, involving multi-stage solutions of increasing complexity, in Part II of the paper.

A formula sheet will be provided with each examination. Details of the formulas to be provided are published with the examination. The formula sheets for Mathematical Methods and Mathematical Methods (CAS) are exactly the same and common to both examinations 1 and 2.

### Structure and format

#### Examination 1

The examination will consist of short answer questions which are to be answered without the use of technology.

The examination will be out of a total of 40 marks.

#### Examination 2

The examination will consist of two parts. Part 1 will be a multiple-choice section containing 22 questions worth one mark each and Part II will consist of extended answer questions, involving multi-stage solutions of increasing complexity worth 58 marks. Examination 2 will be out of a total of 80 marks.

### Approved materials

#### Examination 1

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- A calculator is not allowed in this examination.
- Notes are not permitted in this examination.

Note: protractors, set squares, aids for curve sketching are no longer required for this examination and have been **removed** from the list of approved materials.

#### Examination 2

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- One bound reference that may be annotated. The reference may be a textbook.
- Protractors, set squares, aids for curve sketching.
- Graphics calculator and, if desired, one scientific calculator.

The memories of calculators do not need to be cleared for this examination.

The VCAA publishes details of approved technology for use in mathematics examinations annually. Details of approved calculators for 2006 were published in the October 2005 *VCAA Bulletin*, No. 31. The current list may be found at the VCE Mathematical Methods (CAS) Study page on the VCAA website. Details concerning VCAA approved reference material and technology for use in the 2006 Mathematical Methods examinations were published in the October 2005 *VCAA Bulletin*, No. 31 and November 2005 *VCAA Bulletin*, No. 32.

### **Other resources**

Teachers should refer to the Examination section of the *VCE and VCAL Administrative Handbook 2006*, *VCE Mathematics Assessment Handbook*, the VCE Mathematical Methods (CAS) Study page on the VCAA website and to the *VCAA Bulletin* for further advice during the year.

### **Sample examinations**

The sample examination papers for Mathematical Methods examinations 1 and 2 address content that remains unchanged and new content areas including continuous probability distributions, composition of functions, the modulus function and related rates of change.



**Victorian Certificate of Education  
2006**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

**STUDENT NUMBER**

Figures  
Words


Letter

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**MATHEMATICAL METHODS  
Written examination 1**

**Day Date 2006**

**Reading time: \*.\*.\* to \*.\*.\* (15 minutes)**

**Writing time: \*.\*.\* to \*.\*.\* (1 hour)**

**QUESTION AND ANSWER BOOK**

**Structure of book**

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape, a calculator.

**Materials supplied**

- Question and answer book of 7 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

**Instructions**

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

For the function  $f: (-1, \infty) \rightarrow R, f(x) = 2 \log_e(x + 1)$ ,

- a. find the rule of the inverse function  $f^{-1}$ .

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- b. find the domain of the inverse function  $f^{-1}$ .

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2 + 1 = 3 marks

**Question 2**

- a. Find  $\frac{dy}{dx}$  if  $y = 3x^4 \tan(x)$ .

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- b. If  $f: (-\infty, 2) \rightarrow R$  is such that  $f'(x) = \frac{1}{x-2}$  and  $f(1) = 6$ . Find the rule of  $f$ .

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2 + 2 = 4 marks

**Question 3**

Solve the equation  $\sin(x) = \sqrt{3} \cos(x)$  for  $x \in [-\pi, \pi]$ , giving exact values in terms of  $\pi$ .

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3 marks

**Question 4**

For the function  $f: [-\pi, \pi] \rightarrow \mathbb{R}$ ,  $f(x) = 3 \sin\left(2\left(x + \frac{\pi}{3}\right)\right)$

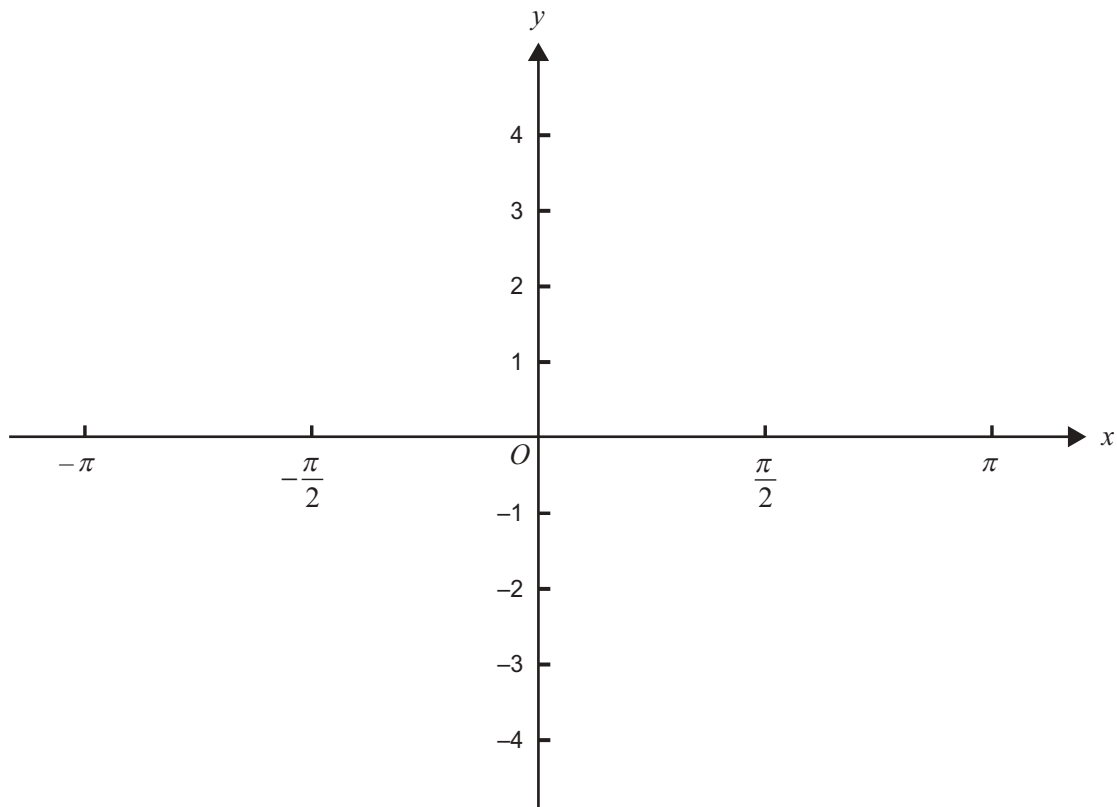
- a. write down the amplitude and period of the function.

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- b. sketch the graph of the function  $f$  on the set of axes below. Label axes intercepts and endpoints with their coordinates.



2 + 3 = 5 marks

**TURN OVER**

**Question 5**

Let  $X$  be a random variable with a normal distribution with mean 4 and standard deviation 2 and let  $Z$  be a random variable with the standard normal distribution.

- a. Find  $\Pr(X > 4)$ .

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- b. Find  $b$  such that  $\Pr(X > 5) = \Pr(Z < b)$ .

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1 + 1 = 2 marks

**Question 6**

The random variable  $X$  has a distribution with probability density function given by

$$f(x) = \begin{cases} ax(2-x) & \text{if } 0 \leq x \leq 2 \\ 0 & \text{if } x < 0 \text{ or } x > 2 \end{cases}$$

where  $a$  is a positive constant.

- a. Find the value of  $a$ .

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- b. Find  $\Pr\left(X < \frac{1}{2}\right)$ .

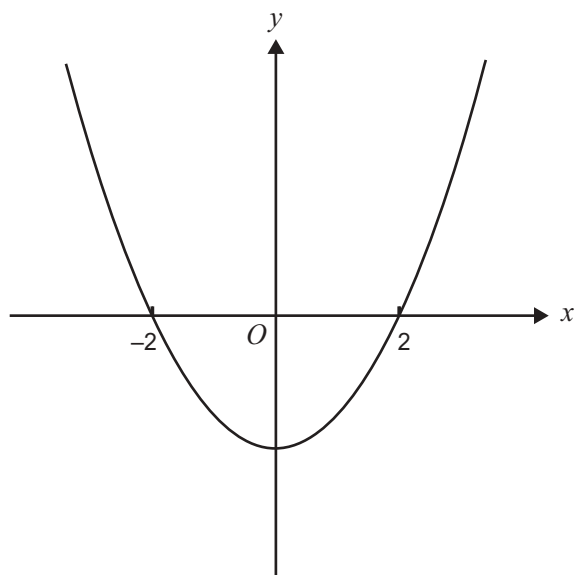
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3 + 2 = 5 marks

**Question 7**

Part of the graph of  $y = x^2 - 4$  is shown below.



- a. On the same set of axes sketch the graph of  $y = |x^2 - 4|$ .
- b. Hence find the area of the region bounded by the two curves.

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1 + 2 = 3 marks

**TURN OVER**

**Question 8**

Let  $f(x) = x^2 + 1$  and  $g(x) = \log_e(x)$ .

- a. Write down the rule of  $g(f(x))$ .

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- b. Find the derivative of  $g(f(x))$ .

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- c. Hence find an anti-derivative of  $\frac{x}{x^2 + 1}$ .

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1 + 1 + 1 = 3 marks

**Question 9**

The line  $y = 4x - 1$  is a tangent to the curve  $y = x^4 + c$ .

Find the exact value of  $c$ .

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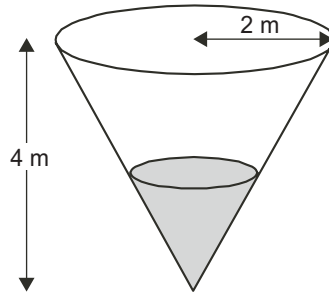
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4 marks



**Question 10**

A right conical vessel with base radius 2 m and height 4 m is being filled with water at a constant rate of  $3 \text{ m}^3/\text{min}$ . At what rate is the water rising when the depth is 3 m?




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4 marks

**Question 11**

Suppose that the probability of snow at a resort is dependent on whether or not it has snowed on the previous day. If it has snowed the day before the probability of snow is 0.6. If it has not snowed on the previous day then the probability of snow is 0.1.

If it has snowed on Thursday, what is the probability that it will **not** snow on the following Saturday?

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4 marks