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MATHS METHODS 3 & 4 TRIAL EXAMINATION 2 SOLUTIONS 2007

1. D	9. D	17. B
2. B	10. C	18. C
3. C	11. E	19. C
4. C	12. B	20. E
5. D	13. D	21. D
6. B	14. A	22. E
7. E	15. A	
8. A	16. E	

Section 1 – Multiple-choice solutions

Question 1

For $f(x)=\frac{2}{\sqrt{4x-3}}$, the maximal domain is given by

$$\begin{aligned}4x-3&>0\\4x&>3\\x&>\frac{3}{4}\end{aligned}$$

The answer is D.

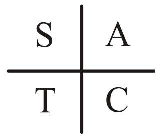
Question 2

$$\begin{aligned}f(x)&=3\cos\left(2x+\frac{\pi}{2}\right)+1\\&=3\cos\left(2\left(x+\frac{\pi}{4}\right)\right)+1\end{aligned}$$

The period is $\frac{2\pi}{2}=\pi$ and the amplitude is 3.
The answer is B.

Question 3

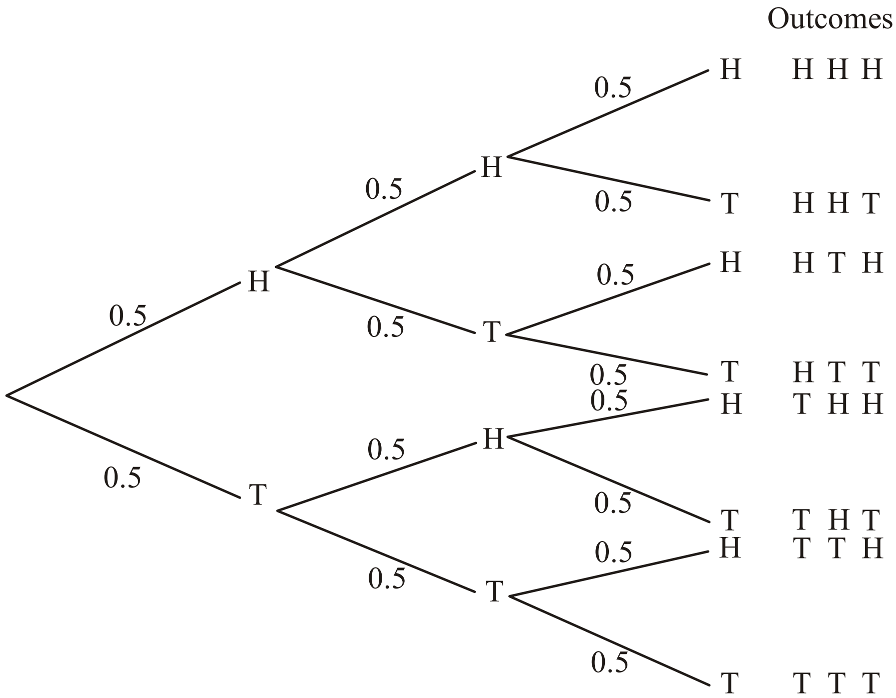
$$4\sin(2x) - 2\sqrt{3} = 0 \qquad 0 \le x \le 2\pi$$
$$4\sin(2x) = 2\sqrt{3} \qquad 0 \le 2x \le 4\pi$$
$$\sin(2x) = \frac{\sqrt{3}}{2}$$
$$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$$
$$x = \frac{\pi}{6}, \frac{2\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6}$$



The sum is $\frac{18\pi}{6}$ or 3π.
The answer is C.

Question 4

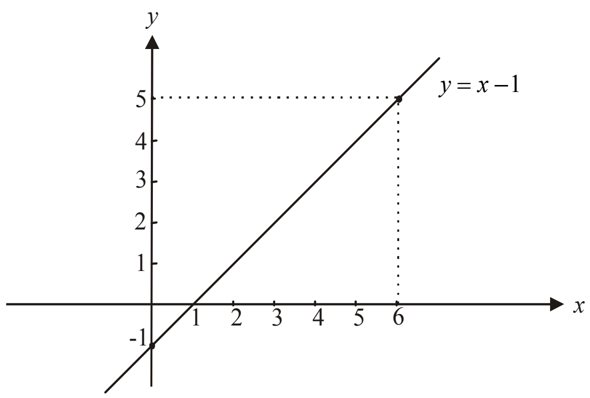
Draw a quick tree diagram.



There are 8 possible outcomes,
1 of those is to get 3 heads,
3 of those are to get 2 heads,
3 of those are to get 1 head,
and 1 of those is to get 0 heads.
The answer is C.

Question 5

Sketch the graph of $y = x - 1$.



For the function *f*, $r_f = [-1,5)$.
Looking at the graph when $y = -1,\ x = 0$. (Check $y = x - 1,\ -1 = 0 - 1$).
Again looking at the graph when $y = 5,\ x = 6$. (Check $y = x - 1,\ 5 = 6 - 1$).
So the domain of $f = [0,\ 6)$
The answer is D.

Question 6

A translation of 2 units to the left changes the rule $y = 1 - e^x$ to become $y = 1 - e^{(x+2)}$.
A reflection in the *y*-axis changes the rule to $y = 1 - e^{(-x+2)}$
The answer is B.

Question 7

$g(f(x))$ exists if $r_f \subseteq d_g$.
Now $d_g = [0,\infty)$.
So we require that $r_f \subseteq [0,\infty)$.
For option **A.** $r_f = R$.
For option **B.** $r_f = R$.
For option **C.** $r_f = R$.
For option **D.** $r_f = [-1,\infty)$.
For option **E.** $r_f = [0,\infty)$.
The answer is E.

Question 8

Let $y=2e^{-x}+1$

Swap x and y

$$x=2e^{-y}+1$$
$$x-1=2e^{-y}$$
$$\frac{x-1}{2}=e^{-y}$$
$$\log_e\left(\frac{x-1}{2}\right)=-y$$
$$y=-\log_e\frac{(x-1)}{2}$$

So $f^{-1}(x)=-\log_e\frac{(x-1)}{2}$

The answer is A.

Question 9

Method 1

$$|3p-2|=p+1$$
$$3p-2=p+1 \qquad \text{if } 3p-2>0$$
$$2p=3 \qquad 3p>2$$
$$p=\frac{3}{2} \qquad p>\frac{2}{3}$$

$$-(3p-2)=p+1 \qquad \text{if } 3p-2<0$$
$$-3p+2=p+1 \qquad 3p<2$$
$$-4p=-1 \qquad p<\frac{2}{3}$$
$$p=\frac{1}{4}$$

The answer is D.

Method 2

Sketch $y_1=\text{abs}(3x-2)$ and $y_2=x+1$.

Points of intersection occur at $x=\frac{1}{4}$ and at $x=\frac{3}{2}$, so, $p=\frac{1}{4}$ or $p=\frac{3}{2}$.

The answer is D.

Question 10

$$y=e^{-x}\sin(3x)$$
$$=-e^{-x}\sin(3x)+e^{-x}\times 3\cos(3x)$$
$$=e^{-x}(3\cos(3x)-\sin(3x))$$

The answer is C.

Question 11

Method 1

$$y = \sqrt{x + 2x^2}$$
$$= \left(x + 2x^2\right)^{\frac{1}{2}}$$
$$= u^{\frac{1}{2}} \qquad \text{where } u = x + 2x^2$$
$$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} \qquad \frac{du}{dx} = 1 + 4x$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{(chain rule)}$$
$$= \frac{1}{2}u^{-\frac{1}{2}}\left(1 + 4x\right)$$
$$= \frac{1}{2} \cdot \frac{1}{\sqrt{u}}\left(1 + 4x\right)$$
$$= \frac{1 + 4x}{2\sqrt{x + 2x^2}}$$

At $x = 1$

$$\frac{dy}{dx} = \frac{5}{2\sqrt{3}}$$

The answer is E.

Method 2

Sketch $y = (x + 2x^2)^{\frac{1}{2}}$.

Calculate $\frac{dy}{dx}$ when $x = 1$.

$$\frac{dy}{dx} = 1.4433757$$
$$= \frac{5}{2\sqrt{3}}$$

The answer is E.

Question 12

Let v equal the volume of the cylinder.

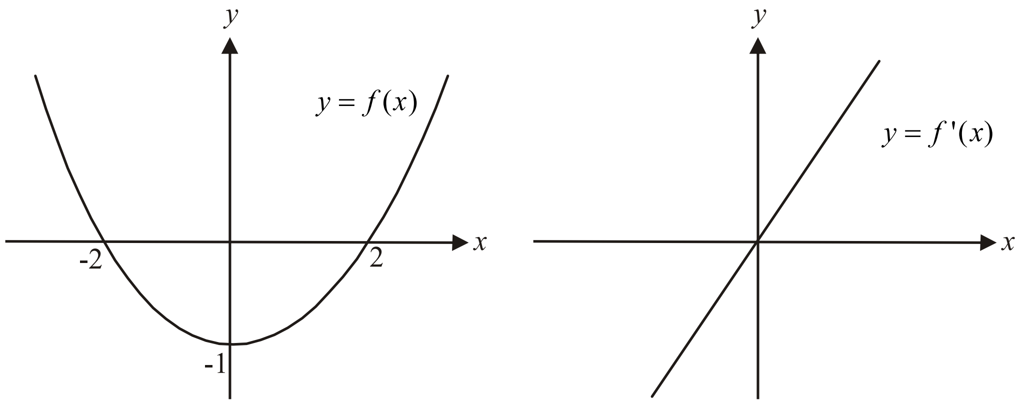
$$v = \pi r^2 h$$
$$\text{since} \quad r = 5$$
$$v = 25\pi h$$
$$\frac{dv}{dh} = 25\pi$$
$$\text{Also} \quad \frac{dh}{dt} = \frac{1}{t}$$

We are required to find $\frac{dv}{dt}$ when $t = 10$.

$$\text{So} \quad \frac{dv}{dt} = \frac{dv}{dh} \cdot \frac{dh}{dt} \quad \text{(Chain rule)}$$
$$= 25\pi \times \frac{1}{t}$$
$$\text{When} \quad t = 10, \; \frac{dv}{dt} = 2.5\pi$$

The answer is B.

Question 13



$f(x)$ is positive for $x < -2 \cup x > 2$.
$f'(x)$ is the gradient of $f(x)$ and is positive for $x > 0$.
So $f(x)$ and $f'(x)$ are both positive for $x > 2$.
The answer is D.

Question 14

Let the antiderivative function of f be called g.
So $g'(x) = f(x)$.
For $x \in (-a, a)$, $f(x) > 0$ so the graph of g will have a positive gradient.
Note that for there to be a stationary point (which may be a local min/max or point of inflection) $f(x) = 0$. For $x \in (-a, a)$, $f(x) \neq 0$.
The answer is A.

Question 15

$$\int (2x-1)^6 \, dx = \frac{1}{2 \times 7}(2x-1)^7 + c$$
$$= \frac{1}{14}(2x-1)^7 + c$$

The answer is A.

Question 16

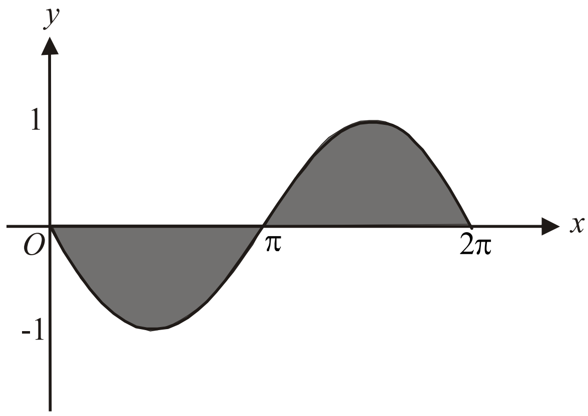
$$h'(x) = 2x - 3g'(x)$$
$$\int h'(x)dx = \int 2x dx - 3\int g'(x)dx$$
$$h(x) = x^2 - 3g(x) + c$$

Now $h(2) = 4$ and $g(2) = 3$
So when $x = 2$,
$$4 = 4 - 9 + c$$
$$c = 9$$

So $h(x) = x^2 - 3g(x) + 9$
The answer is E.

Question 17

Sketch the graph.



$$\text{Area } = -\int_0^{\pi} -\sin(x)\,dx + \int_{\pi}^{2\pi} -\sin(x)\,dx$$

$$= 2\int_0^{\pi} \sin(x)\,dx \text{ because of the symmetry of the sine curve}$$

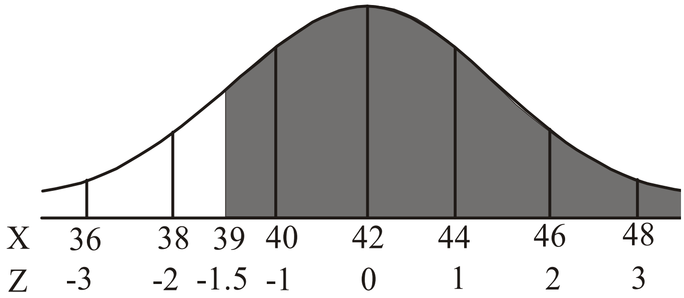
The answer is B.

Question 18

$$2\log_2(3) - \log_2(6)$$
$$= \log_2(3^2) - \log_2(6)$$
$$= \log_2\left(\frac{9}{6}\right)$$
$$= \log_2(1.5)$$
$$= \frac{\log_{10}(1.5)}{\log_{10}(2)} \qquad \text{OR} \qquad \frac{\log_e(1.5)}{\log_e(2)} \quad \text{(change of base rule)}$$
$$= 0.5850 \text{ (to 4 dec. places)} \qquad = 0.5850 \text{ (to 4 dec. places)}$$

The answer is C.

Question 19



The shaded area is given by normalcdf(39, 100, 42,2) = 0.93319…
= 93% to the nearest percent
(Alternatively, normalcdf(-1.5,100,0,1) = 0.93319…)
The answer is C.

Question 20

This represents a binomial distribution because there is a fixed probability of an event happening i.e. 0.78 and there is a fixed number of "trials" i.e. 4 and we want a particular number of events to occur i.e. 3 or 4.

Method 1

$$\begin{aligned}&\Pr(3\text{ goals}) + \Pr(4\text{ goals})\\&= {}^4C_3(0\cdot 78)^3(0\cdot 22)^1 + {}^4C_4(0\cdot 78)^4(0\cdot 22)^0\\&= 0\cdot 7878\end{aligned}$$

The answer is E.

Method 2

binompdf(4,0.78,3) + binompdf(4,0.78,4)
= 0.7878

The answer is E.

Question 21

The mode is the value of x for which the maximum value of f occurs.

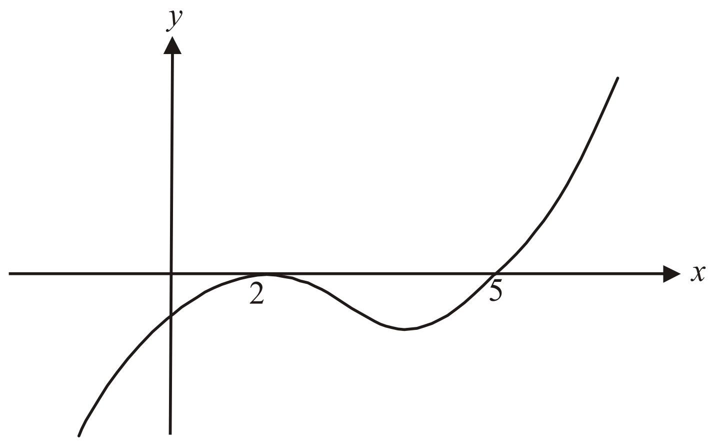
Now, $y = \frac{3}{4}\left(1 - (x-1)^2\right)$ for $0 \le x \le 2$, describes the graph of an inverted parabola with a maximum turning point at $\left(1, \frac{3}{4}\right)$.

So the maximum value of f is $\frac{3}{4}$ and it occurs at $x = 1$.

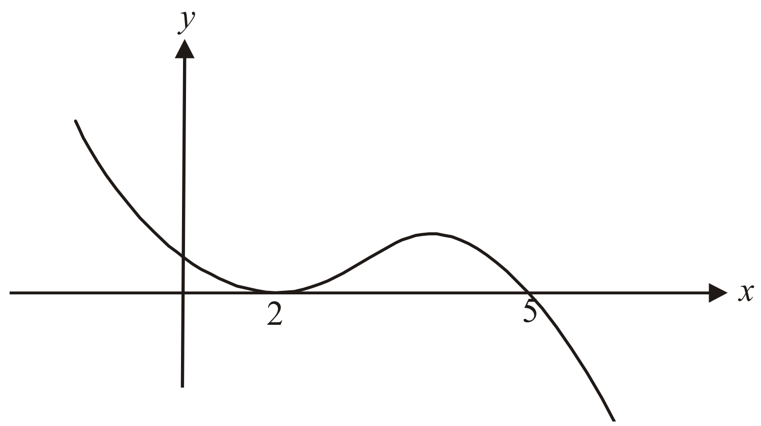
The mode (that is the most popular value) is 1.
The answer is D.

Question 22

The graph of $y = f(x)$ could be



or



Both of these graphs produce the graph of $y = |f(x)|$.
The equation of the first could be $y = (x-2)^2(x-5)$. This is option E.
The equation of the second could be $y = -(x-2)^2(x-5)$. This is not offered.
The answer is E.

SECTION 2

Question 1

a. $y = 6x^2 \log_e(2x)$

x-intercept occurs when $y = 0$

$$0 = 6x^2 \log_e(2x)$$

Either $6x^2 = 0$

$x = 0$ but we reject this

since $d_f = (0, \infty)$

or $\log_e(2x) = 0$

$$e^0 = 2x$$

$$2x = 1$$

$$x = \frac{1}{2}$$

The x-intercept occurs at $\left(\frac{1}{2}, 0\right)$ as required

(1 mark)

b. The function $f(x) = 6x^2 \log_e(2x)$ is only defined for values of x such that $2x > 0$; that is, for $x > 0$ because $\log_e(2x)$ is only defined for these values.

(1 mark)

c. Use a graphics calculator to locate the turning point. It is the point $(0 \cdot 3, -0 \cdot 3)$ where each coordinate is correct to one decimal place.

(1 mark)

d. From part **c.** the range is $[-0 \cdot 2759..., \infty)$ or $[-0 \cdot 3, \infty)$ correct to one decimal place.

(1 mark)

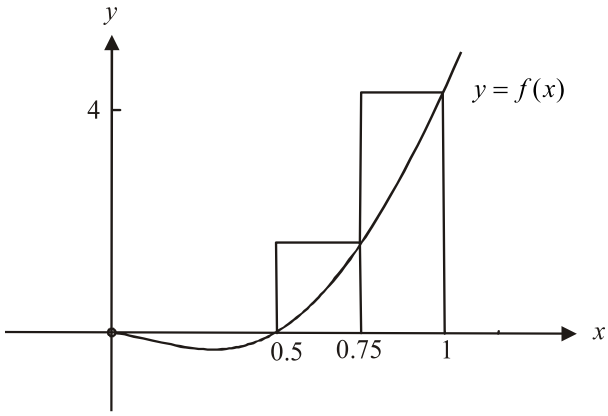
e. **i.** The function f does not have an inverse function because it is not a $1:1$ function.

(1 mark)

ii. The function g must be a $1:1$ function since g^{-1} exists.
The maximal domain of g is therefore $[0 \cdot 3032..., \infty)$ where the value of a is the x-coordinate of the turning point of the curve. The answer is $a = 0 \cdot 3$ where a is correct to one decimal place.

(1 mark)

f. **i.**



(1 mark)

Approximate area using right rectangles method

$= 0 \cdot 25 \times f(0 \cdot 75) + 0 \cdot 25 \times f(1)$

$= 1 \cdot 3818...$ **(1 mark)**

$= 1 \cdot 38$ square units (correct to 2 decimal places)

ii. Since the right rectangles extend above the function *f* the approximation will be greater than the exact area.

(1 mark)

g. **i.**

$$y = x^3 \log_e(2x)$$

$$\frac{dy}{dx} = 3x^2 \log_e(2x) + x^3 \times \frac{2}{2x} \qquad \text{(product rule)}$$

$$= 3x^2 \log_e(2x) + x^2$$

(1 mark)

So $a = 3$ and $b = 1$

(1 mark)

ii. Hence means use what you have already found.

From part **i.**, we found that $\frac{d}{dx}\left(x^3 \log_e(2x)\right) = 3x^2 \log_e(2x) + x^2$

If we antidifferentiate each term on both sides, we obtain

$x^3 \log_e(2x) + c_1 = \int 3x^2 \log_e(2x)dx + \int x^2 dx$ where c_1 is a constant.

Rearranging, we obtain

$$\int 3x^2 \log_e(2x)dx = x^3 \log_e(2x) + c_1 - \int x^2 dx$$

$$= x^3 \log_e(2x) - \frac{x^3}{3} + c$$

(1 mark)

(Note that c = the sum of c_1 and the constant that arose from $\int x^2 dx$.)

So $\int 6x^2 \log_e(2x)dx = 2\left(x^3 \log_e(2x) - \frac{x^3}{3} + c\right)$

$$= 2x^3 \log_e(2x) - \frac{2x^3}{3} + 2c$$

Now the exact area

$$= \int_{0\cdot 5}^{1} 6x^2 \log_e(2x)dx$$ **(1 mark)**

$$= \left[2x^3 \log_e(2x) - \frac{2x^3}{3} \right]_{0\cdot 5}^{1}$$ **(1 mark)**

$$= \left\{ \left(2\log_e(2) - \frac{2}{3} \right) - \left(\frac{1}{4}\log_e(1) - \frac{1}{12} \right) \right\}$$

$$= 2\log_e(2) - \frac{2}{3} + \frac{1}{12}$$

$$= 2\log_e(2) - \frac{7}{12} \text{ square units}$$

(1 mark)

(Note – if you have time, check your answer using a calculator:

$$\int_{0\cdot 5}^{1} 6x^2 \log_e(2x)dx = 0.80296...$$

and $2\log_e(2) - \frac{7}{12} = 0.80296...$)

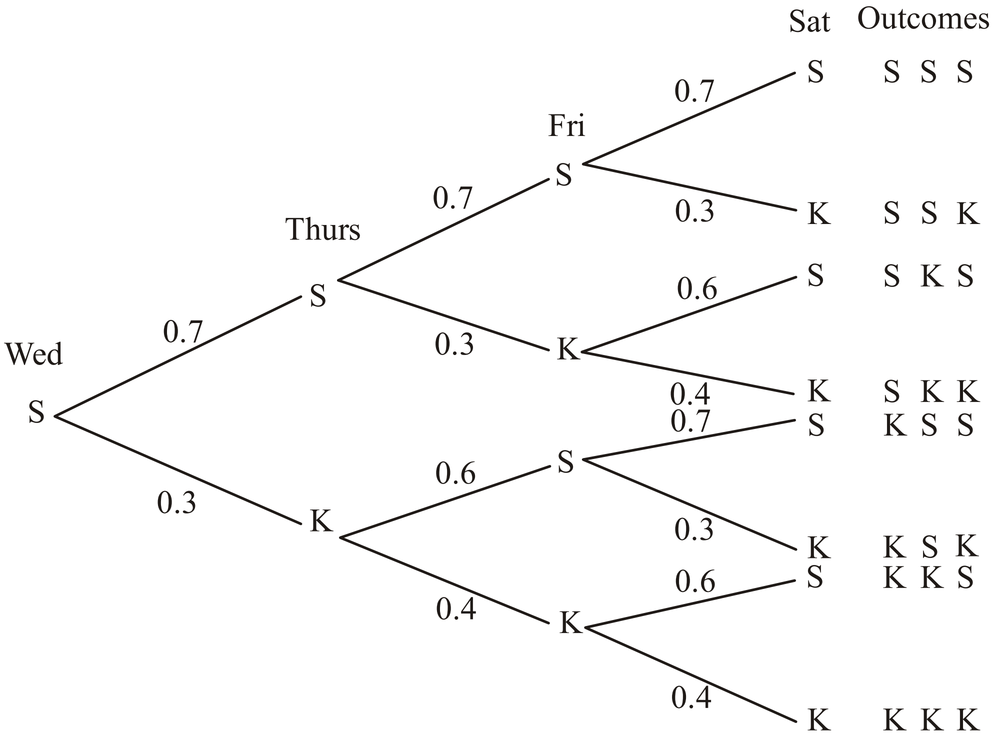
Total 15 marks

Question 2

a. **i.** The probability that Sealord's provides the seafood on Thursday, Friday and Saturday given that it supplied it on Wednesday is $0 \cdot 7 \times 0 \cdot 7 \times 0 \cdot 7 = 0 \cdot 343$.

(1 mark)

ii. Draw a tree diagram to see all the possible outcomes.



(1 mark)

Looking at the possible outcomes the probability that Sealord's supplies on at least two of the next three nights

$= \Pr(SSS) + \Pr(SSK) + \Pr(SKS) + \Pr(KSS)$ **(1 mark)**

$= (0 \cdot 7 \times 0 \cdot 7 \times 0 \cdot 7) + (0 \cdot 7 \times 0 \cdot 7 \times 0 \cdot 3) + (0 \cdot 7 \times 0 \cdot 3 \times 0 \cdot 6) + (0 \cdot 3 \times 0 \cdot 6 \times 0 \cdot 7)$

$= 0 \cdot 343 + 0 \cdot 147 + 0 \cdot 126 + 0 \cdot 126$

$= 0 \cdot 742$

(1 mark)

iii. From the tree diagram, the probability that the first time Kingfisher supplies the restaurant is on Saturday or on Sunday.

$= \Pr(SSK) + \Pr(SSSK)$ (extend the tree by 1 more branch) **(1 mark)**

$= (0 \cdot 7 \times 0 \cdot 7 \times 0 \cdot 3) + (0 \cdot 7 \times 0 \cdot 7 \times 0 \cdot 7 \times 0 \cdot 3)$

$= 0 \cdot 147 + 0 \cdot 1029$

$= 0 \cdot 2499$

(1 mark)

b. Pr(Marco spends more than 5 hours in the kitchen on a particular night)

$= -\frac{6}{125}\int_5^6 (t-1)(t-6)\,dt$ **(1 mark)**

$= 0 \cdot 104$

(1 mark)

c. This is a binomial distribution with $p = 0\cdot104$ (from part **b.**), $n = 4$ and $x = 3$.

Method 1

Required probability $= {}^nC_x p^x q^{n-x}$

$= {}^4C_3(0\cdot104)^3(0\cdot896)^1$

$= 0\cdot00403...$

$= 0\cdot004$ (correct to 3 decimal places)

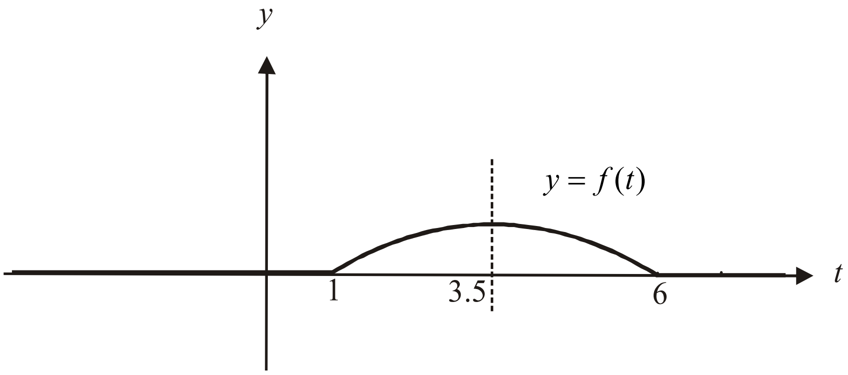
Method 2

Using a calculator, binompdf(4,0.104,3) = 0.004 (correct to 3 decimal places)

(1 mark) for correct answer
(1 mark) for recognition that it was a binomial distribution

d. Method 1

Sketching the graph.



(1 mark)

We have an inverted parabola with its axis of symmetry located at $t = 3\cdot5$.
So the median time spent in the kitchen on a night by Marco is 3.5 hours since the area under the graph for $t < 3\cdot5$ is equal to the area under the graph for $t > 3\cdot5$.

(1 mark)

Method 2

Let *m* be the median.
We require that

$$\int_1^m f(t)dt = 0\cdot5$$

$$-\frac{6}{125}\int_1^m (t-1)(t-6)dt = 0\cdot5$$

(1 mark)

Use your calculator and a trial and error method to find the upper limit *m*.
It is 3.5.

(1 mark)

<u>Method 3</u>
Let m be the median.
We require that

$$\int_1^m f(t)dt = 0\cdot 5$$

$$-\frac{6}{125}\int_1^m ((t-1)(t-6))dt = 0\cdot 5$$ **(1 mark)**

$$-\frac{6}{125}\int_1^m (t^2 - 7t + 6)dt = 0\cdot 5$$

$$-\frac{6}{125}\left[\frac{t^3}{3} - \frac{7t^2}{2} + 6t\right]_1^m = 0\cdot 5$$

$$-\frac{6}{125}\left\{\left(\frac{m^3}{3} - \frac{7m^2}{2} + 6m\right) - \left(\frac{1}{3} - \frac{7}{2} + 6\right)\right\} = 0\cdot 5$$

$$\frac{m^3}{3} - \frac{7m^2}{2} + 6m - 2\cdot 8\dot{3} = -10\cdot 41\dot{6}$$

$$\frac{m^3}{3} - \frac{7m^2}{2} + 6m + 7\cdot 58\dot{3} = 0$$

Solve this by graphing $y = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 7.58\dot{3}$ and see where it intersected with $y = 0$, that is, the x - axis, between $x = 1$ and $x = 6$ using a graphics calculator.
It intersects at $x = 3.5$
So $m = 3\cdot 5$

(1 mark)

Total 12 marks

Question 3

a. **i.** At point A, $x=0$.

$f(0)=\frac{1}{56}\left\{(0-2)^3(0+5)-72\right\}$

$=-2$

A is the point $(0,-2)$.

(1 mark)

ii. At point C, $y=0$.

Use a graphics calculator to solve $0=\frac{1}{56}\left\{(x-2)^3(x+5)-72\right\}$.

The answers are $x=-5\cdot1934...$ and $x=4$.

C is the point $(4,0)$.

(1 mark)

b. $|f(0)|=|-2|$ from part **a.i.**

$=2$

(1 mark)

c. Method 1 – expanding then differentiating

$$f(x)=\frac{1}{56}\left\{(x-2)^3(x+5)-72\right\}$$
$$=\frac{1}{56}\left\{(x^2-4x+4)(x-2)(x+5)-72\right\}$$
$$=\frac{1}{56}\left\{(x^2-4x+4)(x^2+3x-10)-72\right\}$$
$$=\frac{1}{56}\left(x^4+3x^3-10x^2-4x^3-12x^2+40x+4x^2+12x-40-72\right)$$
$$=\frac{1}{56}\left(x^4-x^3-18x^2+52x-112\right)$$

(1 mark)

$$f'(x)=\frac{1}{56}\left(4x^3-3x^2-36x+52\right)$$

(1 mark)

Method 2 – using the product rule

$$f(x)=\frac{1}{56}\left\{(x-2)^3(x+5)-72\right\}$$
$$f'(x)=\frac{1}{56}\left\{(x-2)^3\times1+3(x-2)^2(x+5)\right\}$$

(1 mark)

$$=\frac{1}{56}\left\{(x-2)^2((x-2)+3(x+5))\right\}$$
$$=\frac{1}{56}\left\{(x-2)^2(4x+13)\right\}$$
$$=\frac{1}{56}\left\{(x^2-4x+4)(4x+13)\right\}$$
$$=\frac{1}{56}\left(4x^3-3x^2-36x+52\right)$$

(1 mark)

d. **i.** At point B, $f'(x)=0$.

Method 1 – following on from Method 1 in part **c.**

$$f'(x)=\frac{1}{56}\left(4x^3-3x^2-36x+52\right)=0$$

Use a graphics calculator to solve this. $x=2$ is a solution that corresponds to B's position.

Method 2 – following on from Method 2 in part **c.**

$$\frac{1}{56}\left\{(x-2)^2(4x+13)\right\}=0$$

$x=2$ or $x=-\dfrac{13}{4}$, but $x>0$ so reject this.

Now,

$$\begin{aligned} f(2)&=\frac{1}{56}(0-72)\\ &=-\frac{9}{7}\end{aligned}$$

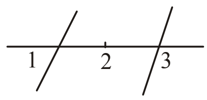
B is the point $\left(2,-\dfrac{9}{7}\right)$

(1 mark)

ii. B is a stationary point because at B, $f'(x)=0$

$$\begin{aligned}\text{At } x=1,\ f'(x)&=\frac{1}{56}(4-3-36+52)\\ &=\frac{17}{56}\\ &>0\end{aligned}$$

$$\begin{aligned}\text{At } x=3,\ f'(x)&=\frac{1}{56}(108-27-108+52)\\ &=\frac{25}{56}\\ &>0\end{aligned}$$



Since the gradient to either side of point B is positive but at point B the gradient is zero, there is a stationary point of inflection at B.

(1 mark)
(1 mark) for first derivative test

e. Area of garden bed

$$=-\int_0^4 f(x)\,dx \qquad \text{(negative because the area falls below the } x\text{-axis)}$$

$$=-\frac{1}{56}\int_0^4\left(x^4-x^3-18x^2+52x-112\right)dx$$

(1 mark) – correct terminals
(1 mark) – correct function

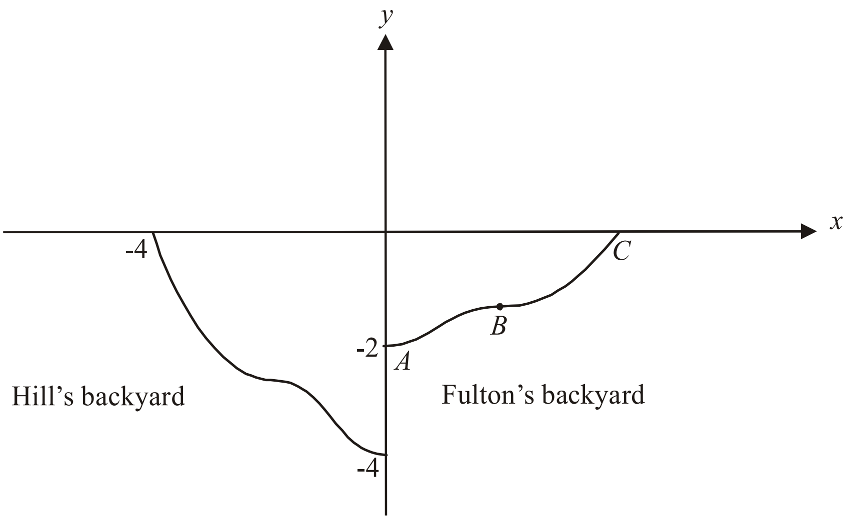
f. **i.** The Fulton's garden border is described by $y=\frac{1}{56}\{(x-2)^3(x+5)-72\}$ so the George's garden border is described by $y=\frac{-1}{56}\{(x-2)^3(x+5)-72\}$.

(1 mark)

ii. Since C is the point $(4,0)$ (from part **a. ii.**) $d_g=[0,\ 4]$.

(1 mark)

g. **i.**



(1 mark)

ii. The rule for the garden border in Fulton's garden is given by

$y=\frac{1}{56}\{(x-2)^3(x+5)-72\}$.

When this function is reflected in the y-axis the rule becomes

$y=\frac{1}{56}\{(-x-2)^3(5-x)-72\}$

When this function is then dilated by a factor of 2 from the x-axis the rule becomes

$$\frac{y}{2}=\frac{1}{56}\{(-x-2)^3(5-x)-72\}$$

$$\text{So } h(x)=\frac{1}{28}\{(-x-2)^3(5-x)-72\}$$

(1 mark)

h. **i.** The function $y=r(x)$ is the inverse function of the function $y=f(x)$ and vice-versa.

(1 mark)

ii. The coordinates of the point B' are $\left(-\frac{9}{7},2\right)$ because the coordinates of B are $\left(2,-\frac{9}{7}\right)$ from part **d.i.**

(1 mark)

Total 16 marks

Question 4

a. Since $d_f = [-1,1]$ and since the point A is at the right hand endpoint of the graph, $x = 1$.

$f(1) = \tan(1)$

$\quad = 1 \cdot 56$ (correct to 2 decimal places)

A is the point $(1,\ 1 \cdot 56)$

(1 mark)

b. The function *f* is not defined for $x = \pm\frac{\pi}{2}$, since an asymptote exists on the graph of $y = \tan(x)$ at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

Since $\frac{\pi}{2} = 1 \cdot 57...$, this would be within 2km in the directions east and west of the intersection.

(1 mark)

c.

$$y = \tan(x)$$

$$\frac{dy}{dx} = \sec^2(x)$$

At $x = 0$

$$\frac{dy}{dx} = \sec^2(0)$$

$$= \frac{1}{\cos^2(0)}$$

$$= \frac{1}{1^2}$$

$$= 1$$

(1 mark)

So at the point of intersection (i.e. $(0,0)$), the gradient of the function $y = \tan(x)$ is 1 and so the rail line makes an angle of 45° or $\frac{\pi}{4}$ with the road heading north (and of course with the road heading east).

(1 mark)

d. At point B,

$\frac{dy}{dx}=\frac{4}{3}$

Since point B lies on the main rail line, we have

$y=\tan(x)$

So $\frac{dy}{dx}=\sec^2(x)$

So $\frac{4}{3}=\sec^2(x)$ **(1 mark)**

$\frac{4}{3}=\frac{1}{\cos^2(x)}$

$\cos^2(x)=\frac{3}{4}$

$\cos(x)=\pm\frac{\sqrt{3}}{2}$

$x=\frac{\pi}{6},\frac{5\pi}{6}...$ **(1 mark)**

Since $d_f=[-1,1]$ and $\frac{5\pi}{6}=2\cdot 6179...$ then $x=\frac{5\pi}{6}$ is outside the domain.

(1 mark)

The value of x is $\frac{\pi}{6}$.

Since $f(x)=\tan(x)$

$f\left(\frac{\pi}{6}\right)=\tan\left(\frac{\pi}{6}\right)$

$=\frac{1}{\sqrt{3}}$

So B is the point $\left(\frac{\pi}{6},\frac{1}{\sqrt{3}}\right)$. **(1 mark)**

e. The gradient of the shunting line is $\frac{4}{3}$ and it passes through $\left(\frac{\pi}{6},\frac{1}{\sqrt{3}}\right)$.

The equation of the shunting line is $y-\frac{1}{\sqrt{3}}=\frac{4}{3}\left(x-\frac{\pi}{6}\right)$. **(1 mark)**

The road running in the east-west direction is represented by the x-axis along which $y=0$.

So $0-\frac{1}{\sqrt{3}}=\frac{4}{3}\left(x-\frac{\pi}{6}\right)$

$\frac{-3}{\sqrt{3}}=4x-\frac{2\pi}{3}$

$4x=\frac{2\pi}{3}-\sqrt{3}$

$x=\frac{\pi}{6}-\frac{\sqrt{3}}{4}$

$=0\cdot 0905...$

(1 mark)

Since $x>0$, the point of intersection occurs on the east side of the dangerous intersection. **(1 mark)**

f. **i.** The curves with equations $y = \tan(x)$ and $y = a - b\sqrt{2}\cos(x)$ meet smoothly at point *C* where $x = \frac{\pi}{4}$.

This means firstly that at $x = \frac{\pi}{4}$

$$\tan(x) = a - b\sqrt{2}\cos(x)$$

that is, $\tan\left(\frac{\pi}{4}\right) = a - b\sqrt{2}\cos\left(\frac{\pi}{4}\right)$

$$1 = a - b\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$1 = a - b \qquad -(1)$$

(1 mark)

Secondly, this means that if

$$y = \tan(x)$$

$$\frac{dy}{dx} = \sec^2(x)$$

and if $y = a - b\sqrt{2}\cos(x)$

$$\frac{dy}{dx} = b\sqrt{2}\sin(x)$$

At $x = \frac{\pi}{4}$, the gradients are equal **(1 mark)**

so $\sec^2(x) = b\sqrt{2}\sin(x)$

that is, $\sec^2\left(\frac{\pi}{4}\right) = b\sqrt{2}\sin\left(\frac{\pi}{4}\right)$

$$\frac{1}{\cos^2\left(\frac{\pi}{4}\right)} = b\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = b$$

$$b = 1 \div \frac{1}{2}$$

$$= 2$$

From (1) $1 = a - b$

so $a = 3$

Have shown.

(1 mark)

ii. We are looking for the *y*-intercept of the graph of $y = 3 - 2\sqrt{2}\cos(x)$.

When $x = 0$,

$$y = 3 - 2\sqrt{2}\cos(0)$$

$$= 3 - 2\sqrt{2}$$

$$= 0 \cdot 17157\ldots\text{km}$$

$$= 172 \text{ metres (to the nearest metre)}$$

(1 mark)

Total 15 marks