Mathematical Methods

2007

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 X is a normally distributed random variable with a mean of 56 and a standard deviation of 5. Z is the standard normal random variable.

a.   Find the value of Z that corresponds to $X = 45$.   1 mark

b.   Given that $\Pr(-1 < Z < 1) = 0.68$,

i. find $\Pr(56 < X < 61)$, correct to 2 decimal places   1 mark

ii. find $\Pr(X < 51 \cup X > 61)$, correct to 2 decimal places.   1 mark

Question 2   The probability density function of a continuous random variable X is given by

$$p(x) = \begin{cases} kx & 0 \le x \le 3 \\ 0 & elsewhere \end{cases}$$

a.   Find the exact value of k.   2 marks

b.   Find the exact value of $\Pr(X > 2)$.   1 mark

Question 3   Find the exact x-intercepts of the graph of $f(x)=x^3-3x^2-8x+24$ for $x\geq 0$.   2 marks

Question 4   Given $f(x)=|3-2x|+1$,
a.   sketch the graph of $y=f(x)$.   2 marks

b.   State a sequence of exactly three transformations, which changes $y=f(x)$ to $y=|x|$.   2 marks

Question 5   Given $f(x)=10^{2\log_{10}(x)-\log_{10}(x+1)-1}$,
a.   show that $y=f(x)$ can be simplified to $y=\dfrac{x^2}{10x+10}$.   2 marks

b.   Find $f^{-1}(x)$.   2 marks

Question 6   Solve $\sin(3\theta)+\sqrt{3}\cos(3\theta)=0$ for the exact values of θ, where $-\pi \le \theta \le \pi$.   3 marks
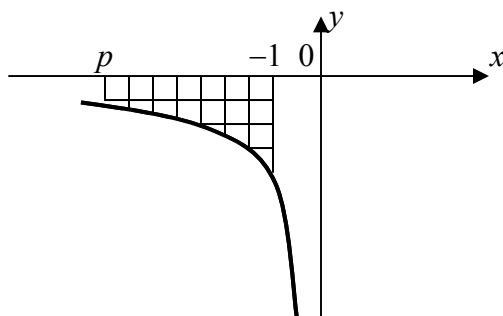
Question 7   Consider $f(x)=e^{2x}-e^{x}-2$.

a.   Solve $e^{2x}-e^{x}-2=0$ for x in exact form.   2 marks

b.   Find the equation of the normal to the curve with equation $y=e^{2x}-e^{x}-2$ where $e^{x}=2$.   3 marks

Question 8   The area of the shaded region is 4 in the following graph of $y=\dfrac{2}{x}$. Find the exact value of p.

3 marks

Question 9 Consider $f(x) = -3x^4 - 4x^3 + 6x^2 + 12x - 3$.

a. Find the coordinates of the stationary points.   2 marks
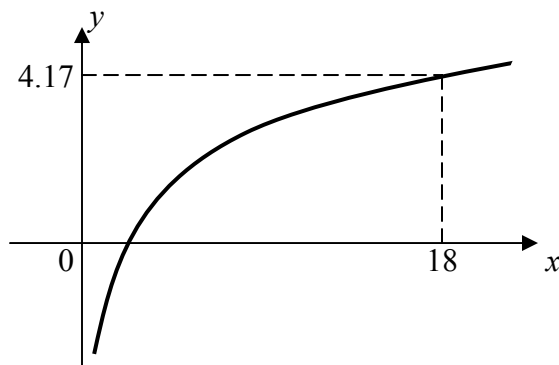
b. Determine the nature of each stationary point found in part a.   2 marks

c. Sketch the graph of $y = f(x)$. (Note: Do not determine the values of the x-intercepts)   2 marks

Question 10 Given $f(x) = \log_2 x$ and the graph of $f(x)$ is shown below.
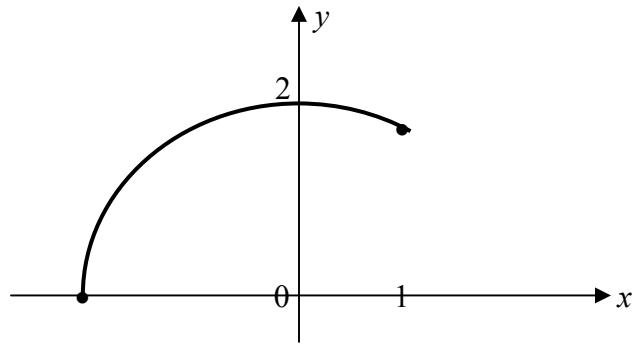

a. Show that $f'(x) = \dfrac{1}{x \log_e 2}$.   1 mark

b. Use Euler's linear approximation method to estimate $\log_2 19$ to 2 decimal places, given $\log_e 2 = 0.69$.   2 marks

Question 11   The following graph shows a portion of a circle with radius of 2 units and centred at the origin.

a.   Use function notation to specify the portion of the circle shown above.   2 marks

b.   Sketch the graph of the gradient function from the graph shown above. Pay particular attention to the end points.

2 marks

End of exam 1