

**Year 2007**

**VCE**

**Mathematical Methods  
and  
Mathematical Methods  
( CAS )**

**Trial Examination 1**



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**Victorian Certificate of Education  
2007**

**STUDENT NUMBER**

Figures  
Words


Letter

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**MATHEMATICAL METHODS  
AND  
MATHEMATICAL METHOD ( CAS )**

**Trial Written Examination 1**

Reading time: 15 minutes  
Total writing time: 1 hour

**QUESTION AND ANSWER BOOK**

**Structure of book**

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

**Materials supplied**

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.  
A decimal approximation will not be accepted if an **exact** answer is required to a question.  
In questions where more than one mark is available, appropriate working must be shown.  
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

For the function  $f : R \setminus \{3\} \rightarrow R$ ,  $f(x) = \frac{2}{x-3} + 4$

a. Find the rule for the inverse function  $f^{-1}$ .

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1 mark

b. Find the domain of the inverse function  $f^{-1}$ .

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1 mark

**Question 2**

Let  $y = \frac{\tan(2x)}{2x}$ . Evaluate  $\frac{dy}{dx}$  when  $x = \frac{\pi}{8}$ .

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3 marks

**Question 3**

- a. State in words, giving scale factors, the transformations required to sketch the graph of  $y = 4 \cos\left(2\left(x - \frac{\pi}{3}\right)\right)$  from the graph of  $y = \cos(x)$ .

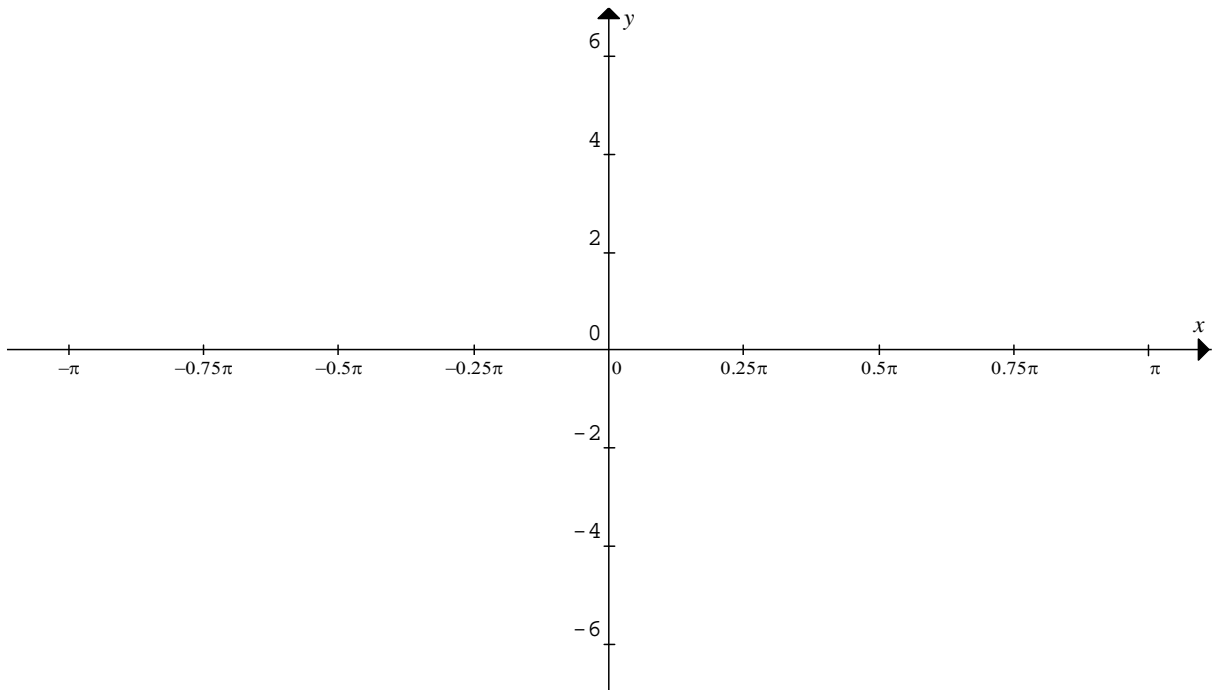
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2 marks

- b. Sketch the graph of the function  $f : [-\pi, \pi] \rightarrow R$ ,  $f(x) = 4 \cos\left(2\left(x - \frac{\pi}{3}\right)\right)$  on the set of axes below. Label axes intercepts with their coordinates. Label endpoints with their coordinates.



3 marks

**Question 4**

The probability density function of a continuous random variable  $X$  is given by

$$f(x) = \begin{cases} k(x-4)^2 & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- a.** Show that  $k = \frac{3}{26}$ .

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1 mark

- b.** Find  $\Pr(X < 2)$ .

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2 marks

**Question 5**

The weights of chocolate bars are normally distributed with a mean of 51 grams and a standard deviation of 4 grams. Let  $Z$  be the standard normal random variable.

Given that  $\Pr(Z < 0.25) = 0.6$ ,

- a.** Find the probability that a randomly selected chocolate bar has a weight of less than 50 grams. Give your answer correct to one decimal place.

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1 mark

- b.** Find the probability that a randomly selected chocolate bar has a weight of between 51 and 52 grams. Give your answer correct to one decimal place.

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1 mark

- c.** One night Lilly eats three such chocolate bars. Find the probability that at least one of the chocolate bars has a weight of more than 50 grams. Give your answer correct to three decimal places.

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2 marks

**Question 6**

Let  $g(x) = \cos(2x)$  and  $f(g(x)) = \log_e(\cos(2x))$

a. Write down the rule for the function  $f(x)$ .

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1 mark

b. Find the derivative of  $f(g(x))$ .

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1 mark

c. **Hence**, find an anti-derivative of  $\tan(2x)$ .

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1 mark

**Question 7**

The line  $3y + x + k = 0$  is a normal to the curve  $y = x^5 + bx$  at the point  $x = -1$ .

Find the values of  $b$  and  $k$ .

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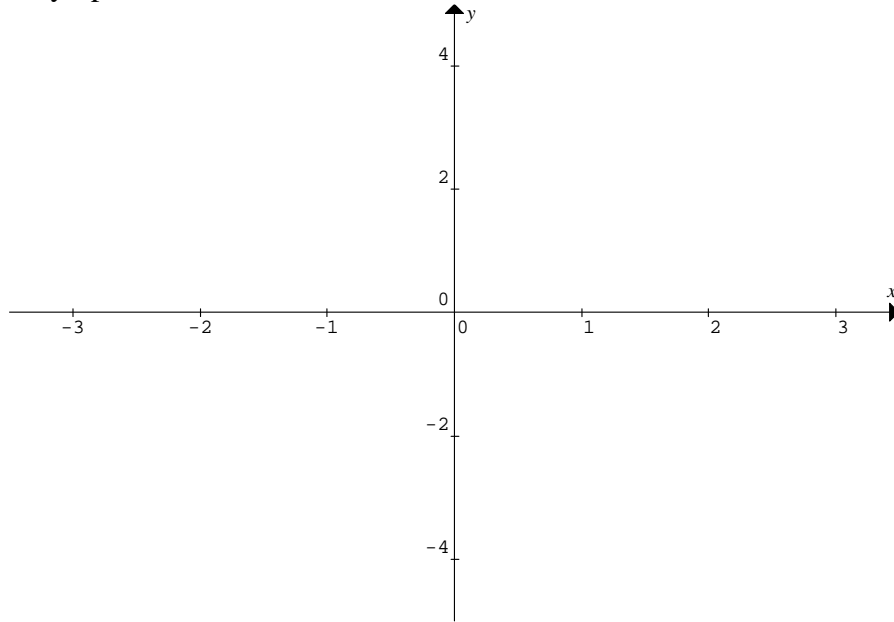
4 marks



**Question 8**

a. Sketch the graph of the function  $f : \mathbb{R} \setminus \left\{ \frac{3}{2} \right\} \rightarrow \mathbb{R}$ ,  $f(x) = \frac{-6}{|2x-3|}$

on the axes below, stating the coordinates of any axial intercepts and the equations of all asymptotes.



2 marks

b. Find the area bounded by the graph of  $y = \frac{-6}{|2x-3|}$ , the coordinate axes and the line  $x = 1$ .

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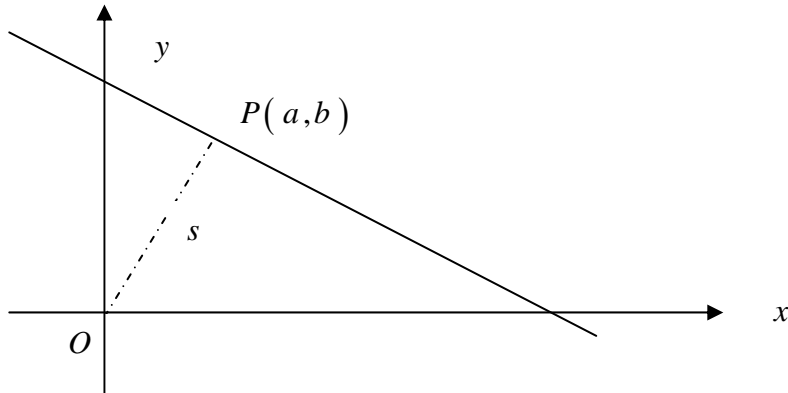
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2 marks

**Question 9**

The point  $P(a, b)$  where  $a$  and  $b$  are positive real numbers lies on the line  $y + 2x - 5 = 0$ .



- a. Let  $s$  be the distance from the origin  $O$  to the point  $P$ .  
Find the distance  $s$  in terms of  $a$ .

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1 mark

- b. Find, using **calculus**, the minimum value of  $s$  and the value of  $a$  for which this occurs.

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3 marks

**Question 10**

Each night at dinner Bill has either a glass of wine or a glass of beer. If he has beer one night, the probability he has a beer the next night is 0.6. If he has wine one night, the probability he has a beer the next night is 0.2. Suppose he has a beer on Thursday night. What is the probability that he has a beer on the following Saturday night?

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3 marks

**Question 11**

- a. Show that the graph of the function  $f : (0, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = 2x + \frac{8}{x^2}$  has a single stationary point and find its coordinates.

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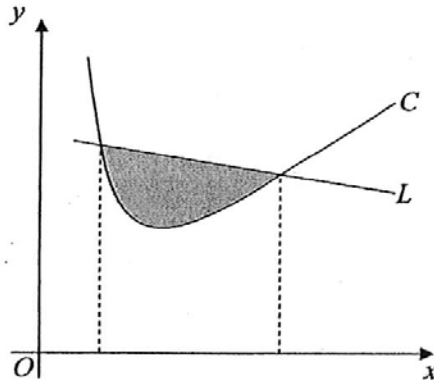
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2 marks

- b. The curve  $C$  with the equation  $y = 2x + \frac{8}{x^2}$  for  $x > 0$  is sketched below.  
 The line  $L$  with the equation  $x + 2y = 21$  intersects the curve  $C$  at the points  $(a, 10)$  and  $(b, 8.5)$ . If the area of the shaded region bounded by the curve  $C$  and the line  $L$  can be expressed as the definite integral

$$\int_a^b \left( p + qx + \frac{r}{x^2} \right) dx, \text{ find the values of } a, b, p, q \text{ and } r.$$

( There is no need to evaluate the definite integral or find the area. )




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3 marks

**END OF QUESTION AND ANSWER BOOKLET**

**EXTRA WORKING SPACE**

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**END OF EXAMINATION**

# **MATHEMATICAL METHODS**

## **Written examination 1**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods and CAS Formulas

### Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc \sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

### Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(e^{ax}) = ae^{ax}$ $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$ $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$ $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$ $\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$ $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ $\int \frac{1}{x} dx = \log_e x  + c$ $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$ $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
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product rule: 
$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule: 
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Chain rule: 
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

approximation: 
$$f(x+h) \approx f(x) + h f'(x)$$

### Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Mean: 
$$\mu = E(X)$$

variance: 
$$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = E(X)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$